Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

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In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

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of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

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The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

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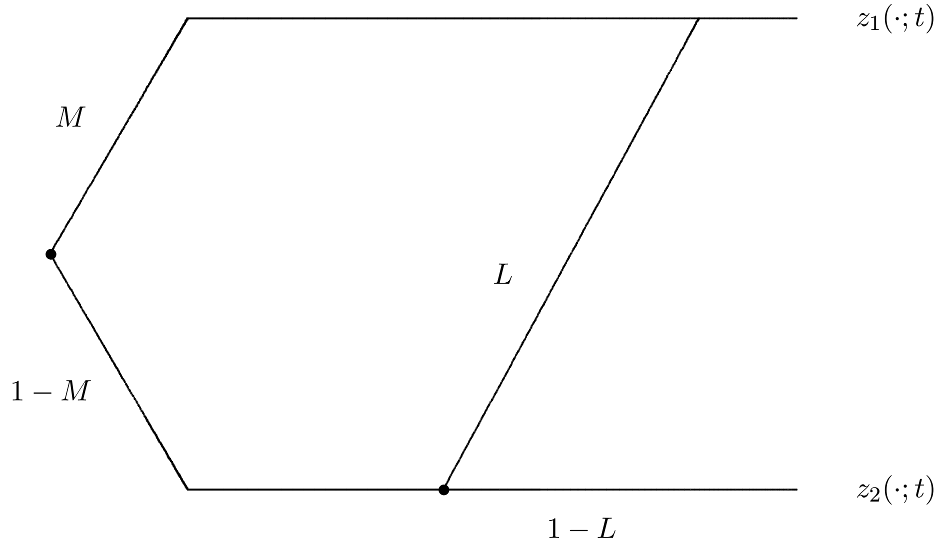

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The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

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Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

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The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

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1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

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Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

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Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

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2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

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In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

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for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

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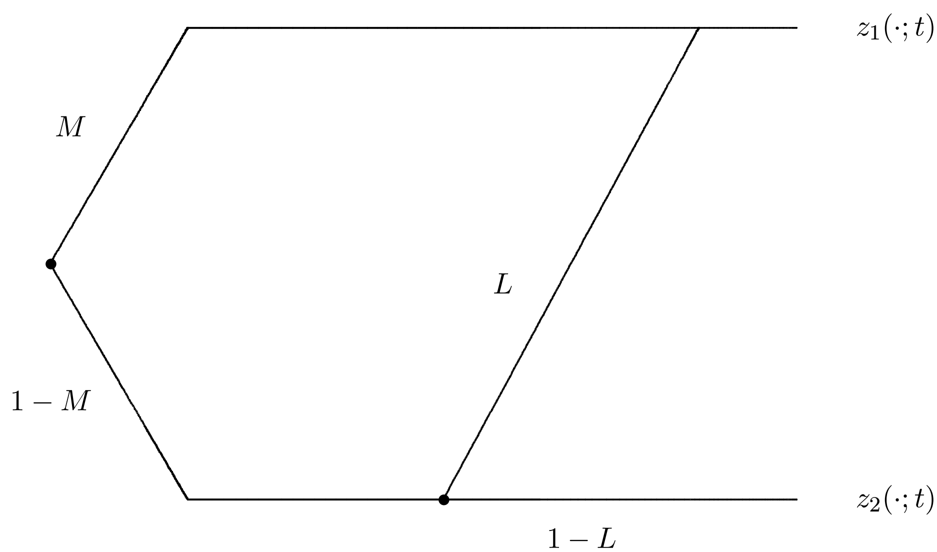

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where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

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[6]For a definition of capacities see Appendix A, Definition 2.
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of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

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Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

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outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

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Payoff in	Period 0	Period 1	Period 2
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Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

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We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

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tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

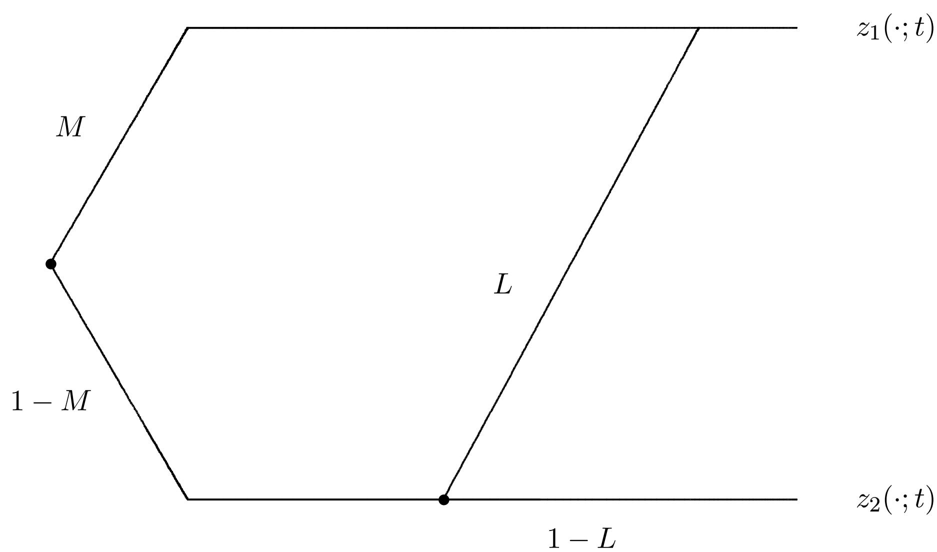

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

First Version: November 1997
This Version: October 2003

Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

[1] We would like to thank seminar participants at the Norwegian School of Management (Sandvika), the University of Oslo, the University of Birmingham, the Bundesbank (Frankfurt a.M.), the Free University in Berlin, the University of Frankfurt a.M., Keele University, University of Hohenheim and Chemnitz University of Technology for helpful comments and suggestions.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

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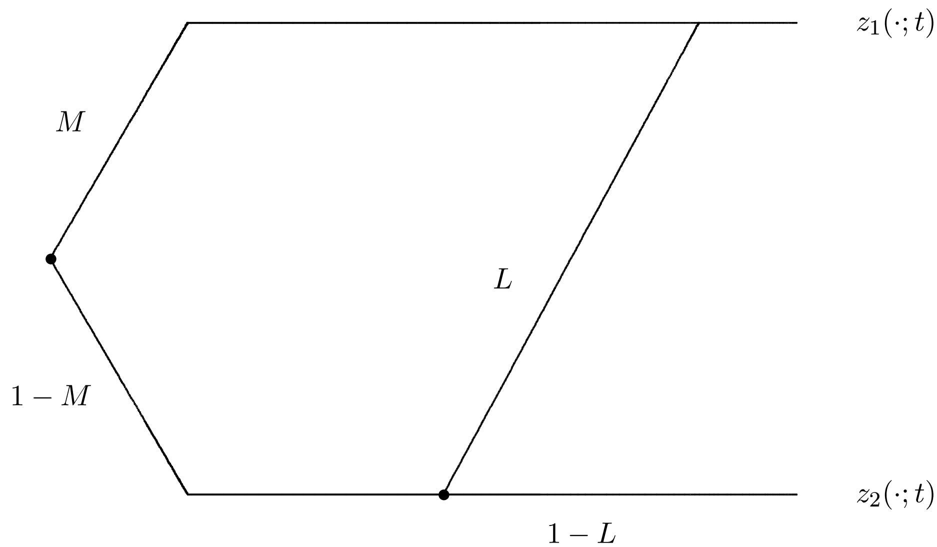

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Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

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Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

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Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau=\pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

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$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

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Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

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Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

First Version: November 1997
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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

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1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

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events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.
[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0,1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

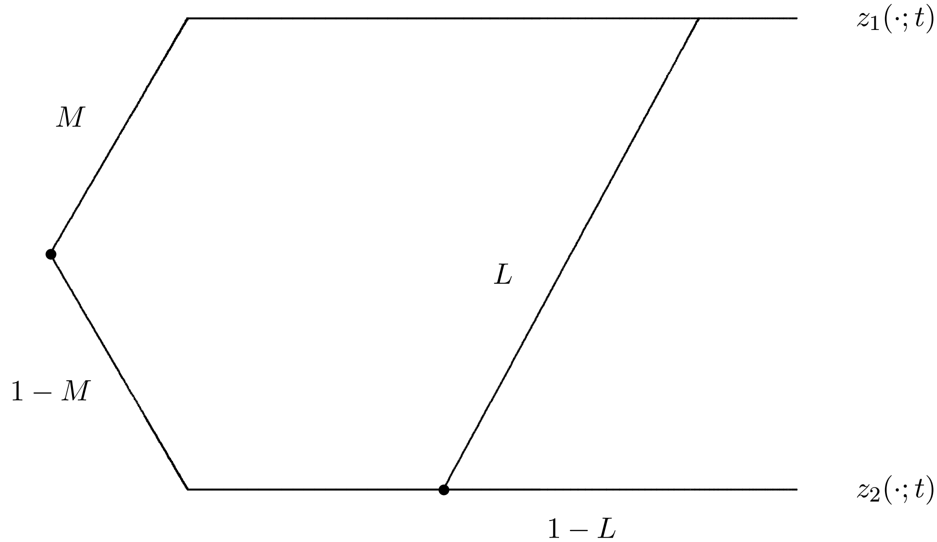

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{array}{rll} z_1(M,L) := & M + \alpha_1 \cdot L \cdot (1-M) & \text{and} \\ z_2(M,L) := & \alpha_2 \cdot (1-L) \cdot (1-M). & \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) :=\ & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ =\ & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

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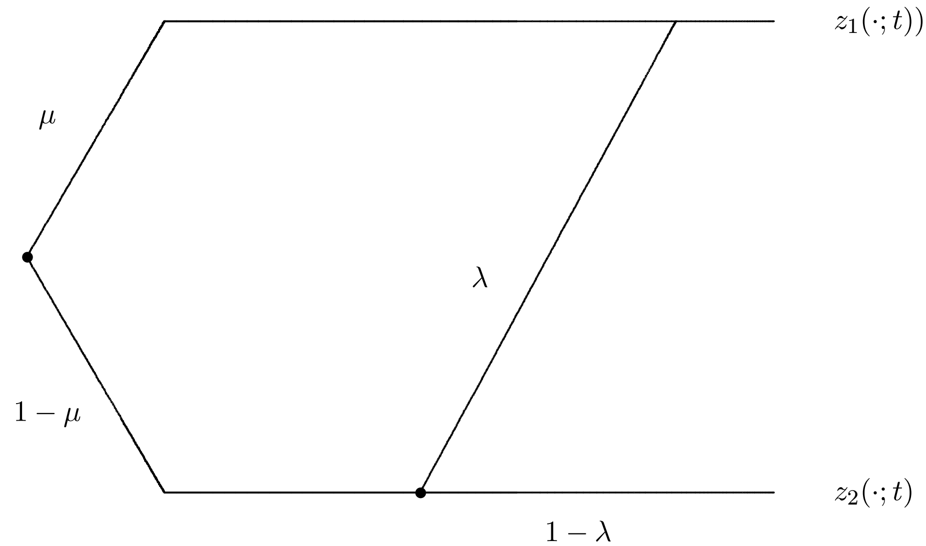

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$U(z_h, z_\ell; \tau) := \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell)$$

$$:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}].$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

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In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}_+^2 \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

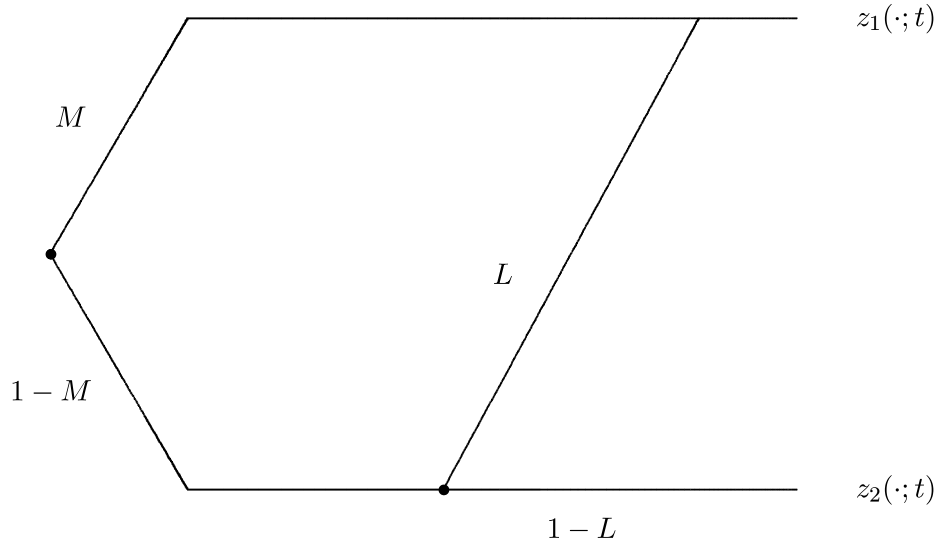

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & \qquad + (1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M)\cdot \alpha_2 \geq \beta_\ell \cdot M + (1-M)\cdot \alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\, , \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

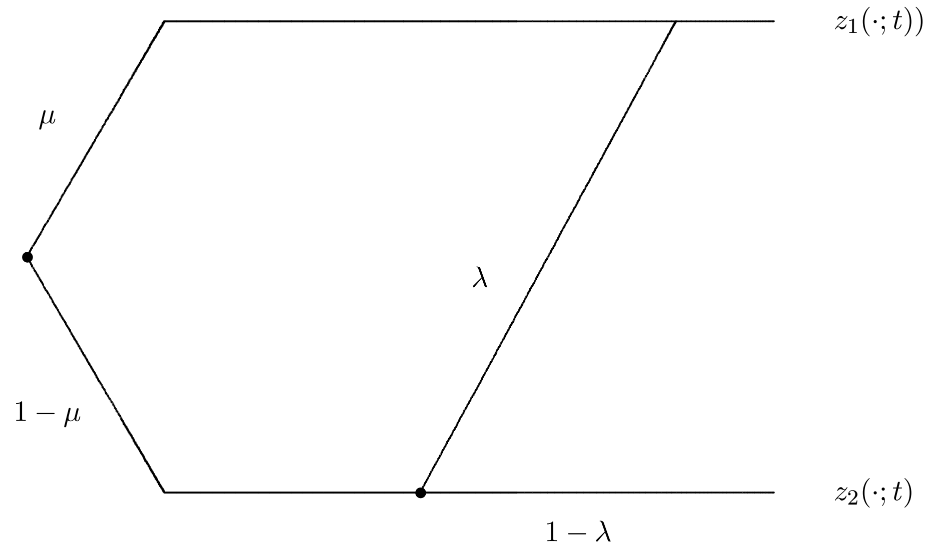

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$
\begin{array}{lllll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}
$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

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and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

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In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

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In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S. *The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

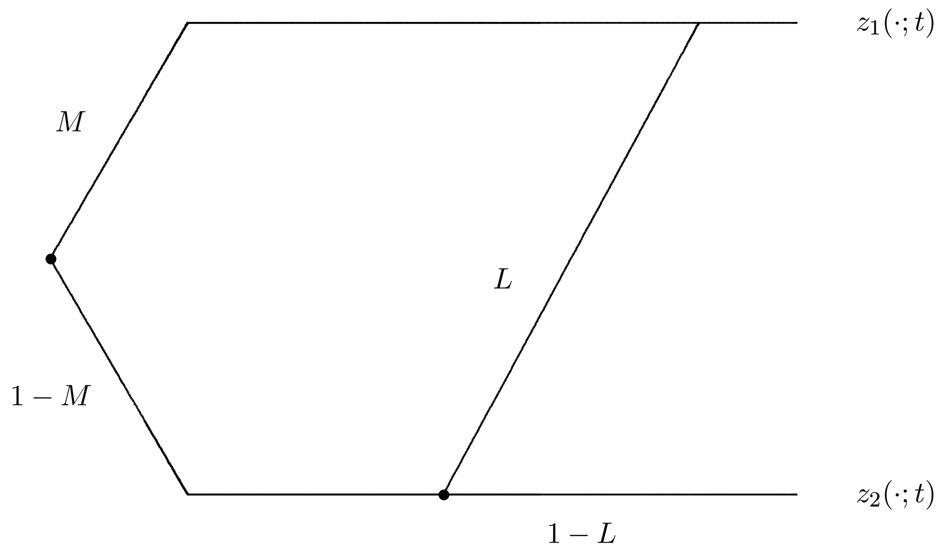

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) :=& \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) :=& \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

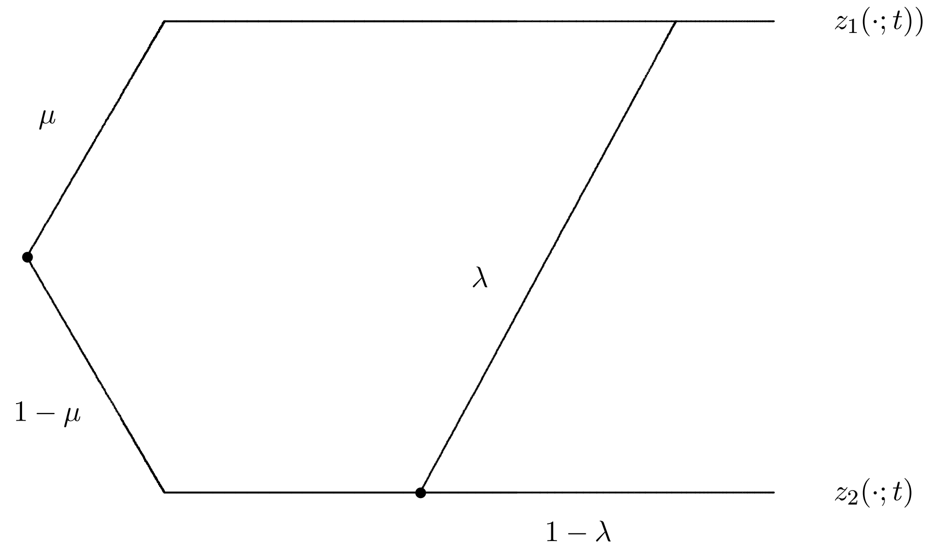

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := & \quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t, \tau) \in \{h, \ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

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Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

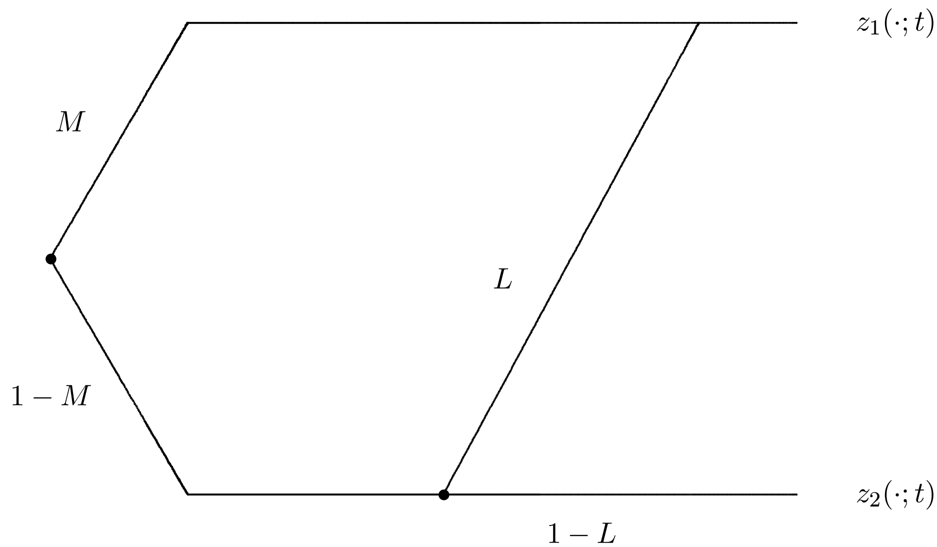

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{array}{rll} z_1(M,L) := & M + \alpha_1 \cdot L \cdot (1-M) & \text{and} \\ z_2(M,L) := & \alpha_2 \cdot (1-L) \cdot (1-M). & \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\, d\tau \\ & \qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & \qquad + (1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

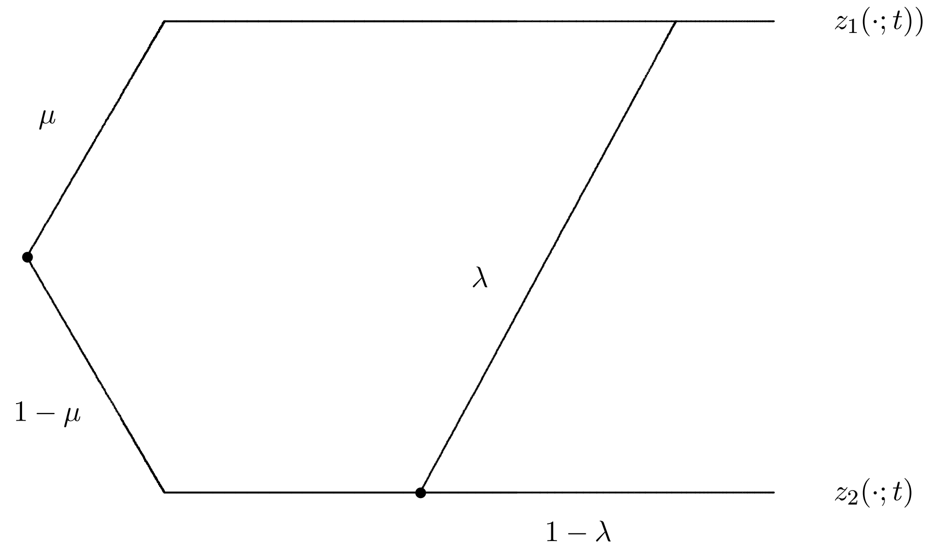

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := &\quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := &\quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \multicolumn{3}{l}{\lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0.} & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

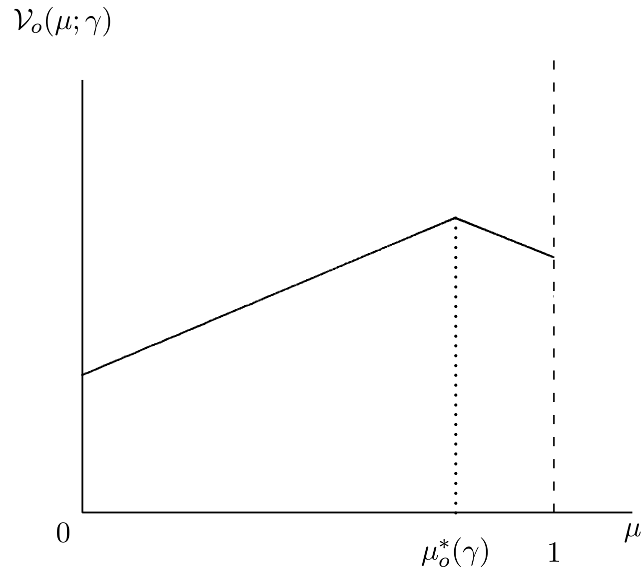

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment***,* which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

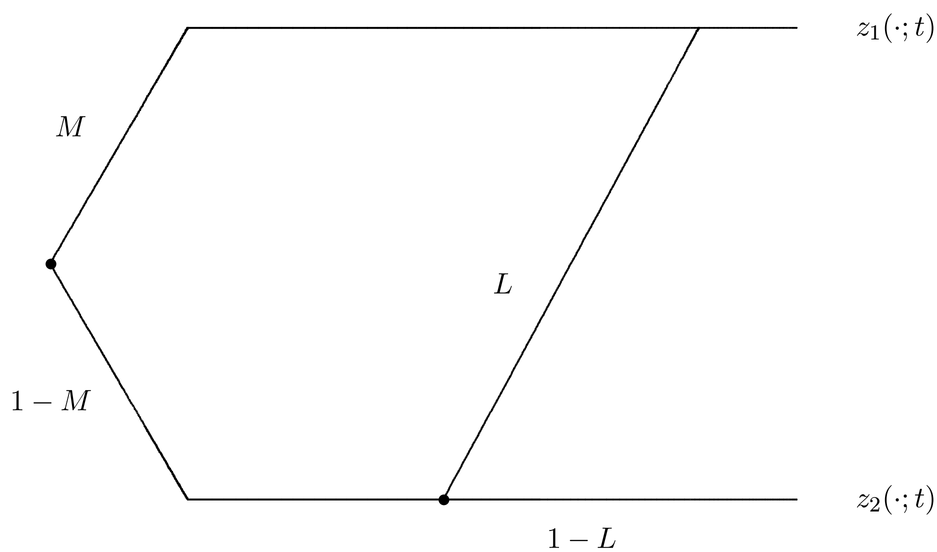

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) &:= \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) &:= \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M)\cdot \alpha_2 \geq \beta_\ell \cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

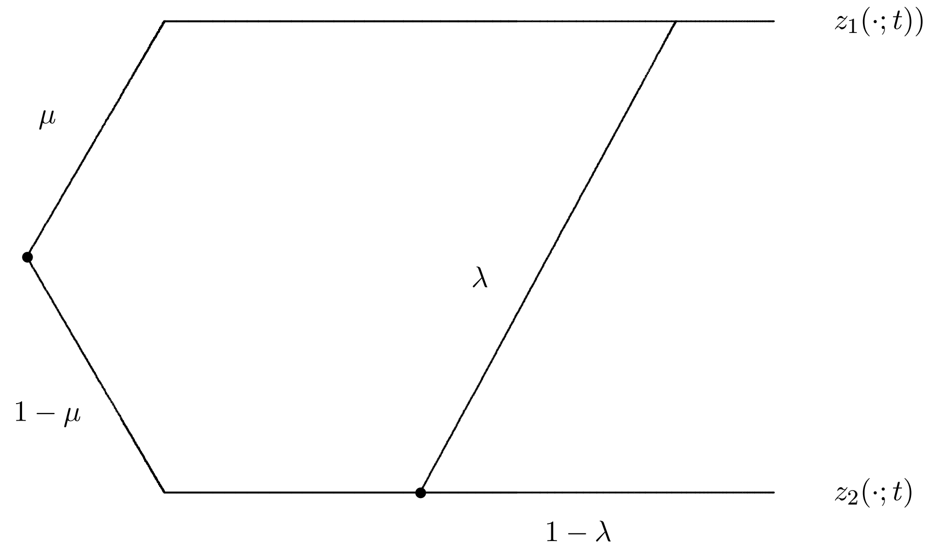

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$
\begin{aligned}
U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\
&:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}].
\end{aligned}
$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$
\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}
$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

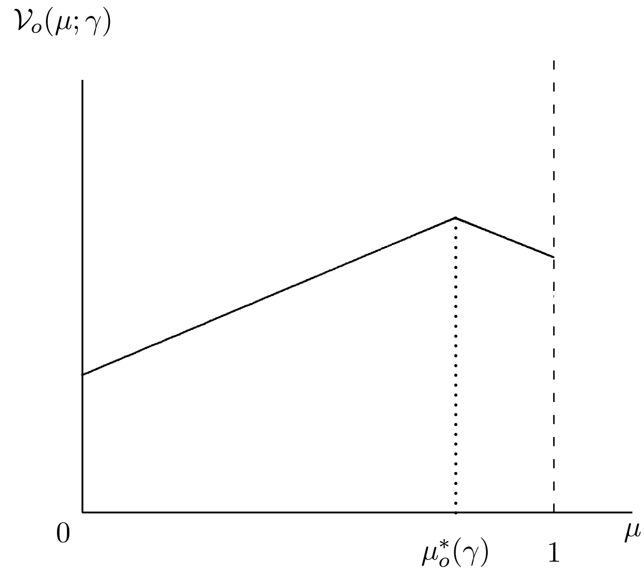

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

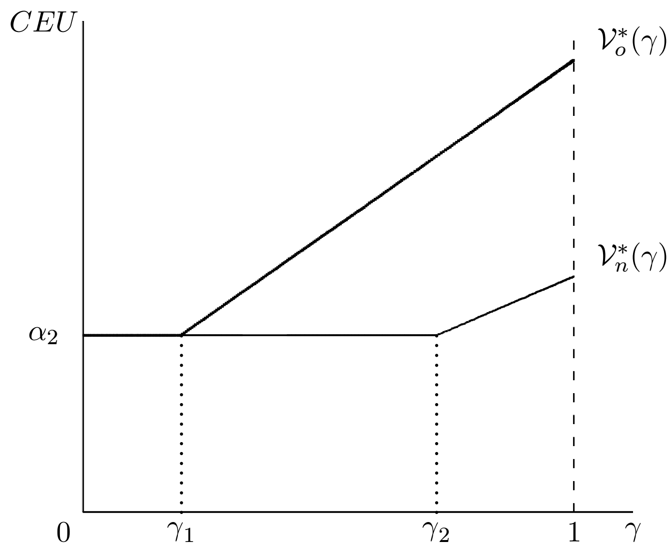

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \; d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \; ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

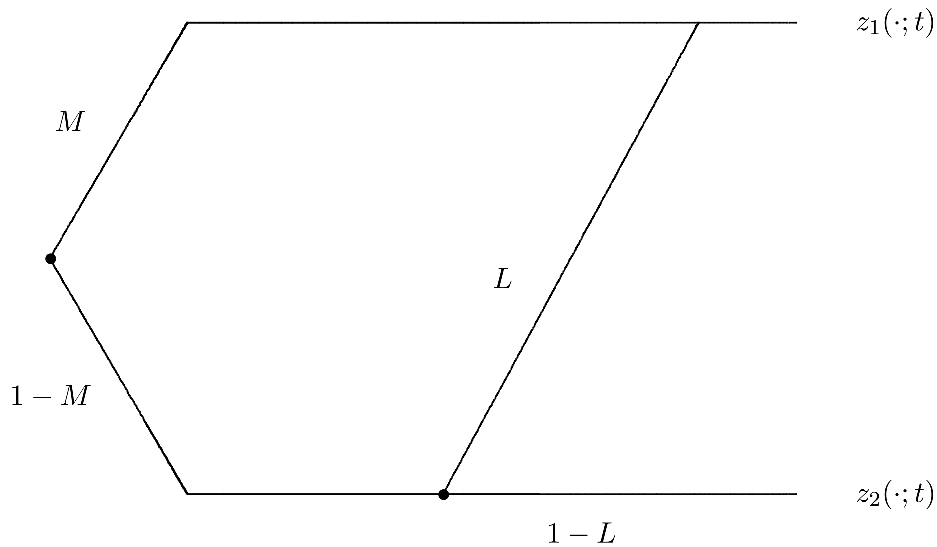

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\, , \, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

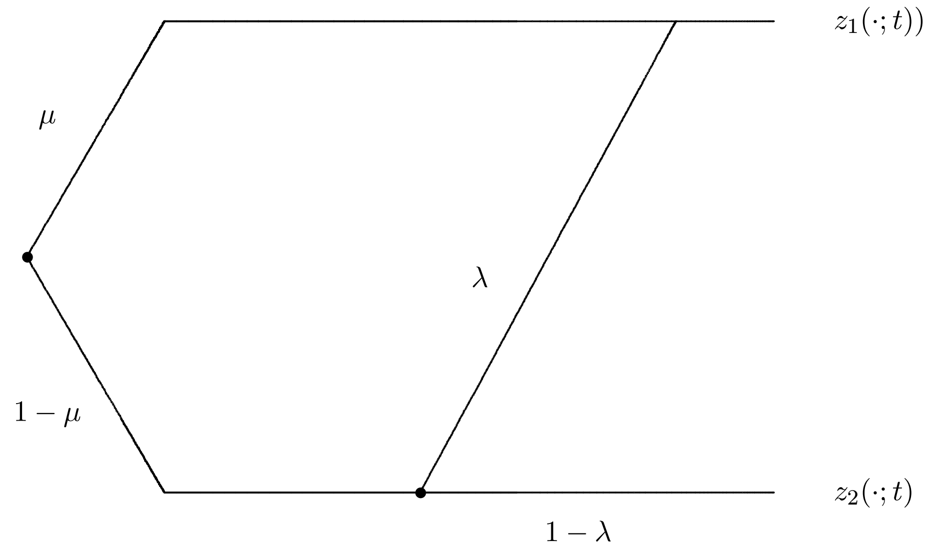

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$U(z_h, z_\ell; \tau) := \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell)$$
$$:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}].$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$
\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}
$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

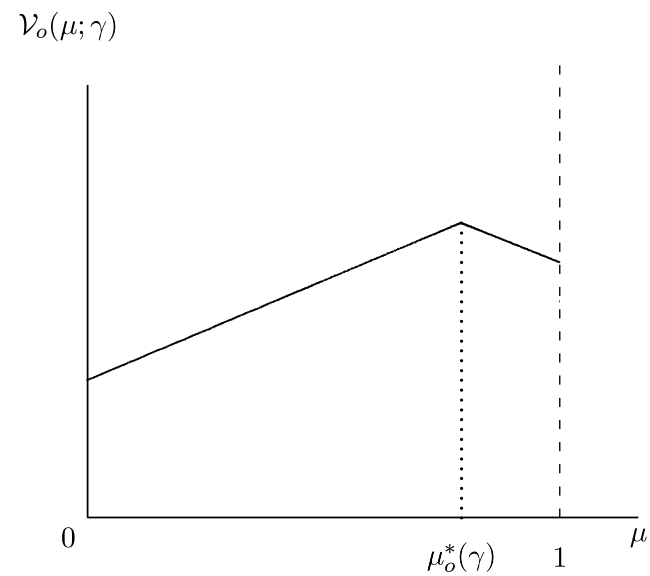

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

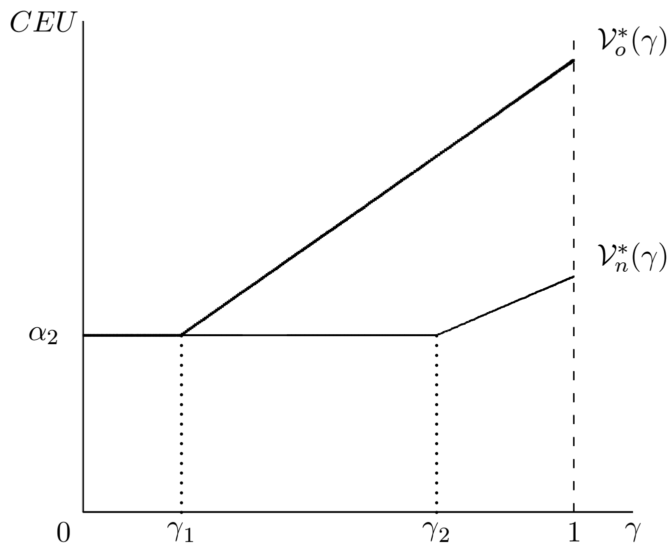

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

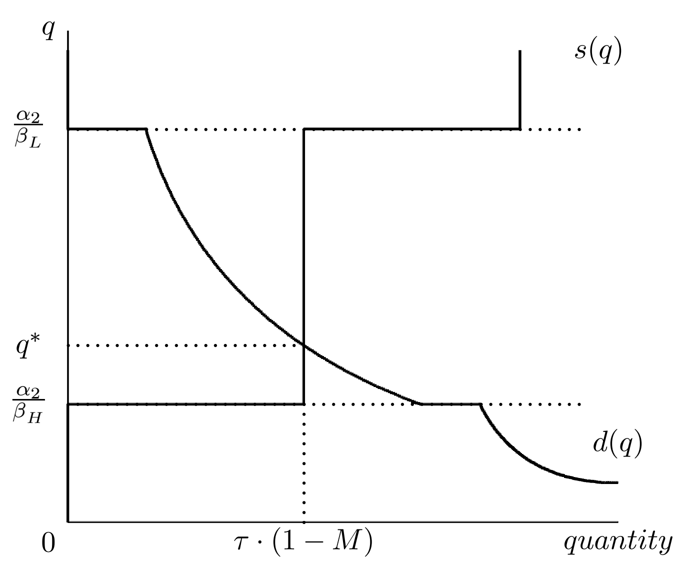

Figure 5: Market for securities

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$, *where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S. *The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

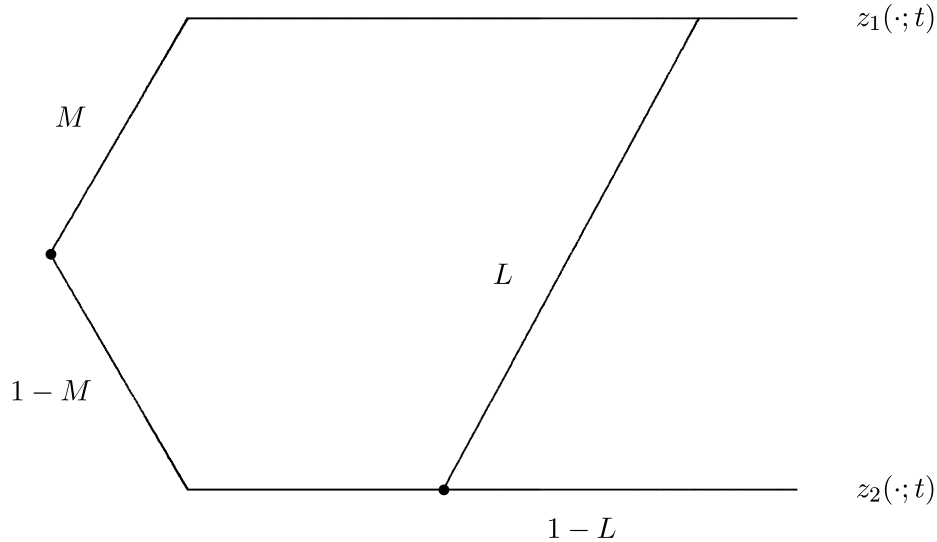

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{array}{ll} z_1(M, L) := & M + \alpha_1 \cdot L \cdot (1 - M) \quad \text{and} \\ z_2(M, L) := & \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

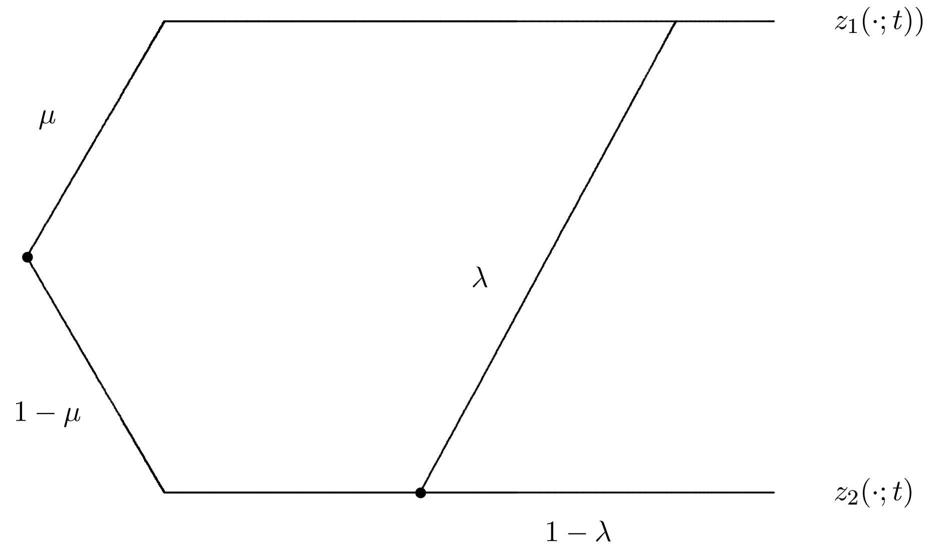

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := & \quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1-\pi) \cdot v_o(\pi, \mu; \ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

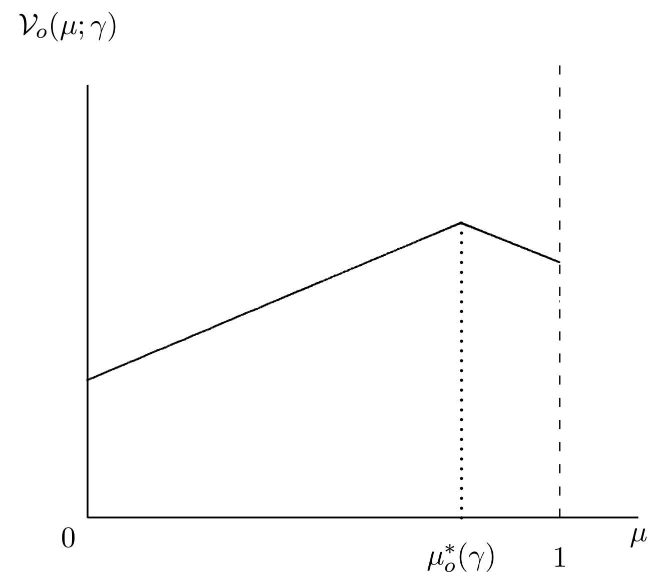

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

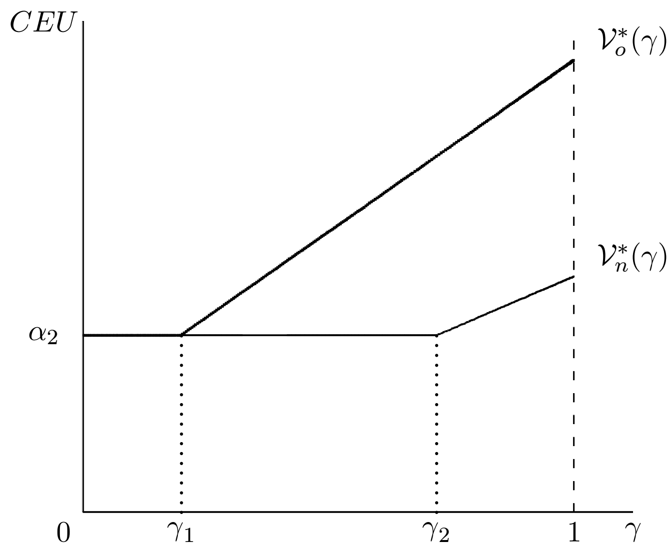

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

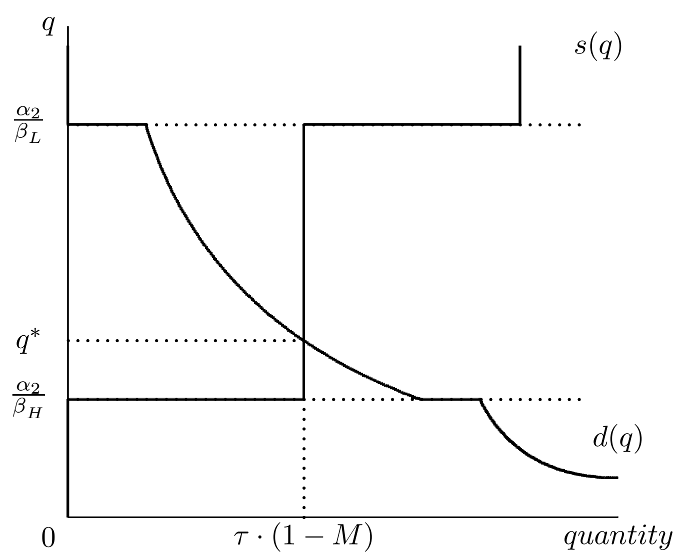

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

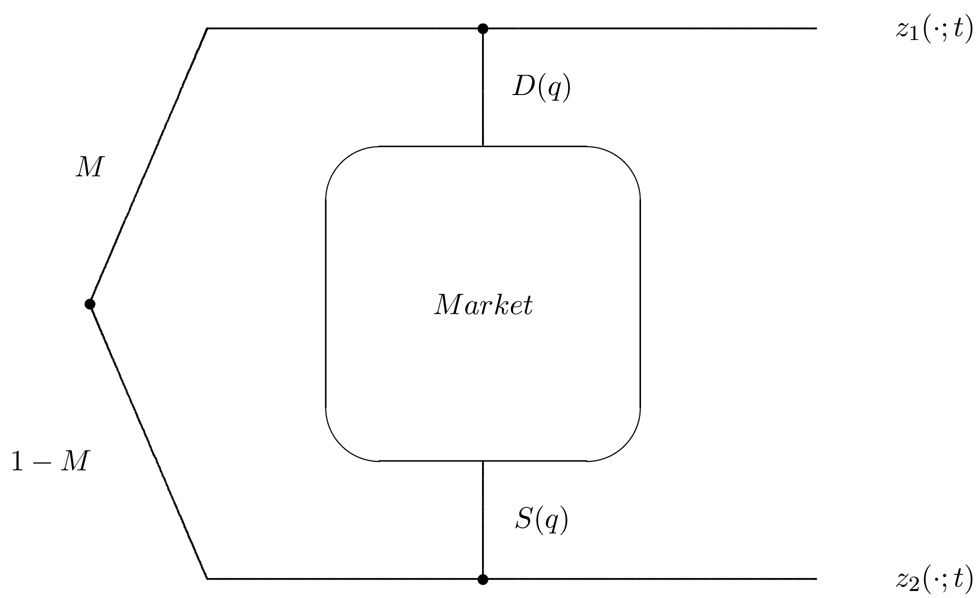

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

First Version: November 1997
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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

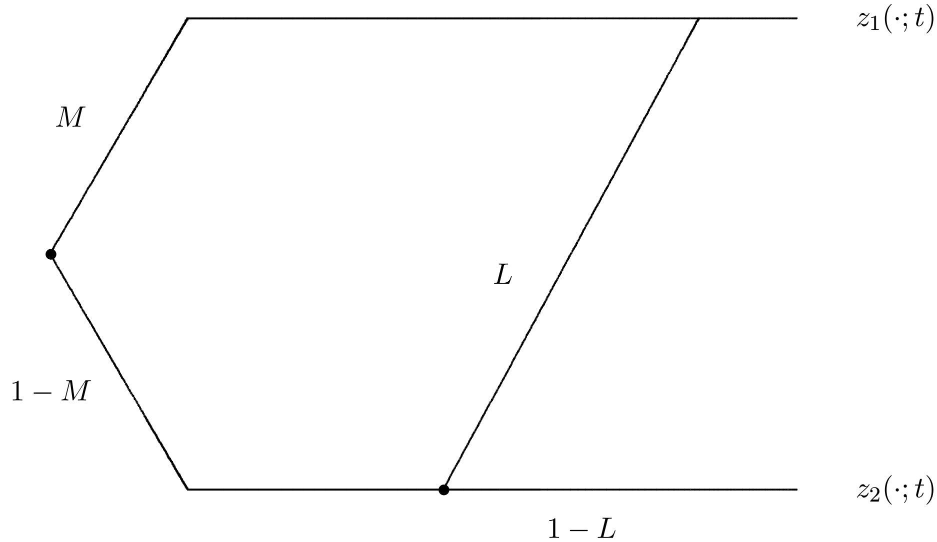

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=& \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) :=& \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \;& \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\; d\tau \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \;& \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & + (1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13]Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

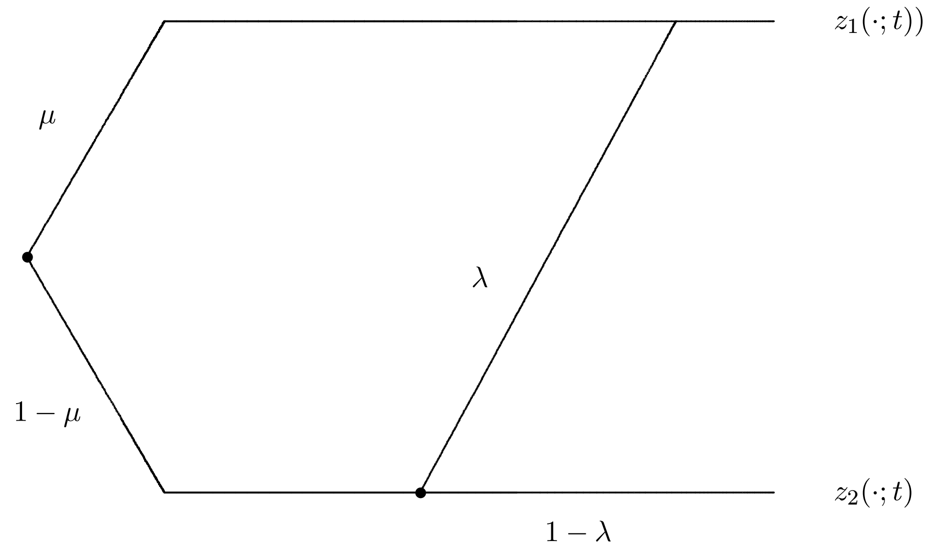

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \text{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \text{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \text{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \text{F}_2 \\
& \lambda \geq 0, \; z_{1h} \geq 0, \; z_{1\ell} \geq 0, \; z_{2h} \geq 0, \; z_{2\ell} \geq 0. & & & \text{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := & \quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

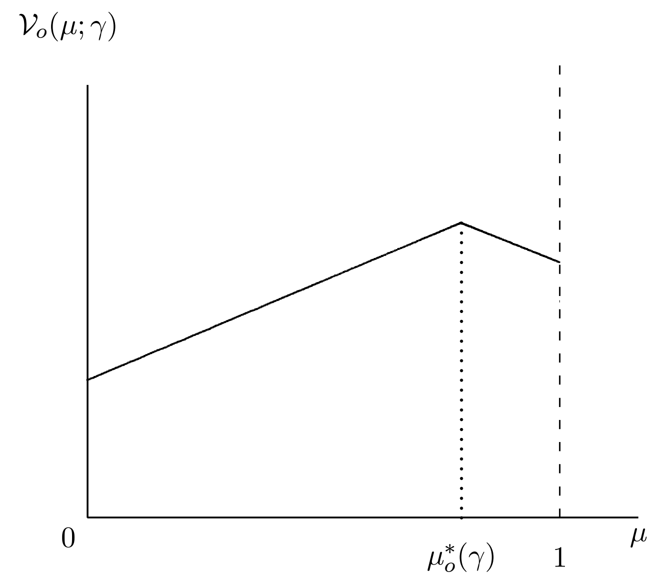

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

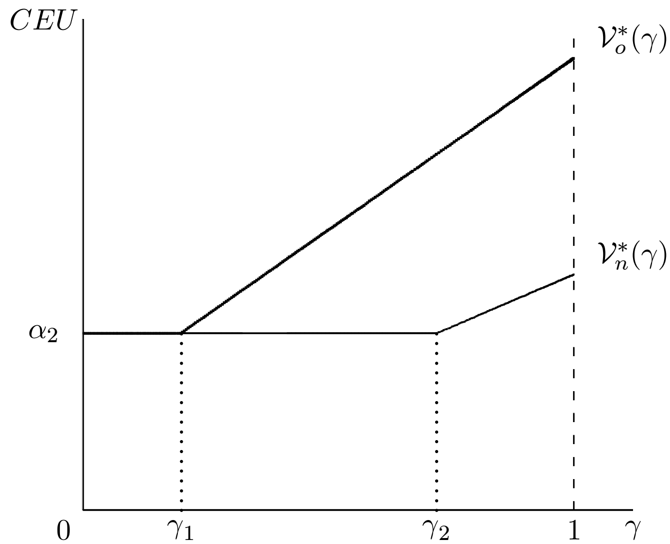

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

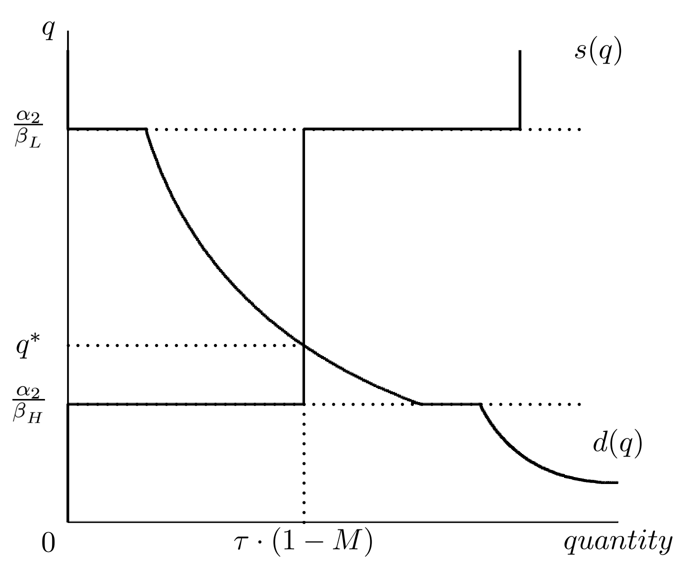

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

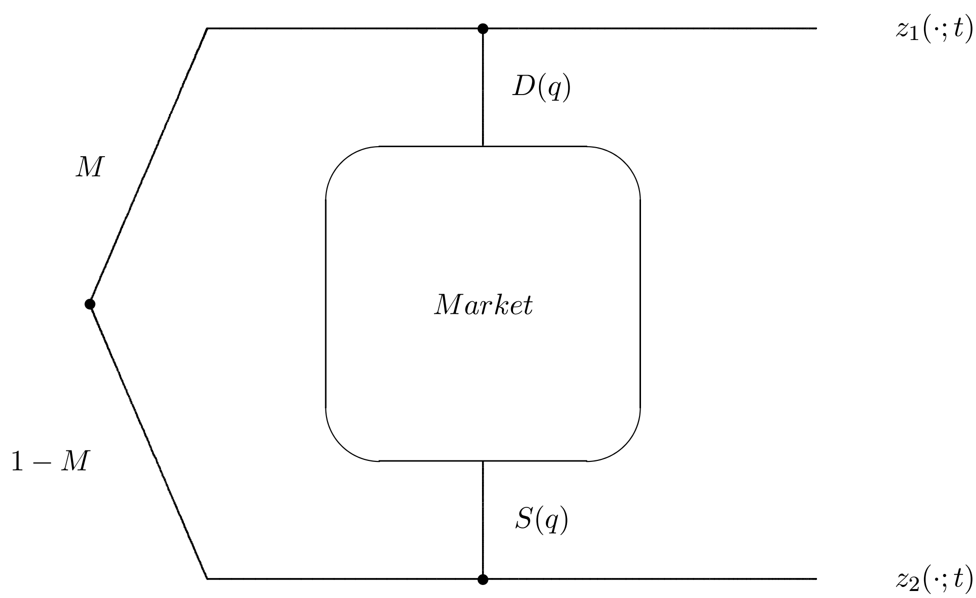

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\mathcal{V}_a(M,\mu;\gamma) := \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ +(1-\gamma)\cdot\min_{(t,\tau)\in\{h,\ell\}\times[0,1]}[v_a(M;\tau,\mu;t)].$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

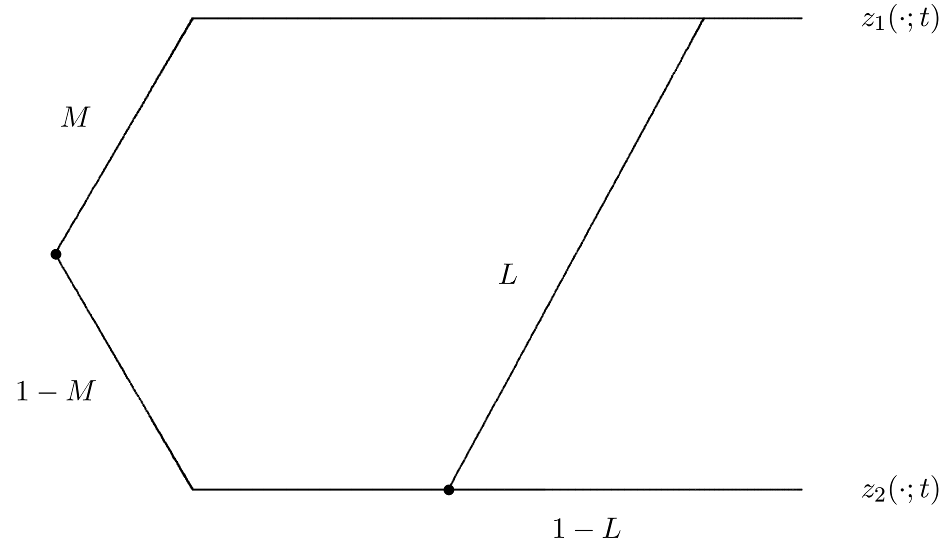

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

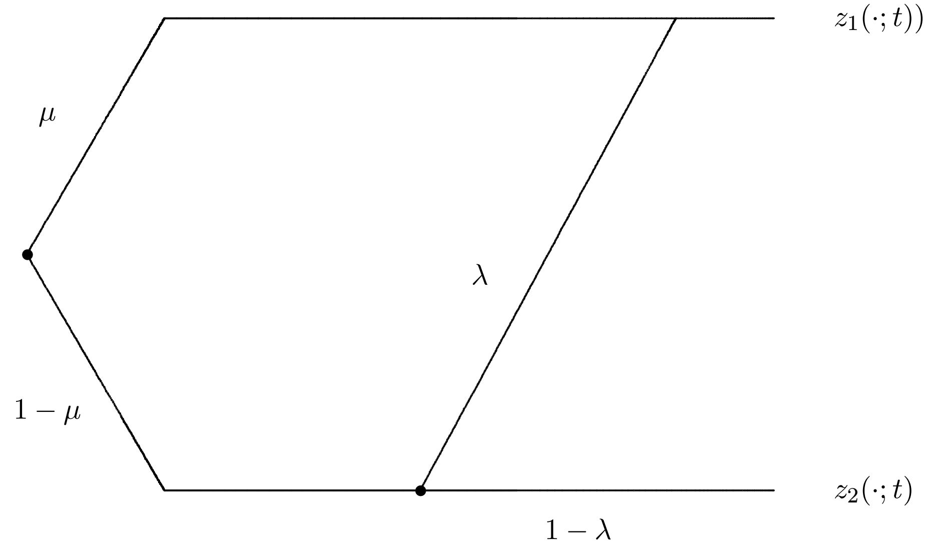

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \multicolumn{3}{l}{\lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0.} & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := \quad & u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = \quad & \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h, \ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

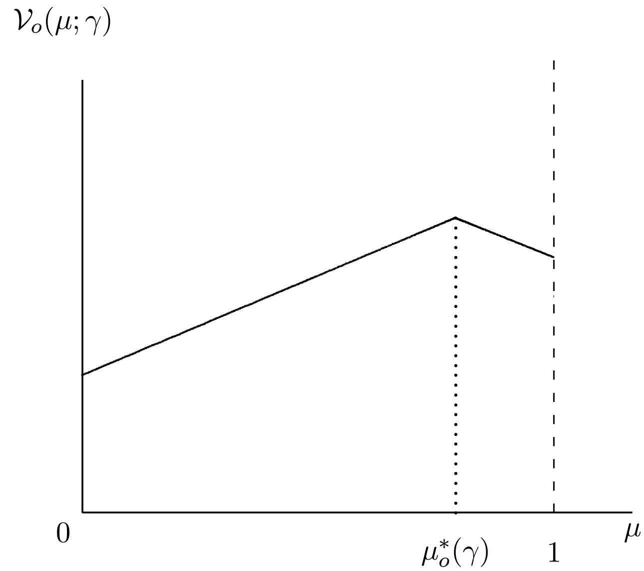

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

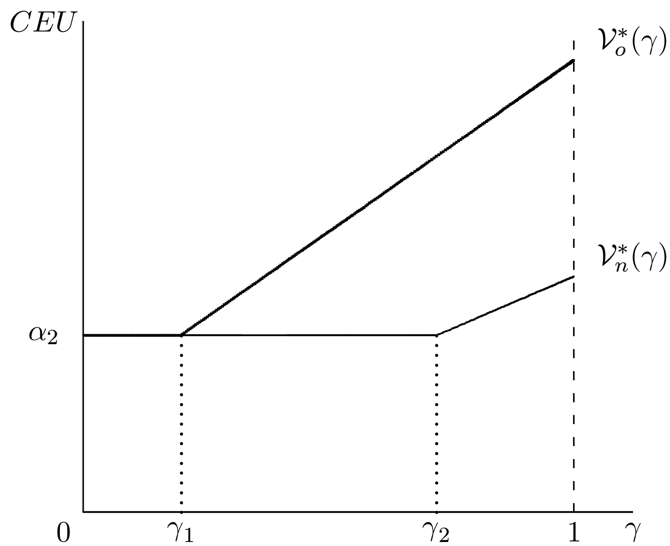

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

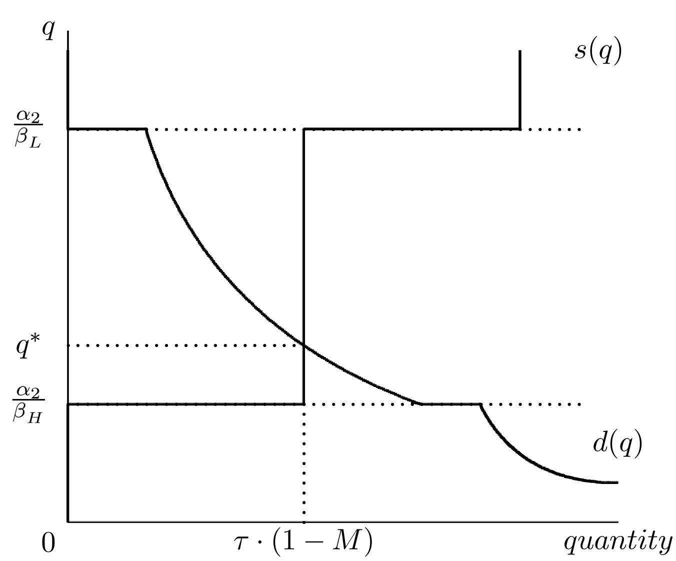

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

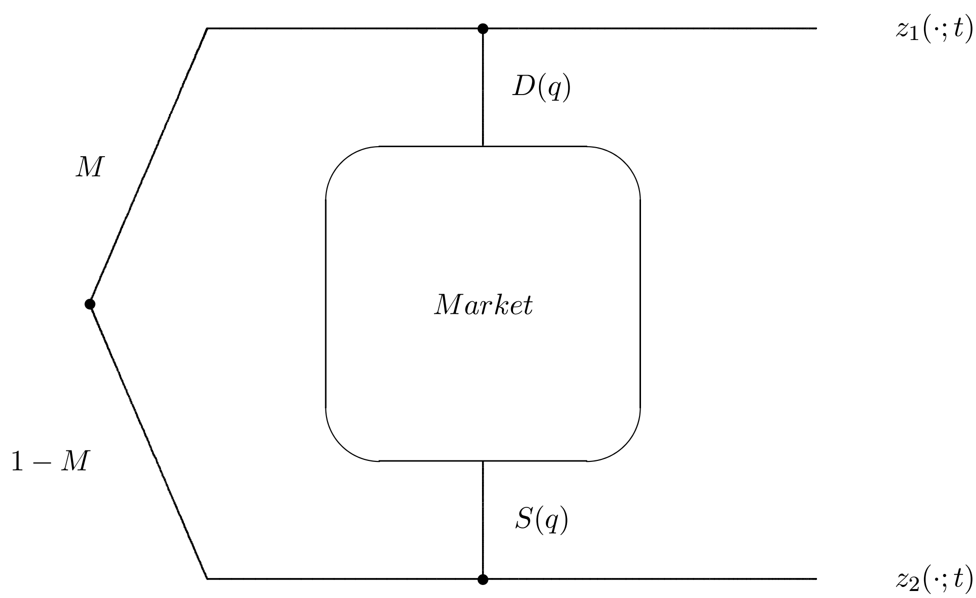

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] \,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

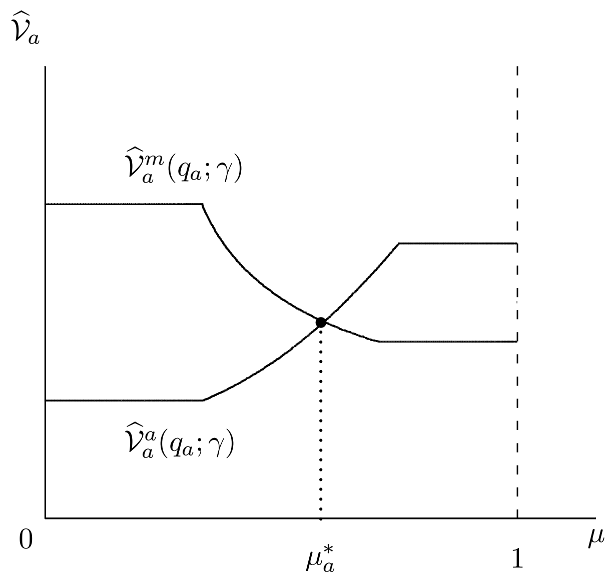

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f\, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s)\, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}_+^2 \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

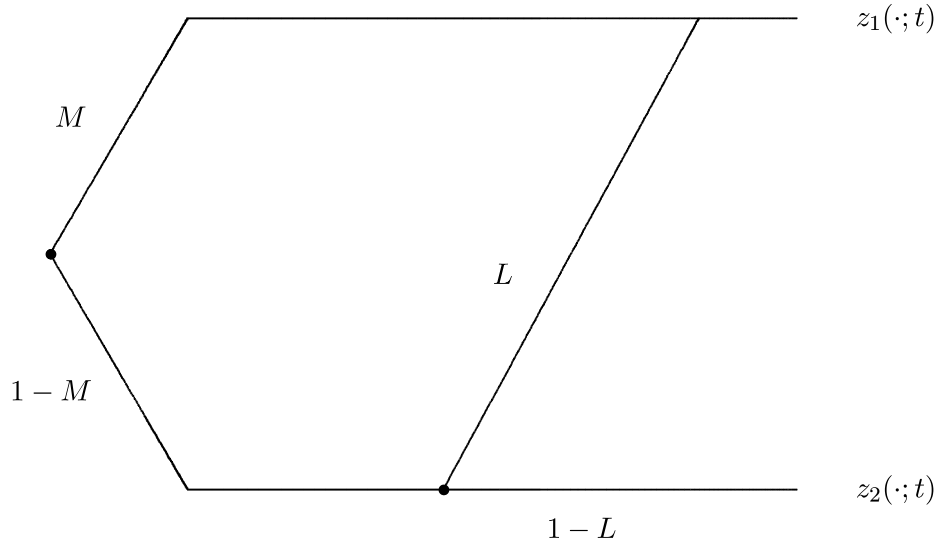

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{array}{rl} z_1(M,L) := & M + \alpha_1 \cdot L \cdot (1-M) \quad \text{and} \\ z_2(M,L) := & \alpha_2 \cdot (1-L) \cdot (1-M). \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) :=\ & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ =\ & \gamma \cdot [\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell \cdot M + (1-M)\cdot \alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

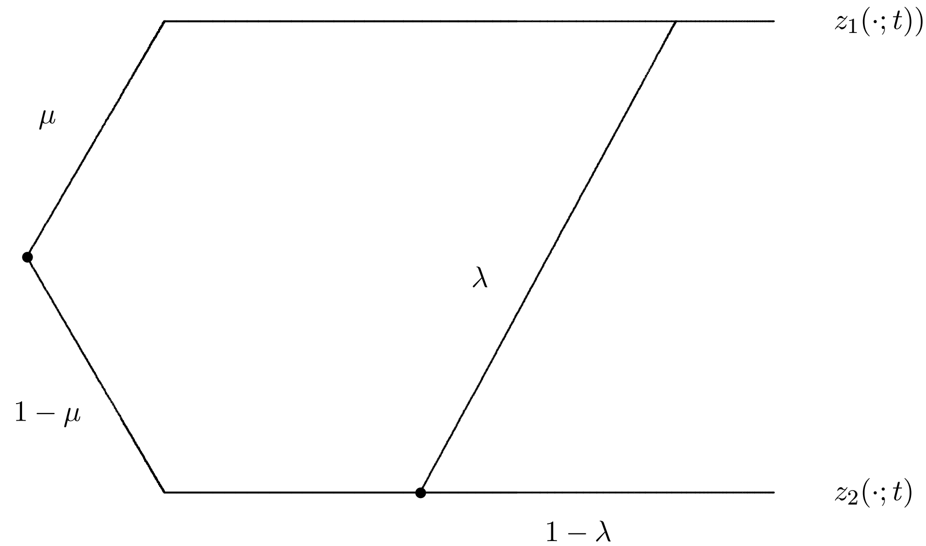

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$U(z_h, z_\ell; \tau) := \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell)$$
$$:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}].$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := \quad & u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = \quad & \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

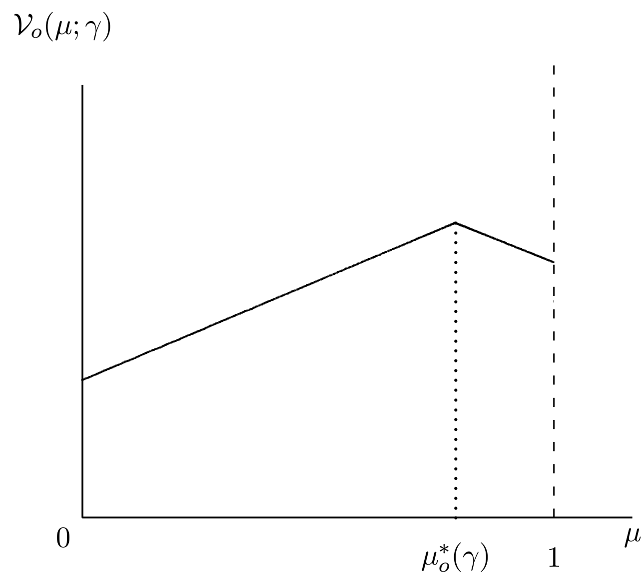

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

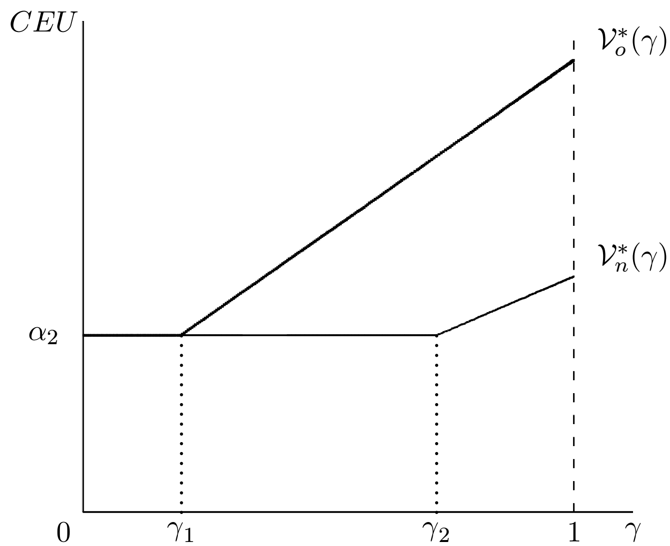

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

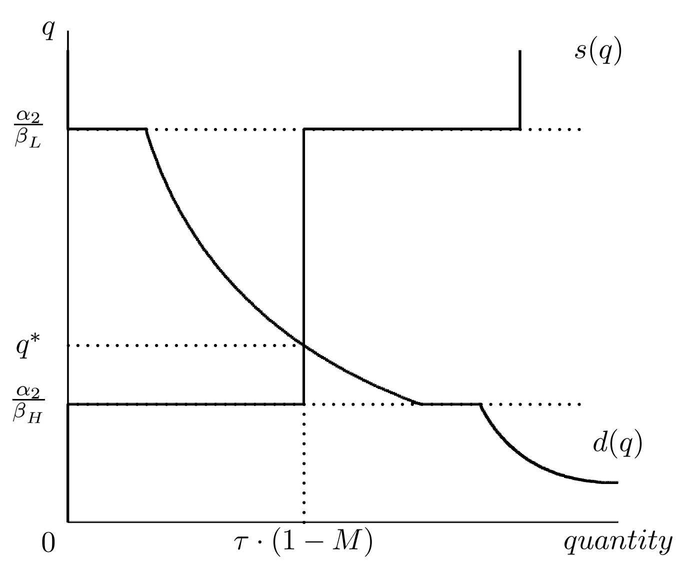

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

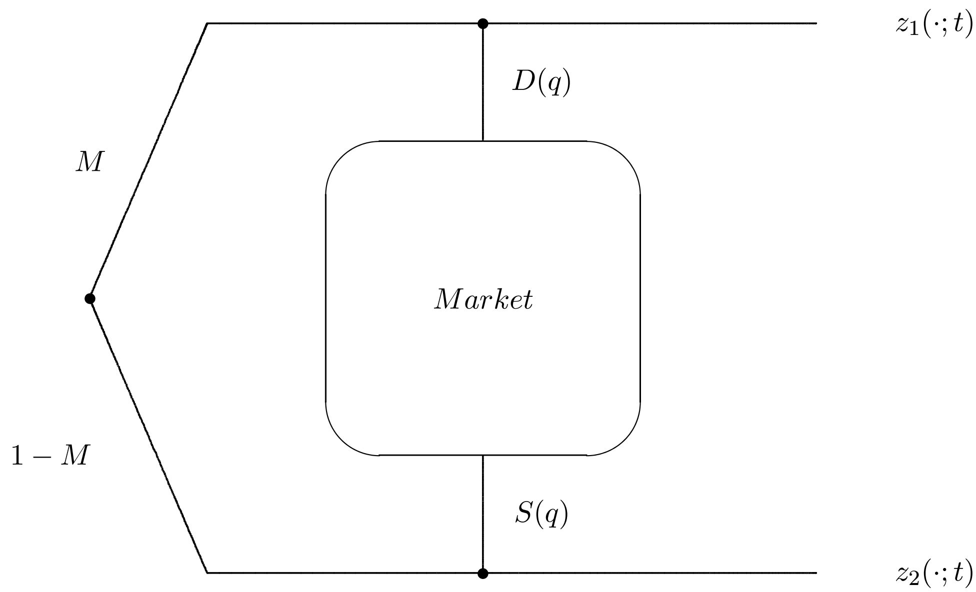

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M) \cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

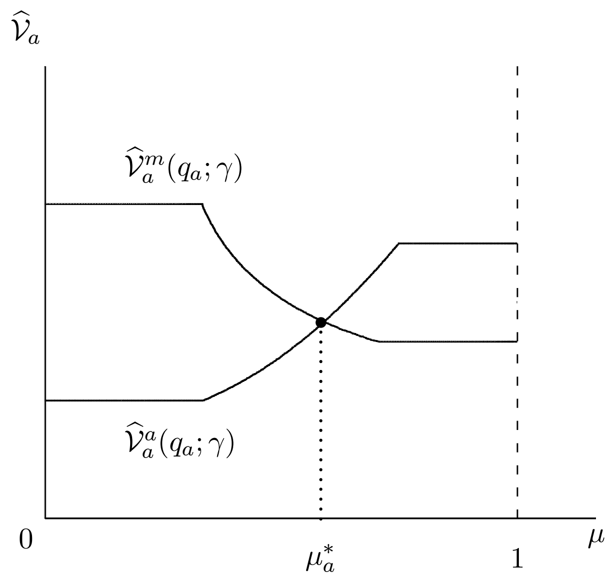

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

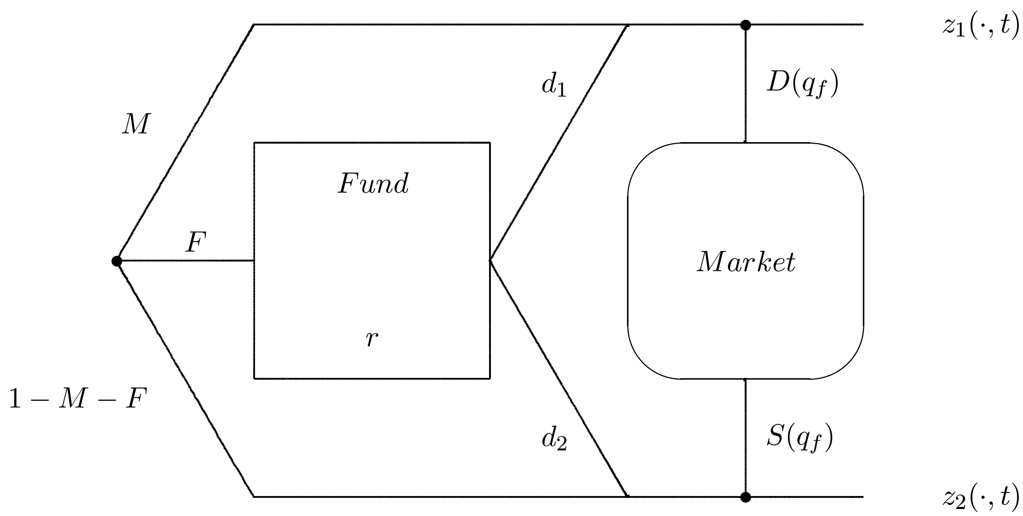

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$*, where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

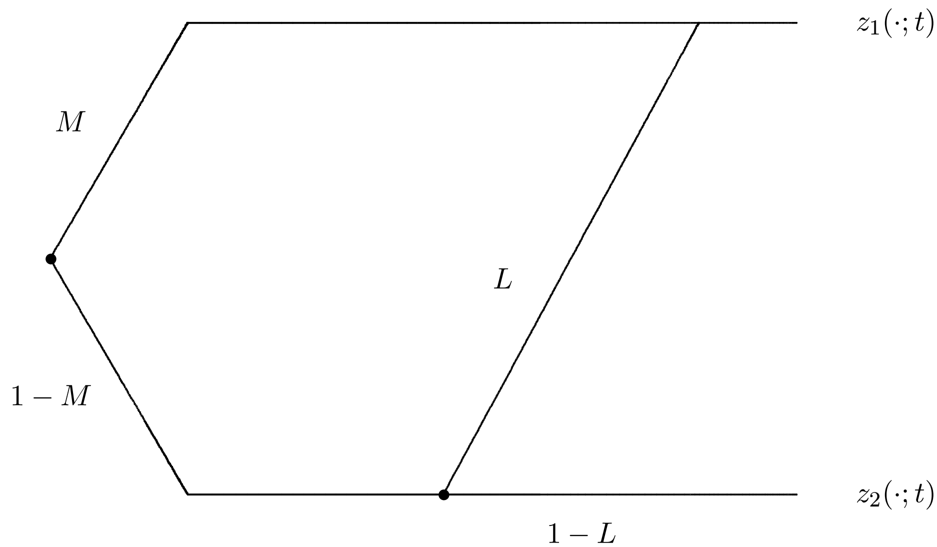

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=& \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) :=& \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1-M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau=\pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13]Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

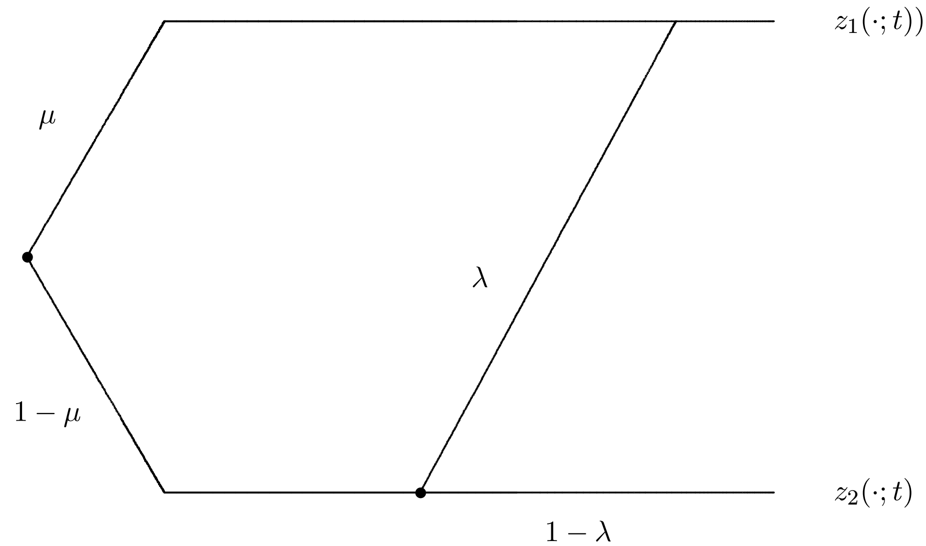

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := &\quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := &\quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

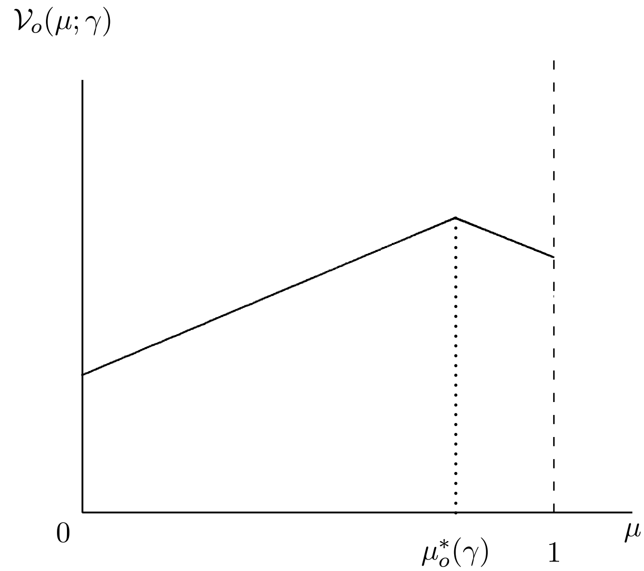

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

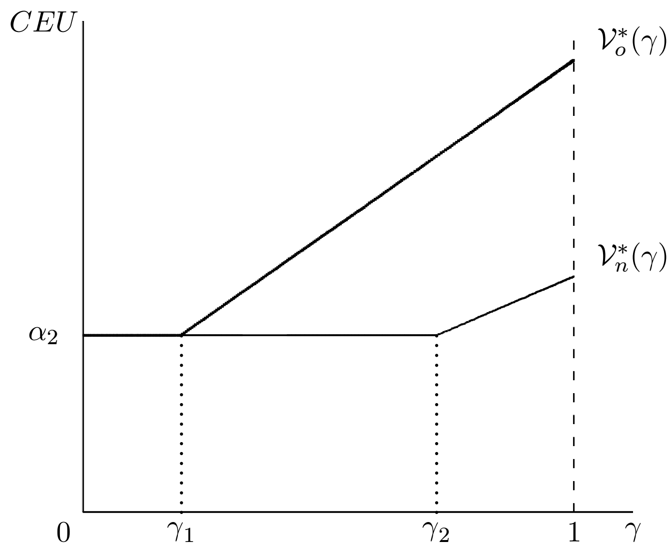

Figure 4: Optimal contract vs. no intermediation

4. ASSET MARKET AND MUTUAL FUND

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

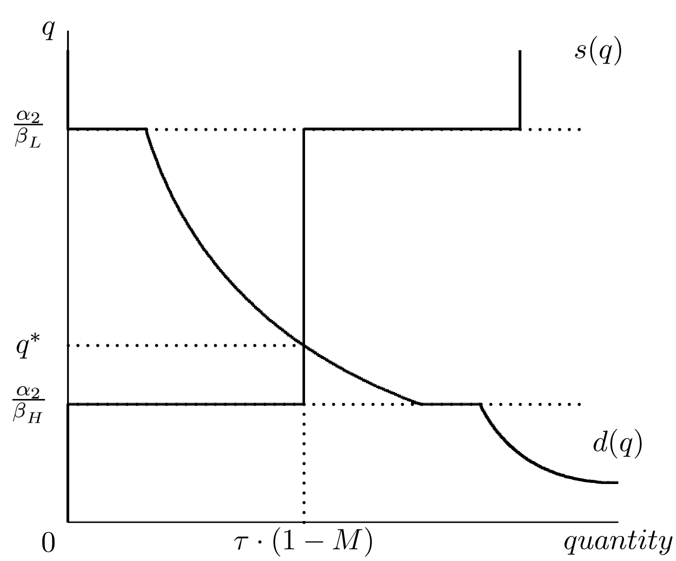

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

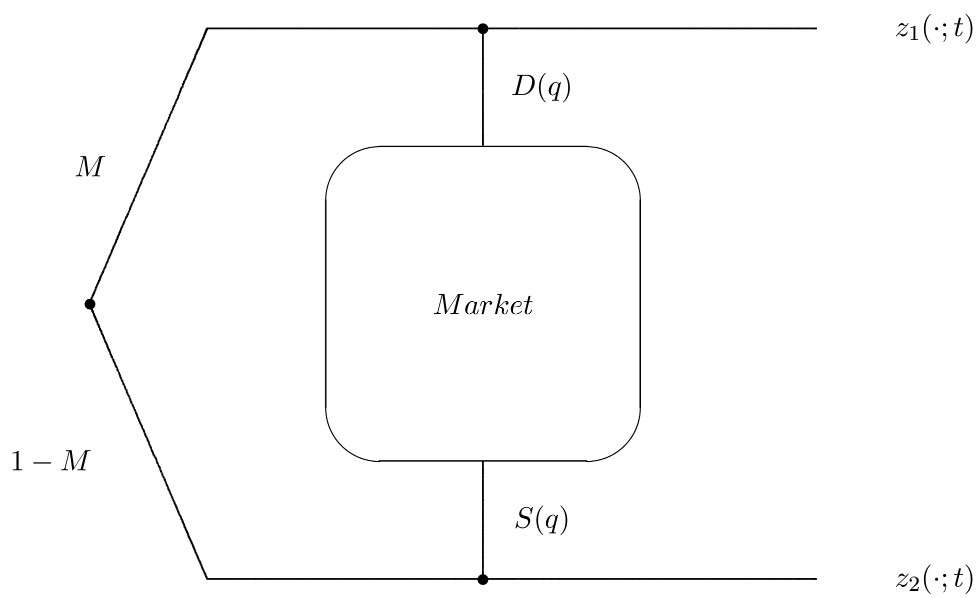

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

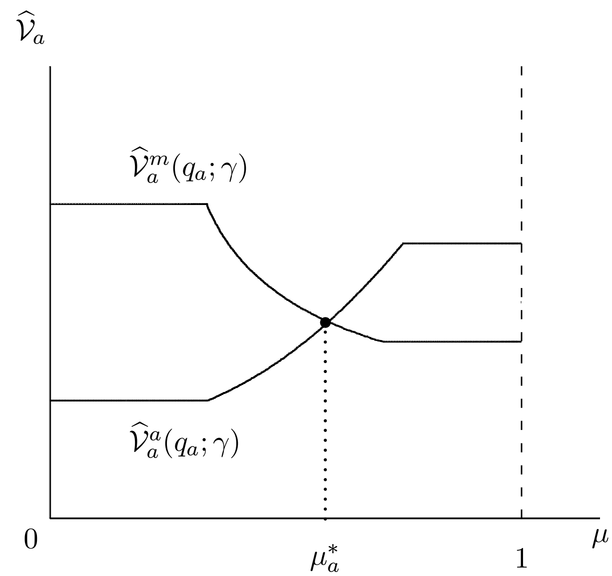

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

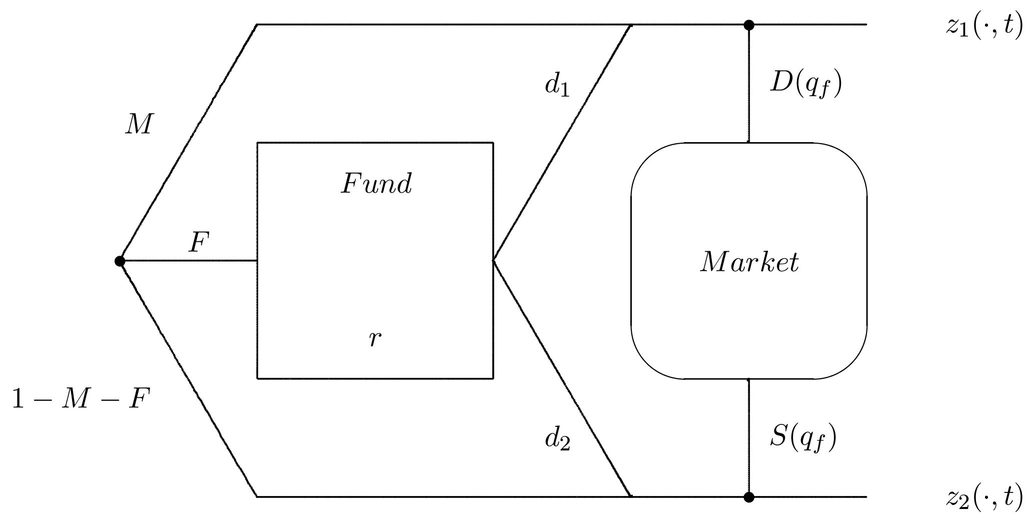

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4]The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5]See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.
[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f\, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s)\, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

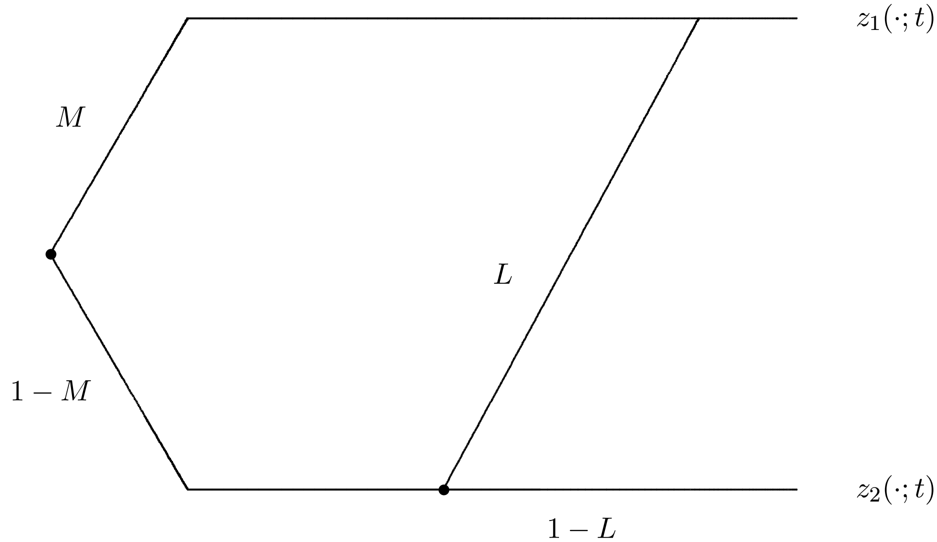

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma\cdot\pi < \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma\cdot\pi = \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 1 & \text{for} \quad \gamma\cdot\pi > \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma\cdot\pi\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

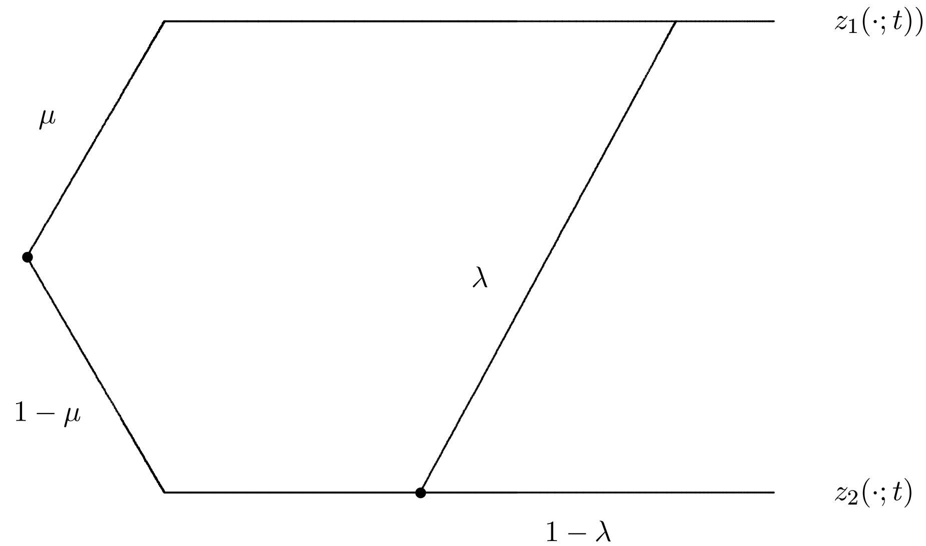

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$
\begin{aligned}
U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\
&:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}].
\end{aligned}
$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

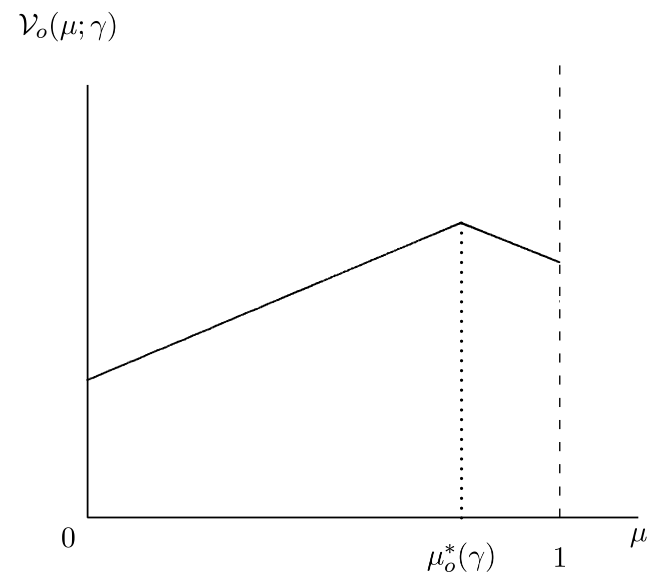

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

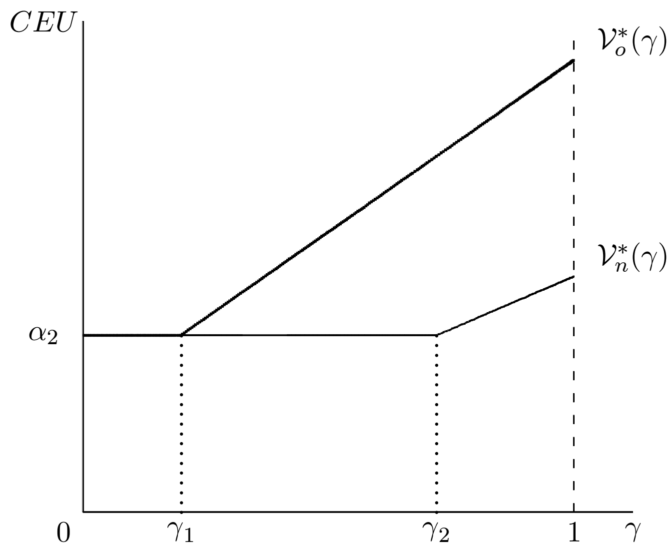

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

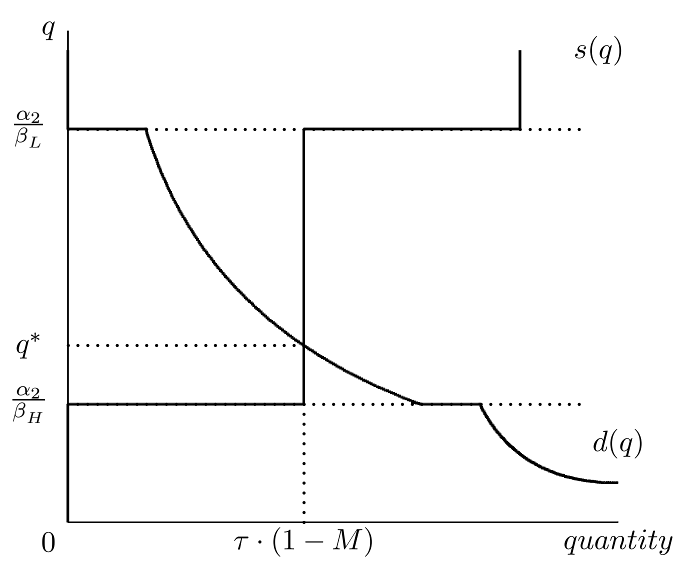

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau, \mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau, \mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

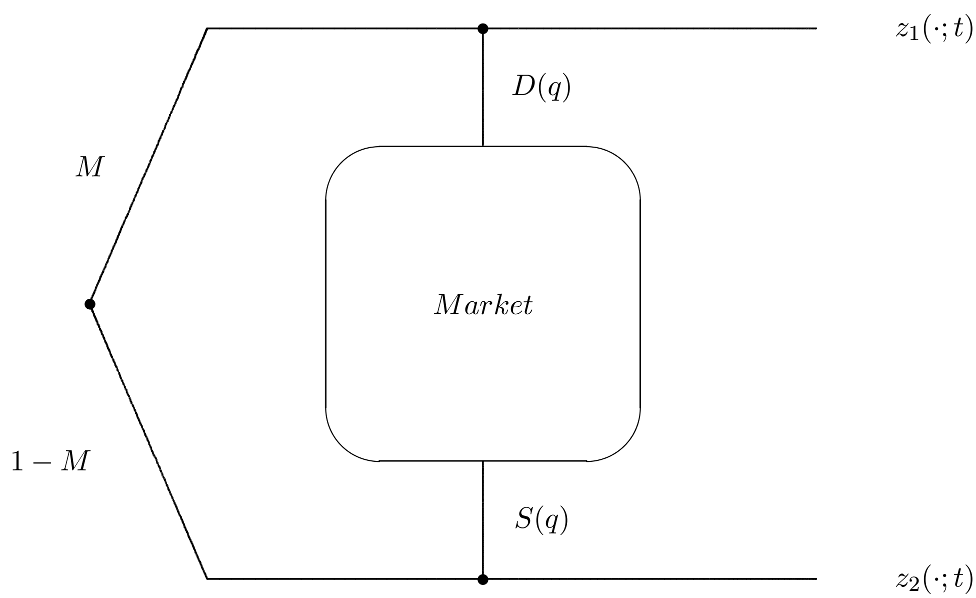

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M; \tau, \mu; t)$, from the investment decision M is easily computed:

$$v_a(M; \tau, \mu; t) := M \cdot R^m(q_a^E(\tau, \mu); t) + (1-M) \cdot R^a(q_a^E(\tau, \mu); t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]}[v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

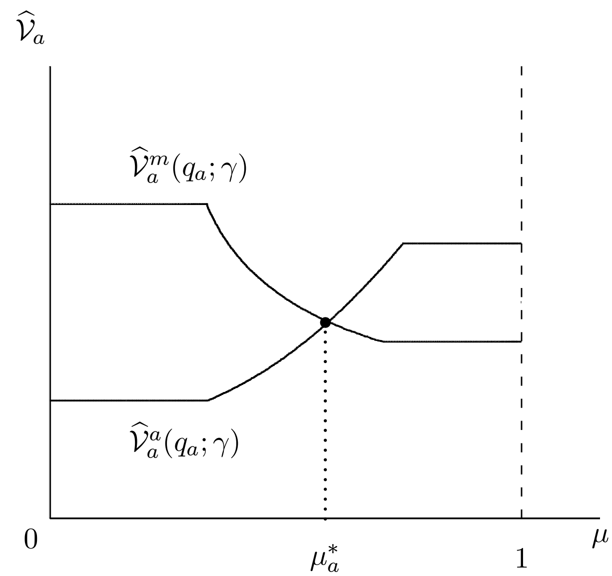

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

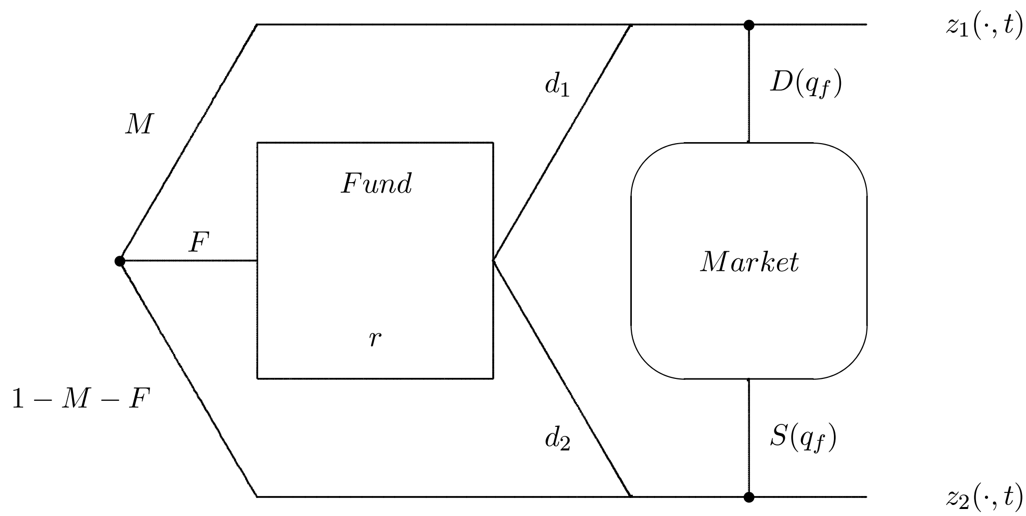

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

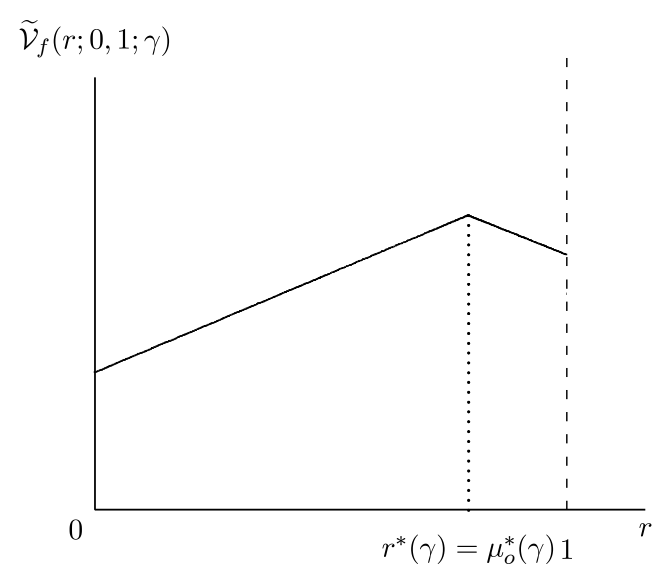

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

[1]We would like to thank seminar participants at the Norwegian School of Management (Sandvika), the University of Oslo, the University of Birmingham, the Bundesbank (Frankfurt a.M.), the Free University in Berlin, the University of Frankfurt a.M., Keele University, University of Hohenheim and Chemnitz University of Technology for helpful comments and suggestions.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$, *where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S. *The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

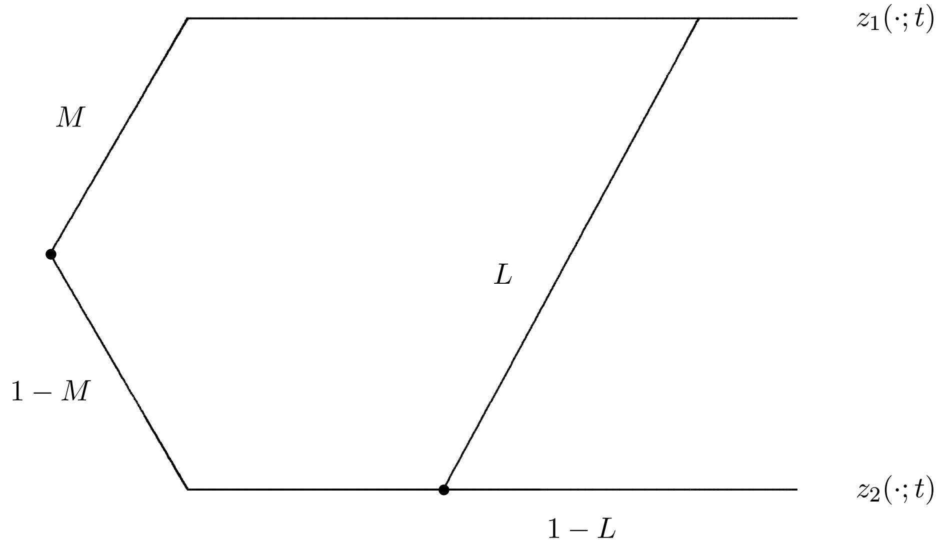

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{array}{rll} z_1(M, L) := & M + \alpha_1 \cdot L \cdot (1 - M) & \text{and} \\ z_2(M, L) := & \alpha_2 \cdot (1 - L) \cdot (1 - M). & \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

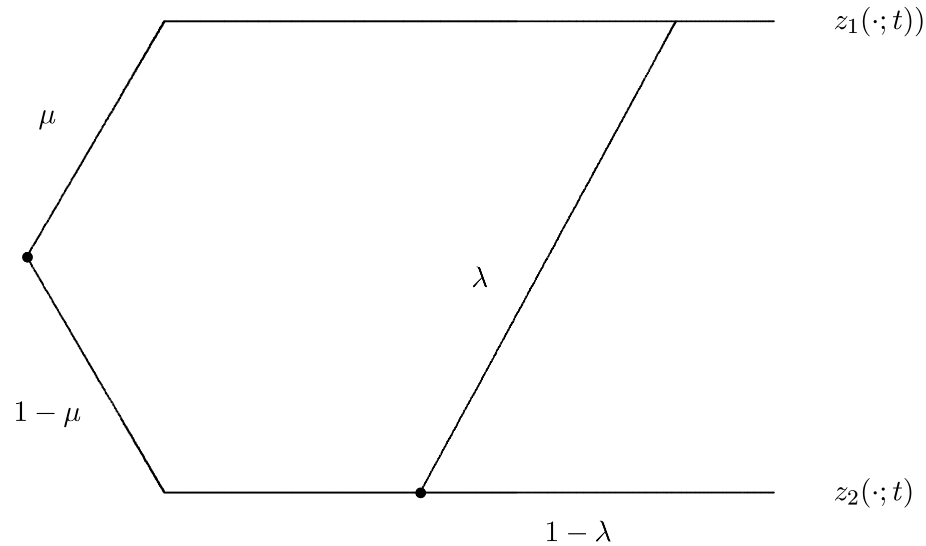

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := &\quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := &\quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$
\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}
$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

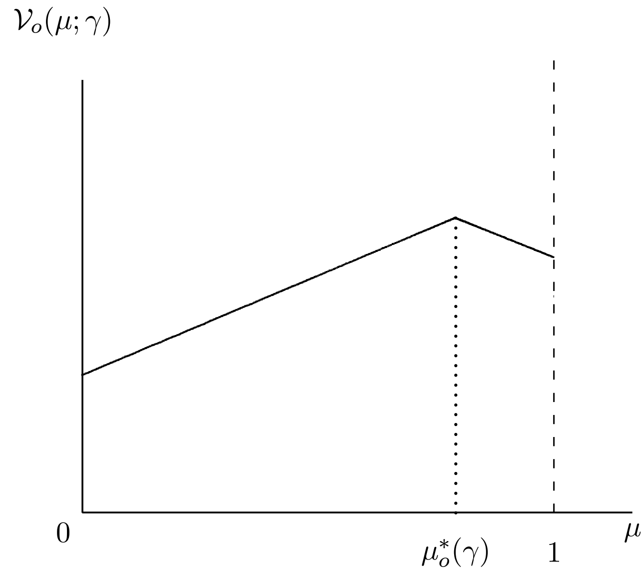

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

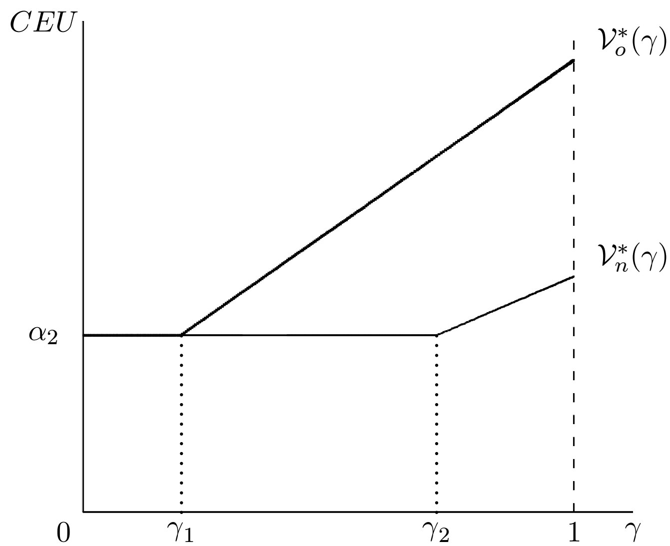

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

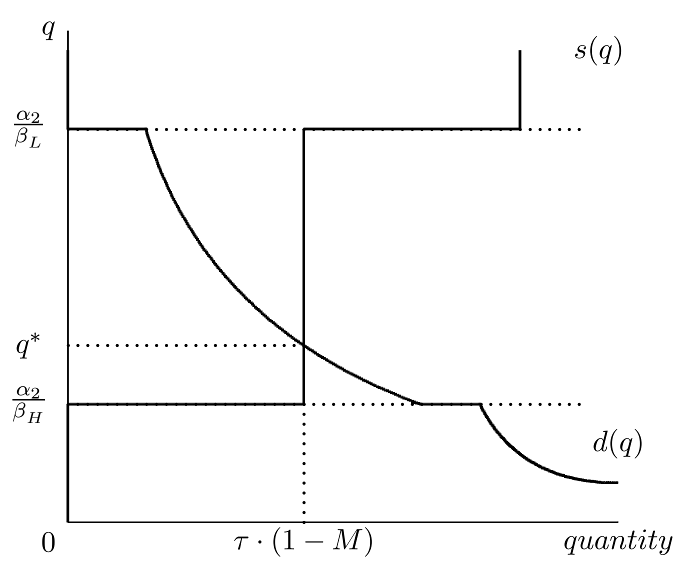

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau, \mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau, \mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

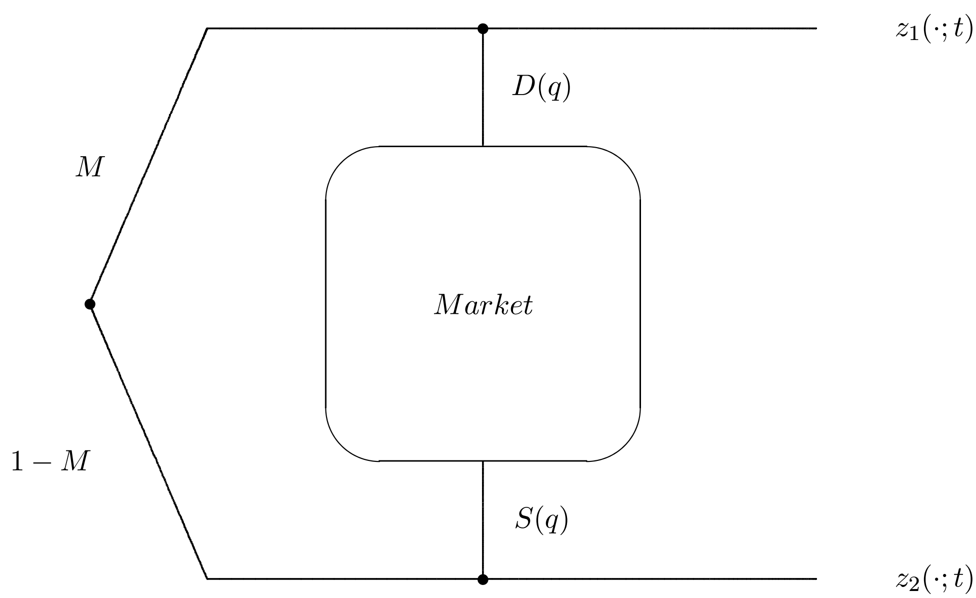

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M) \cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

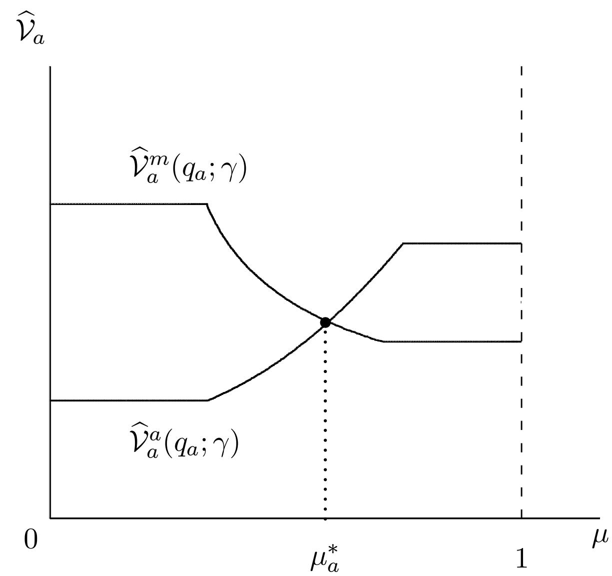

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

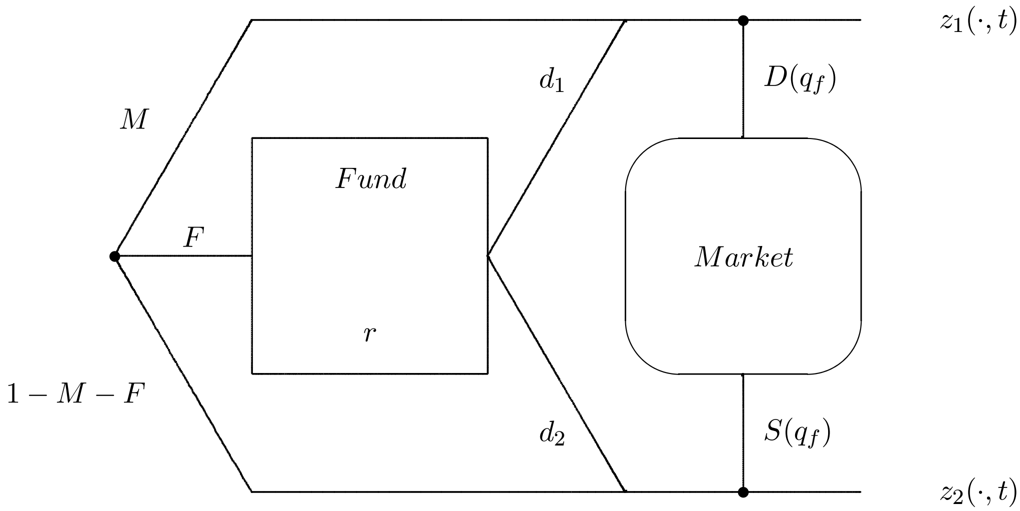

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) \,+\, (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] \\ +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

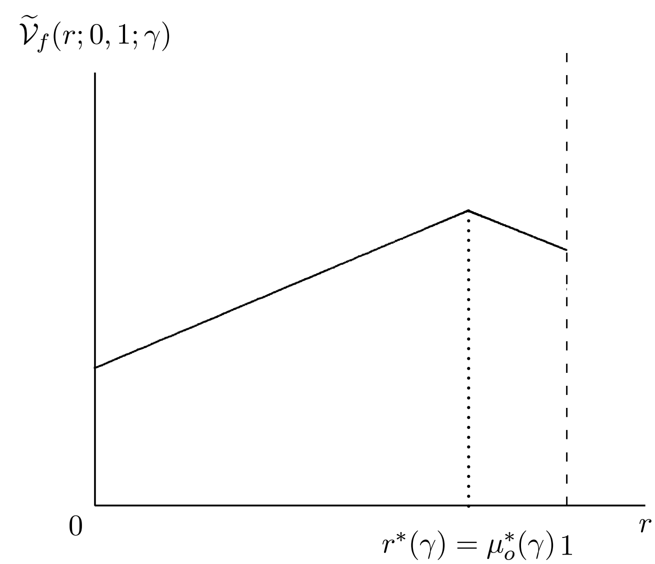

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

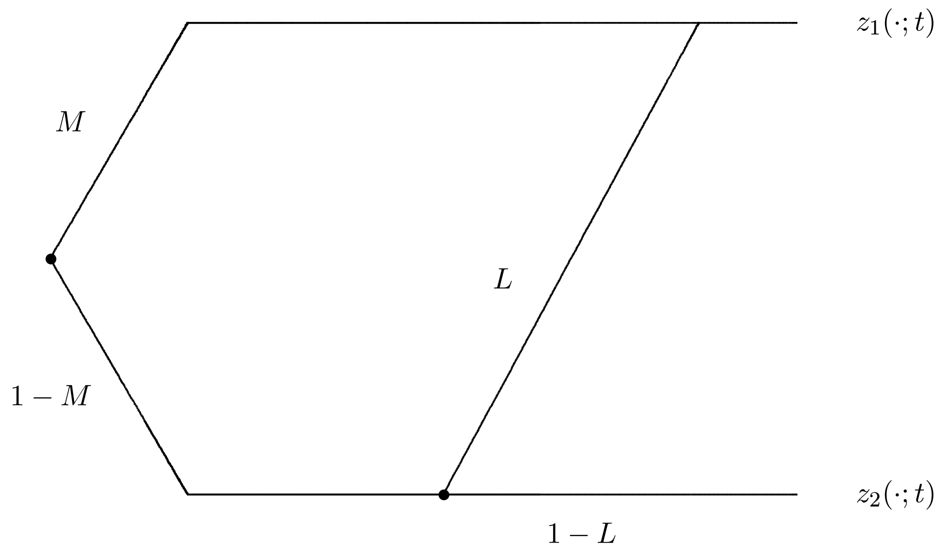

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

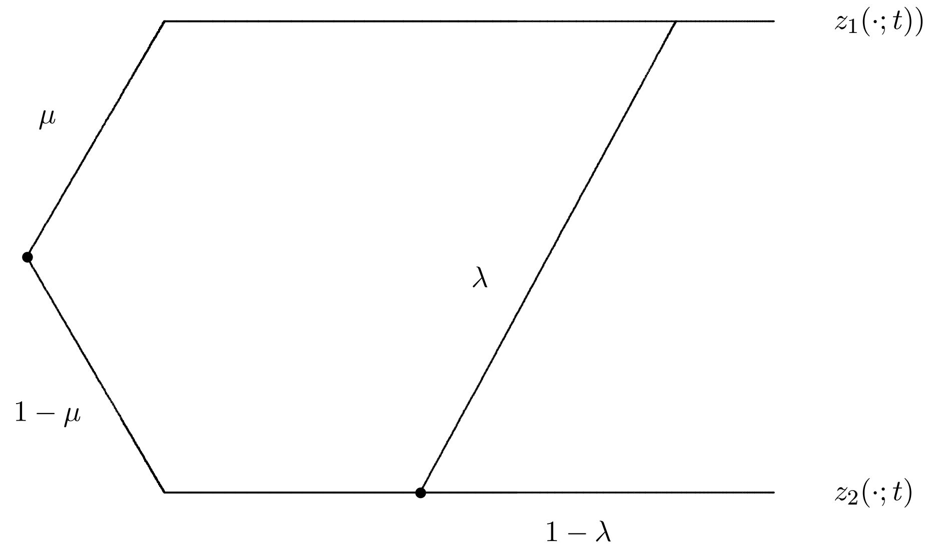

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{lllll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell+(\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

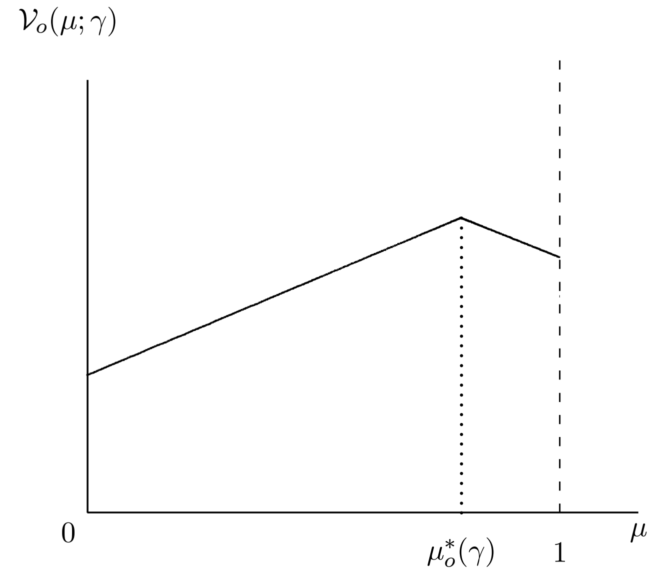

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

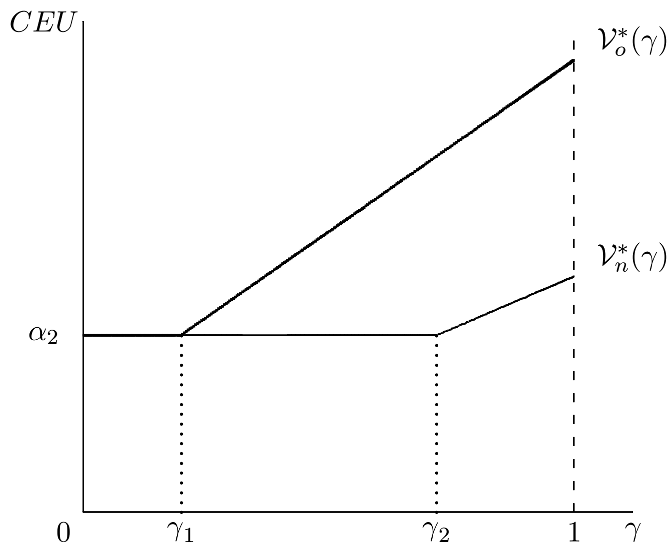

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

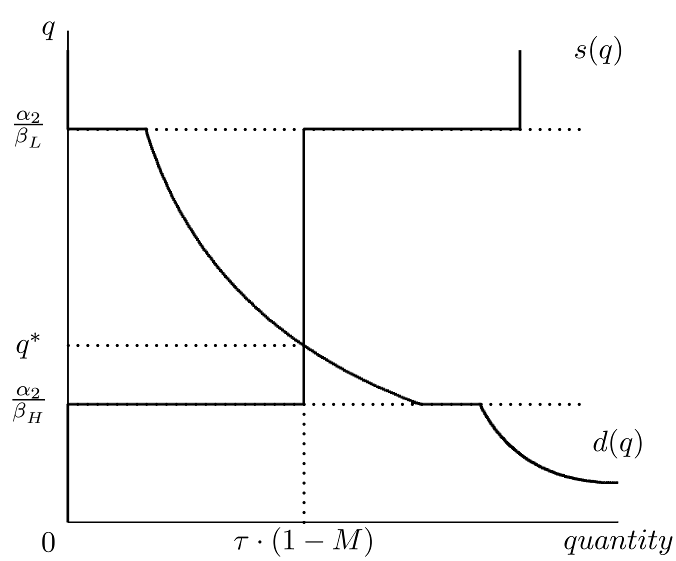

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

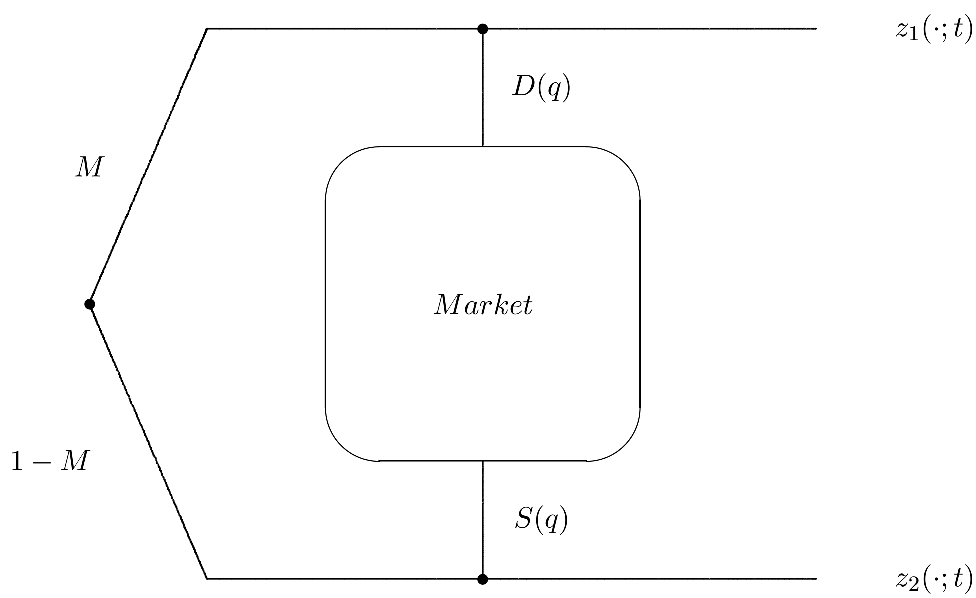

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) :=\quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) :=\quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

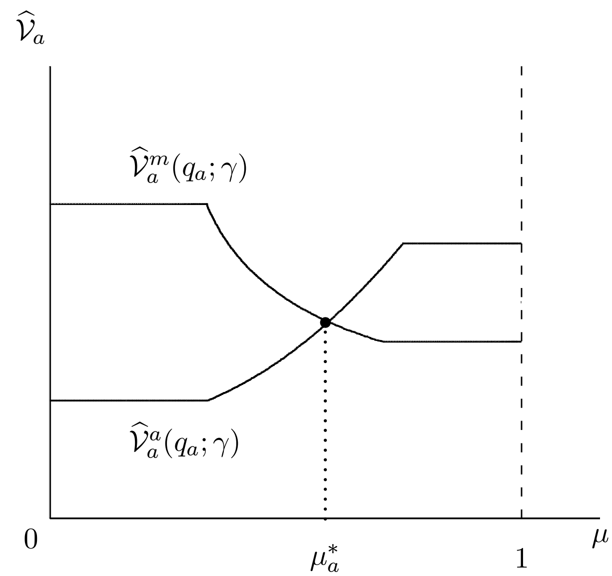

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

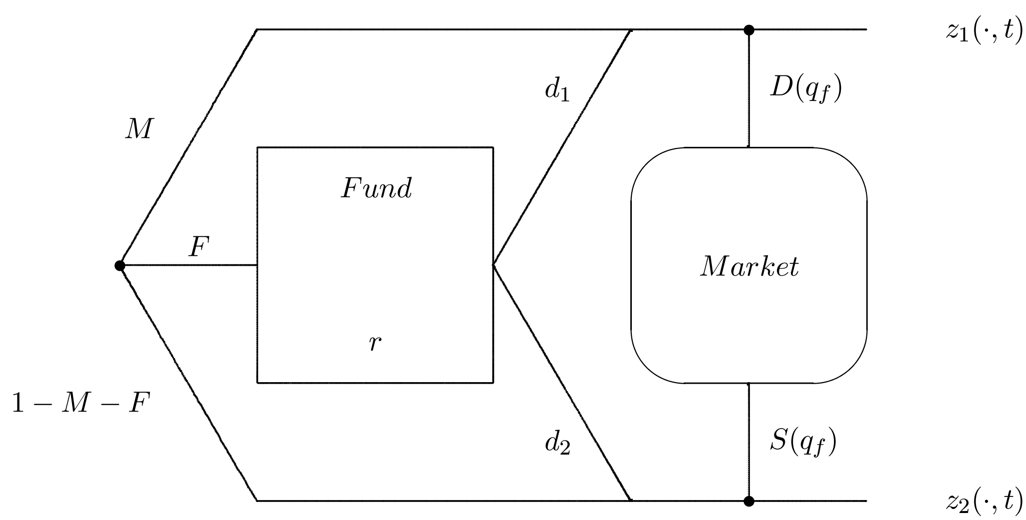

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F, M; \tau, \psi, \varphi, r; \gamma) := \quad \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1-\pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)] \\ + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ, φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{(F,M) \in [0,1] \times [0,1]} \mathcal{V}_f(F, M; \tau, \psi, \varphi, r; \gamma)$$

$$\text{s.t. } F + M \leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ, φ) is [25]

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma).$$

This optimization problem is illustrated in the Figure 9.

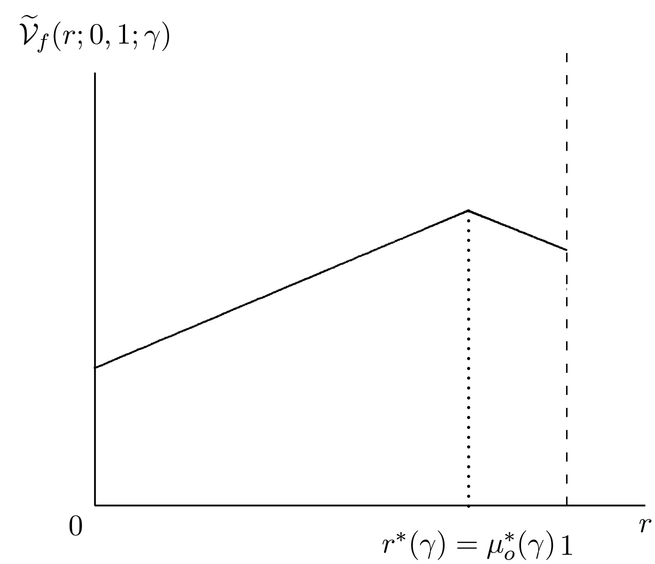

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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ABSTRACT. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. INTRODUCTION

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$*, where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

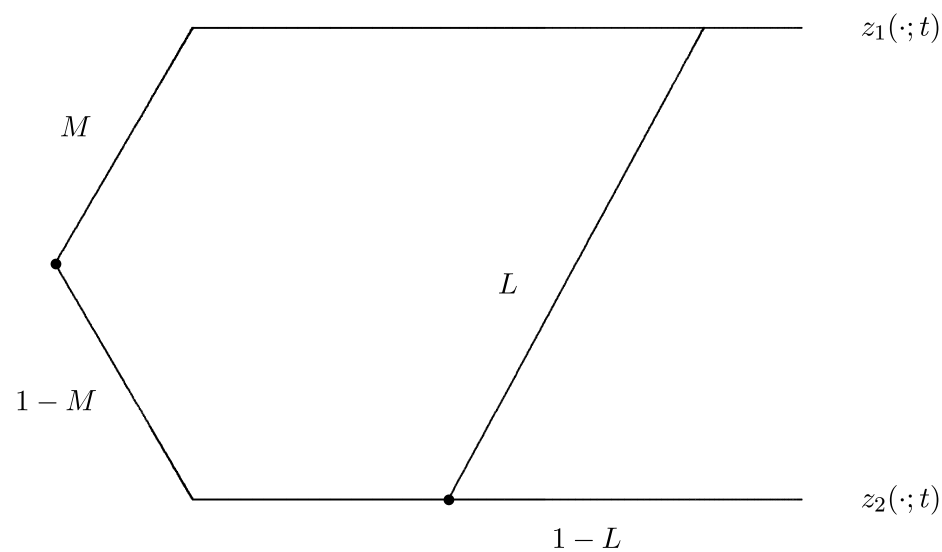

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{array}{ll} z_1(M, L) := & M + \alpha_1 \cdot L \cdot (1 - M) \quad \text{and} \\ z_2(M, L) := & \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & +(1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\} \cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\}) \cdot v_n(M;\ell)] \\ & +(1-\gamma) \cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M) \cdot \alpha_2 \geq \beta_\ell \cdot M + (1-M) \cdot \alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2 , \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

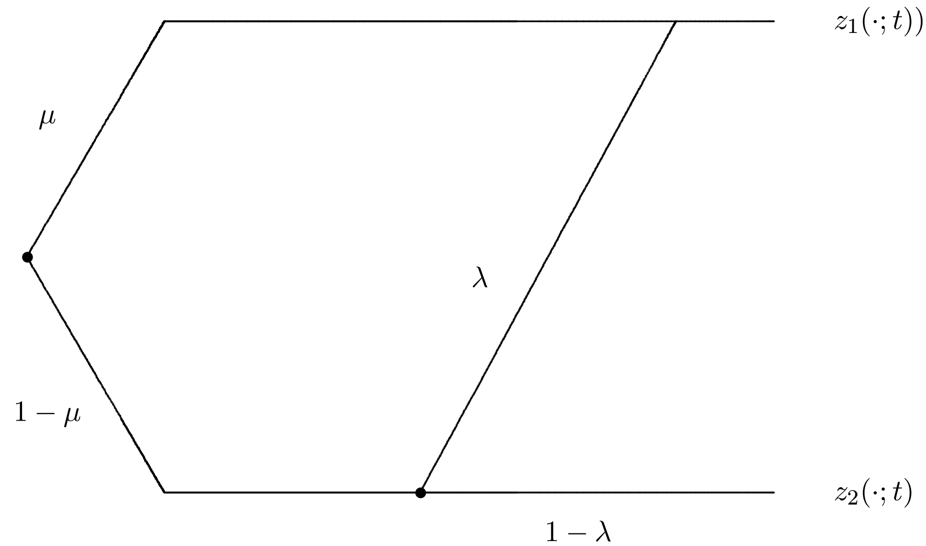

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := \quad & u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = \quad & \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

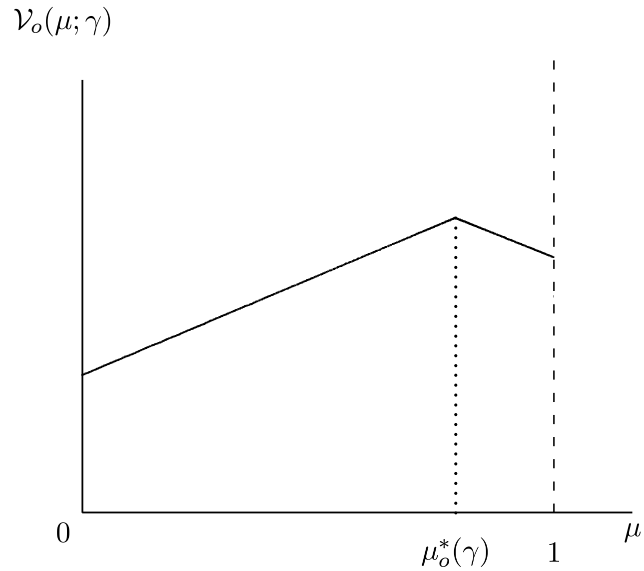

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

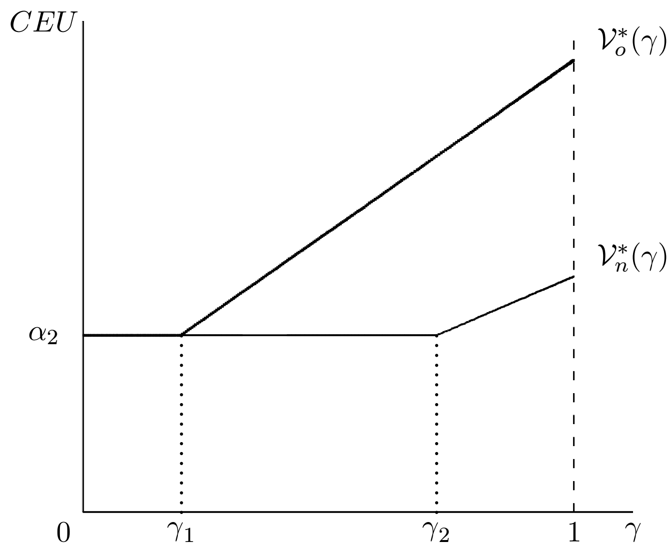

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

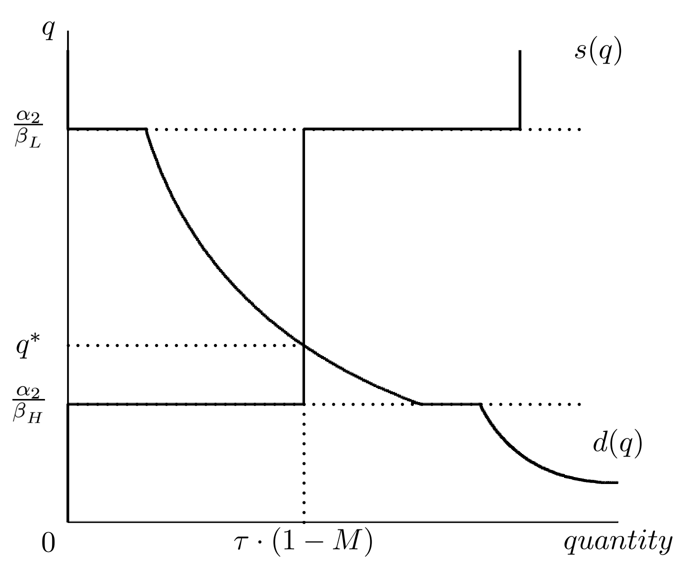

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

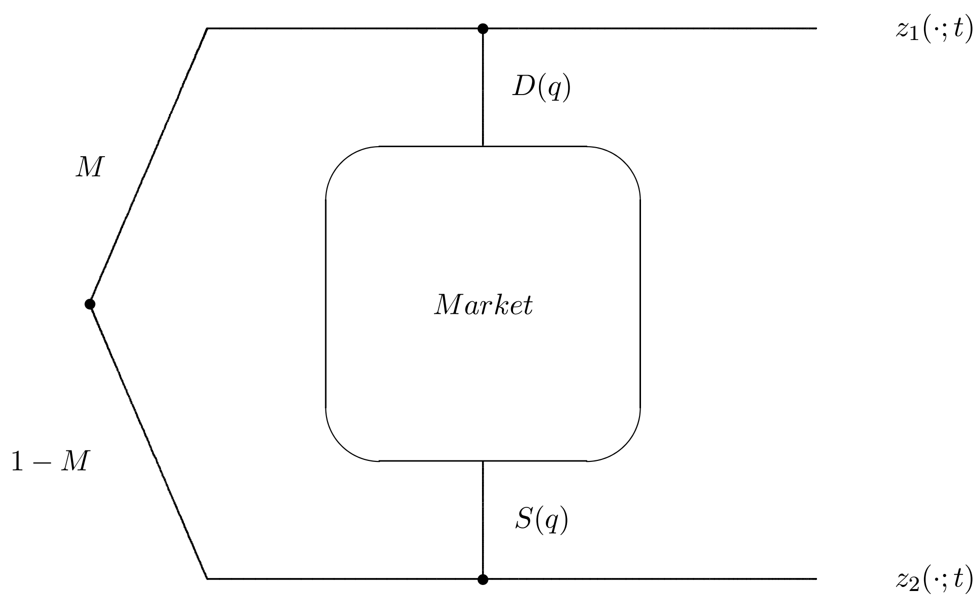

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M) \cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0,1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

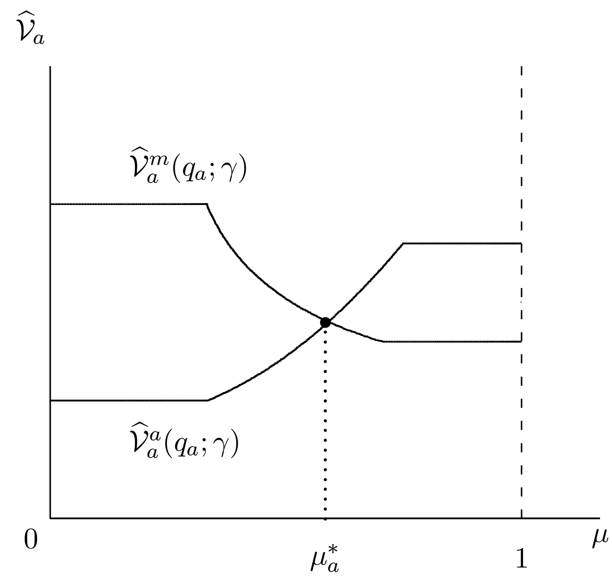

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

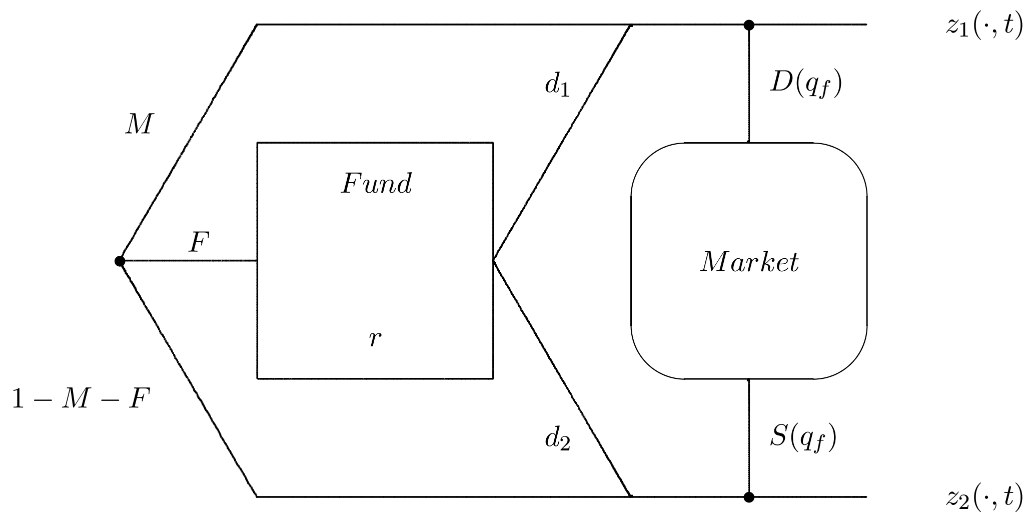

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

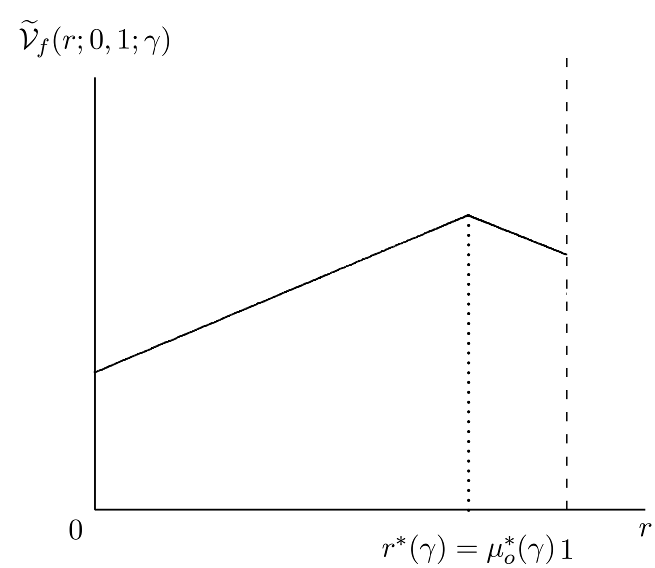

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.
For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) \;:=\; \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) \;=\; \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

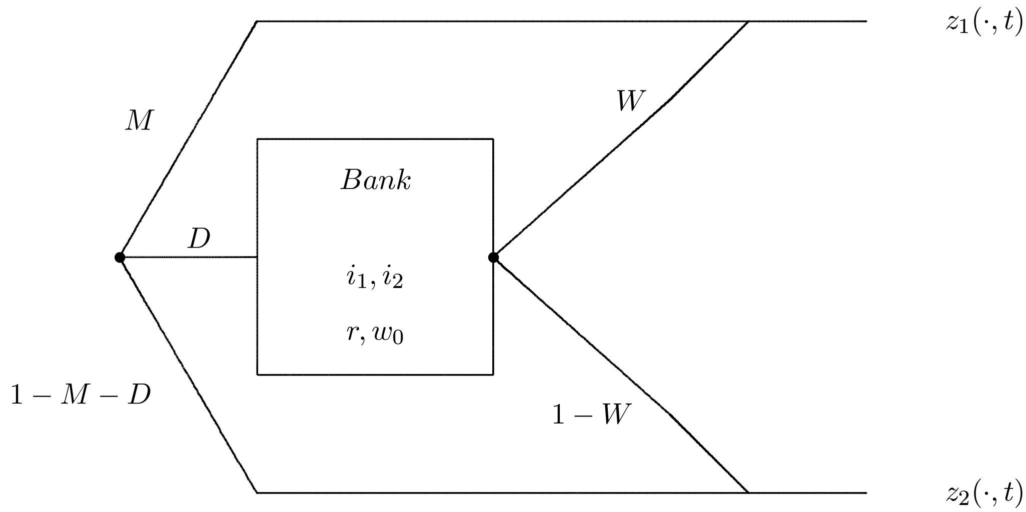

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

First Version: November 1997
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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

[1] We would like to thank seminar participants at the Norwegian School of Management (Sandvika), the University of Oslo, the University of Birmingham, the Bundesbank (Frankfurt a.M.), the Free University in Berlin, the University of Frankfurt a.M., Keele University, University of Hohenheim and Chemnitz University of Technology for helpful comments and suggestions.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

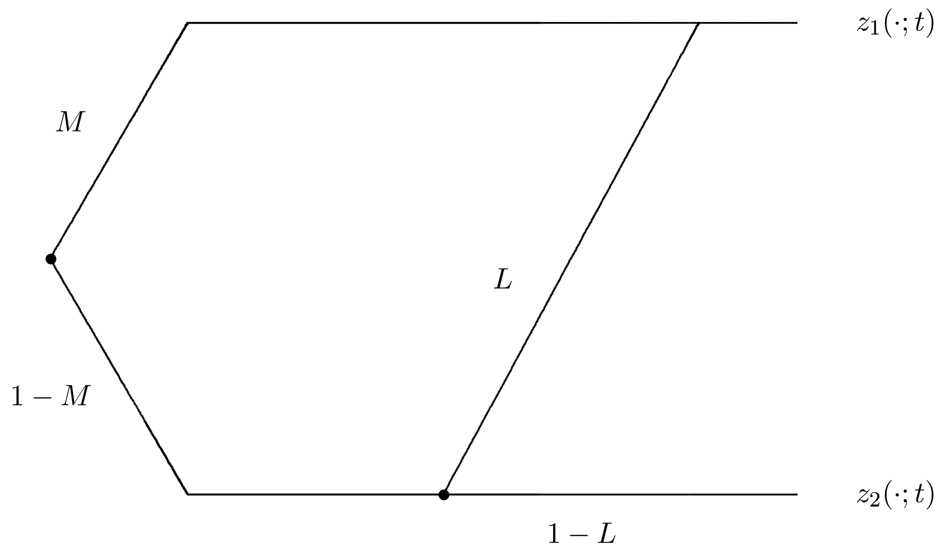

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{array}{rll} z_1(M,L) := & M + \alpha_1 \cdot L \cdot (1-M) & \text{and} \\ z_2(M,L) := & \alpha_2 \cdot (1-L) \cdot (1-M). & \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

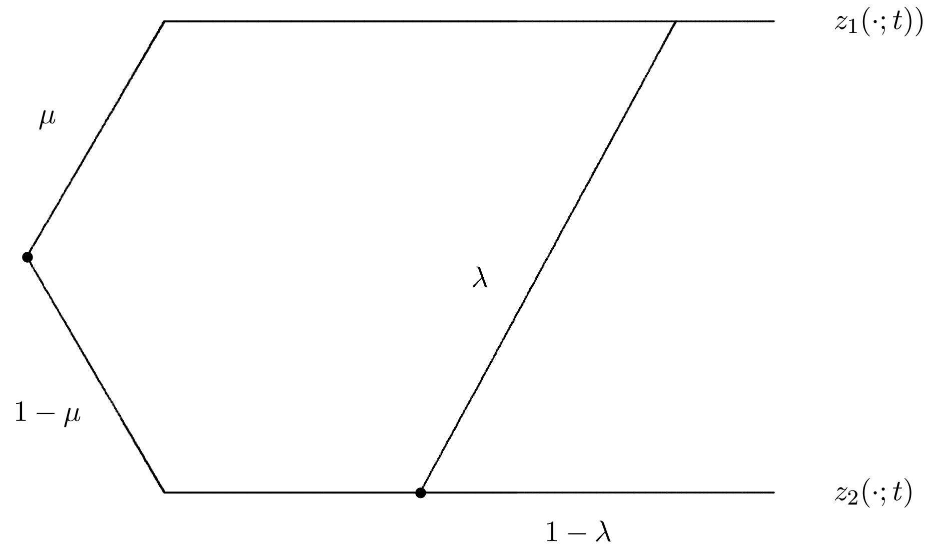

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$
\begin{array}{lllll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}
$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

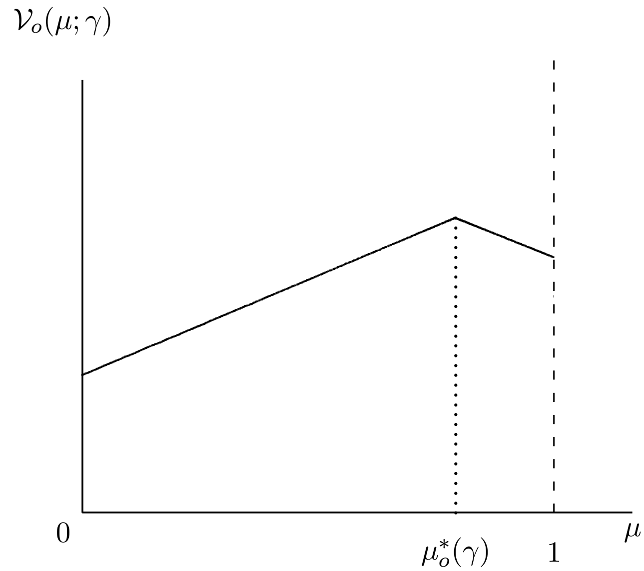

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

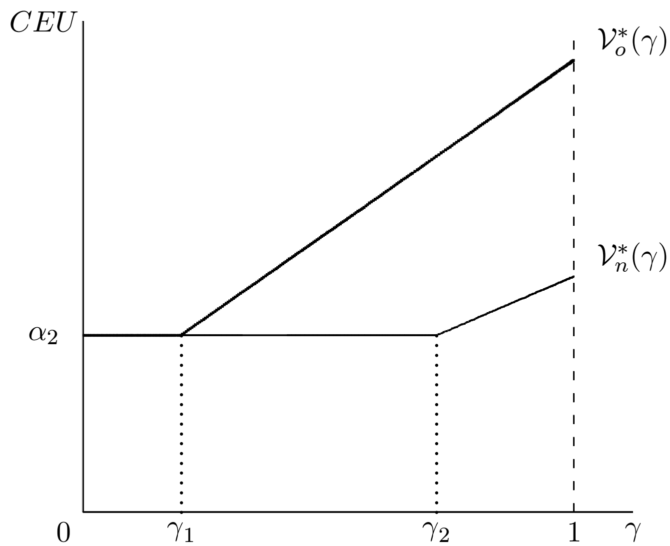

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

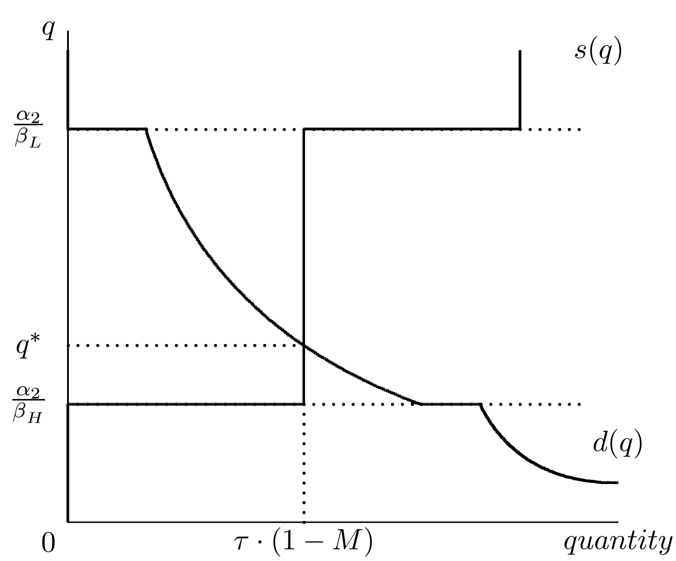

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau, \mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau, \mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

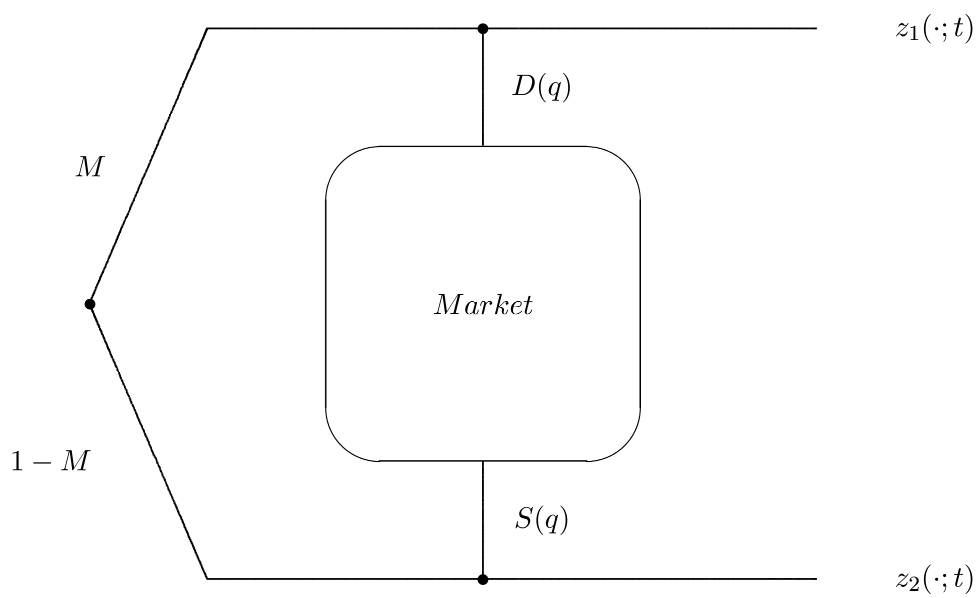

Figure 6: Asset market

Denote by $R^m(q_a; t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a; t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M; \tau, \mu; t)$, from the investment decision M is easily computed:

$$v_a(M; \tau, \mu; t) := M \cdot R^m(q_a^E(\tau, \mu); t) + (1 - M) \cdot R^a(q_a^E(\tau, \mu); t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

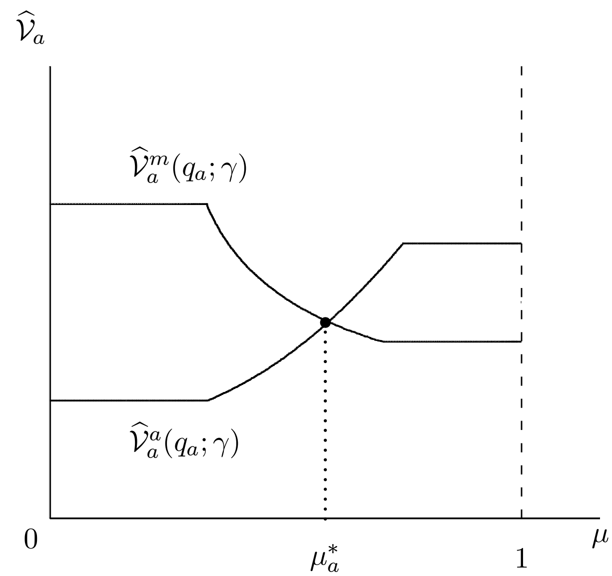

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

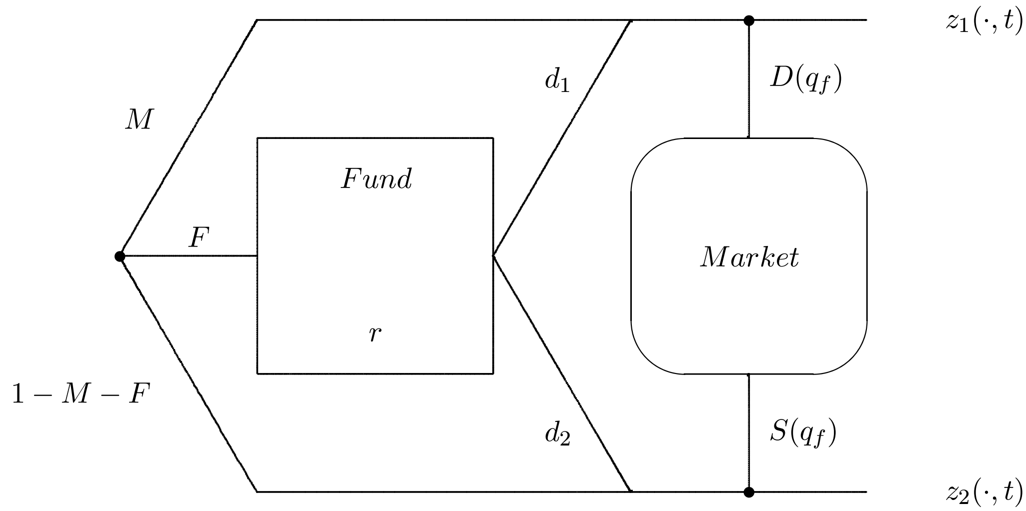

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

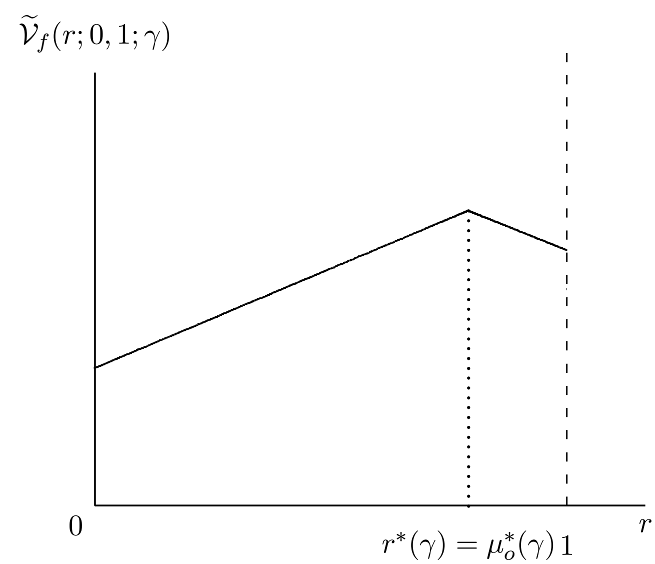

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

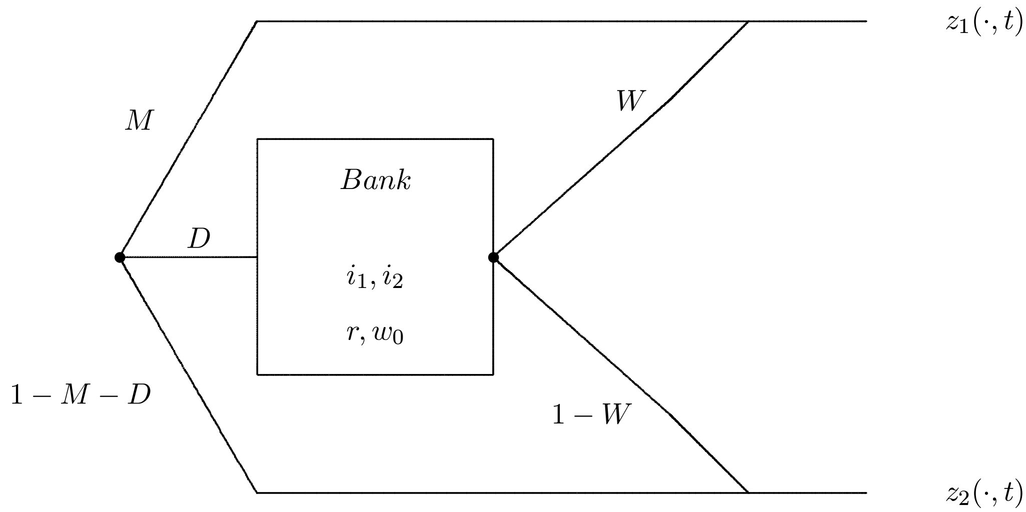

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

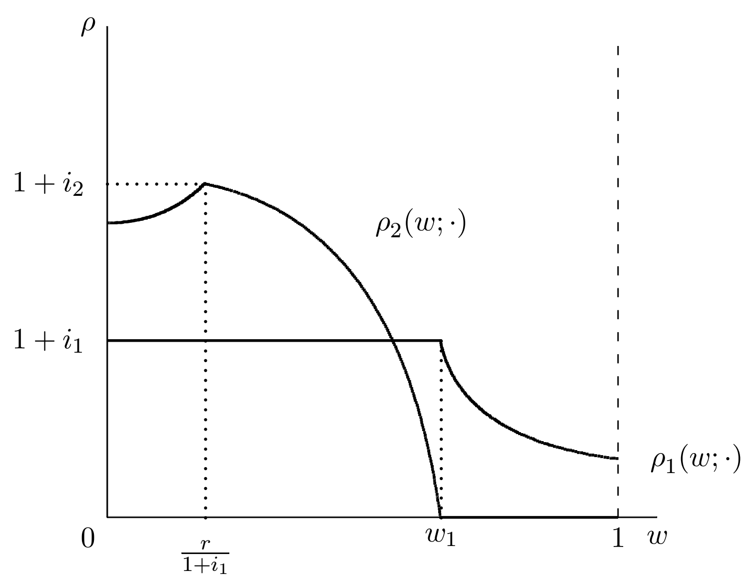

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

[1] We would like to thank seminar participants at the Norwegian School of Management (Sandvika), the University of Oslo, the University of Birmingham, the Bundesbank (Frankfurt a.M.), the Free University in Berlin, the University of Frankfurt a.M., Keele University, University of Hohenheim and Chemnitz University of Technology for helpful comments and suggestions.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0,1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

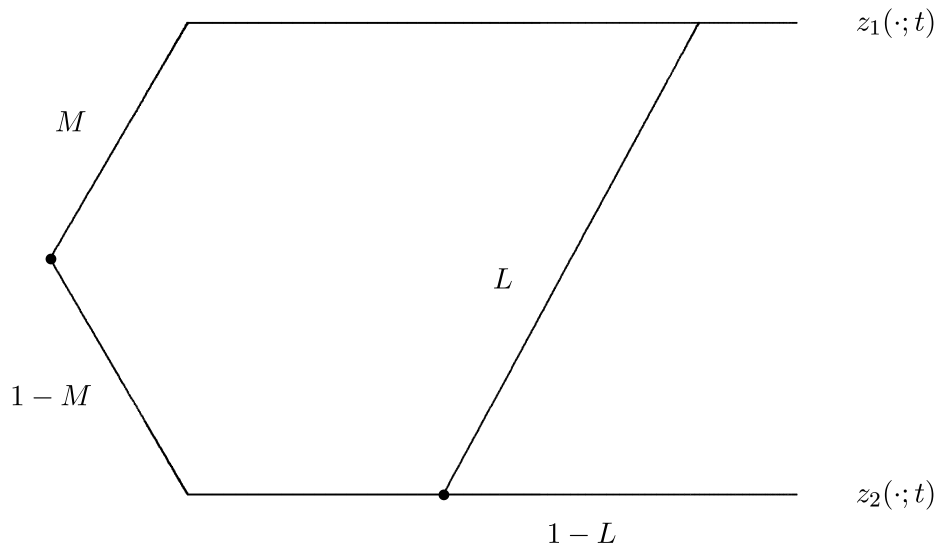

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) &:= \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) &:= \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & +(1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\} \cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\}) \cdot v_n(M;\ell)] \\ & +(1-\gamma) \cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M) \cdot \alpha_2 \geq \beta_\ell \cdot M + (1-M) \cdot \alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2 \, , \, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

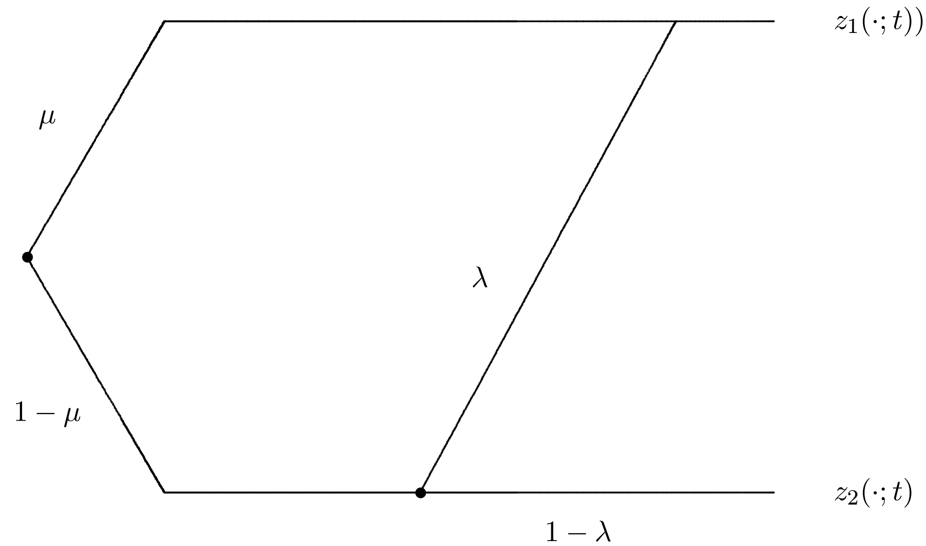

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := \quad & \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := \quad & \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

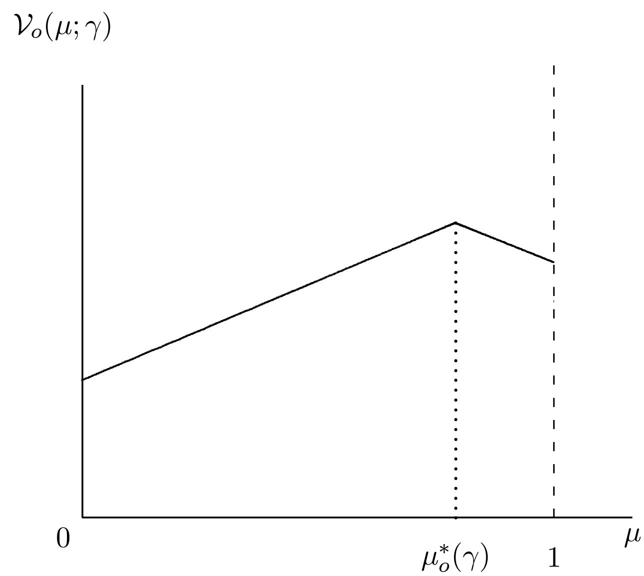

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

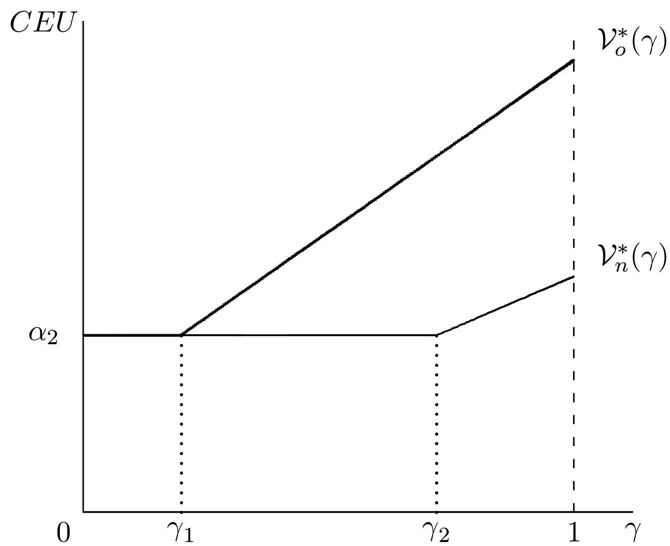

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

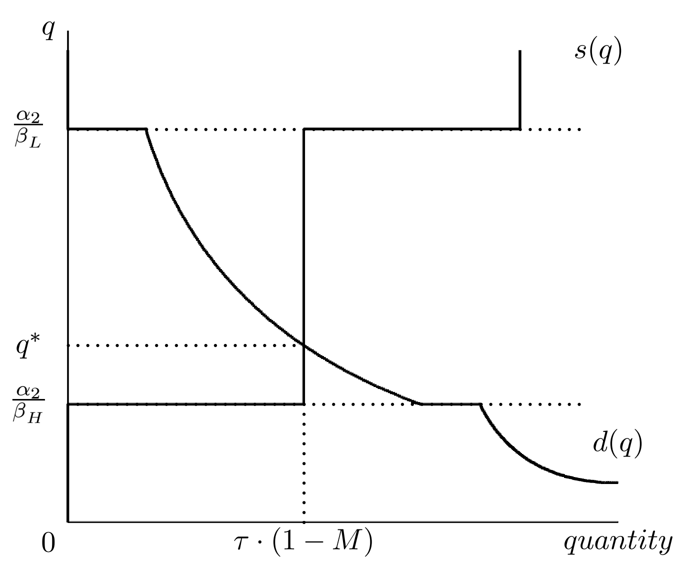

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

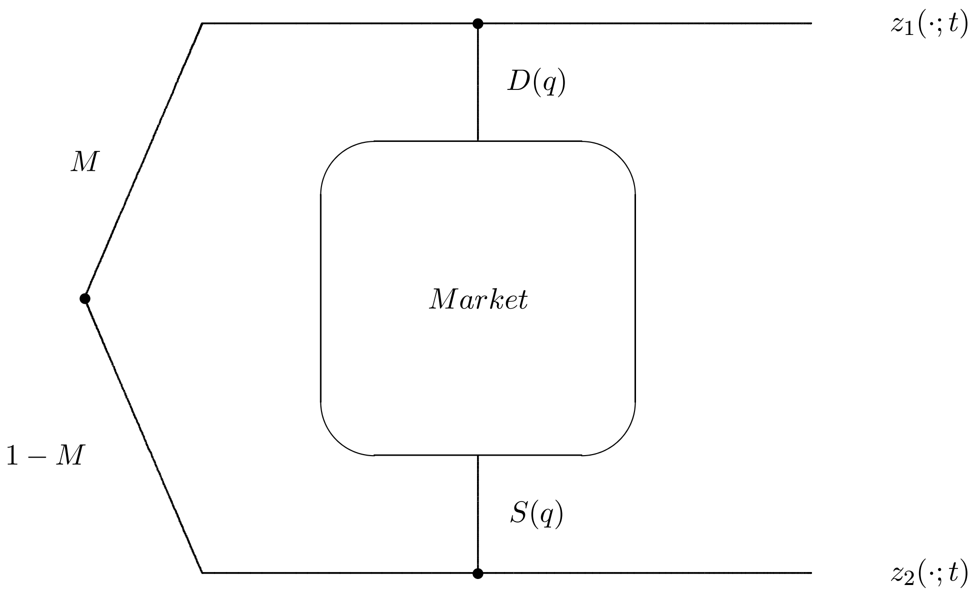

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)] .\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

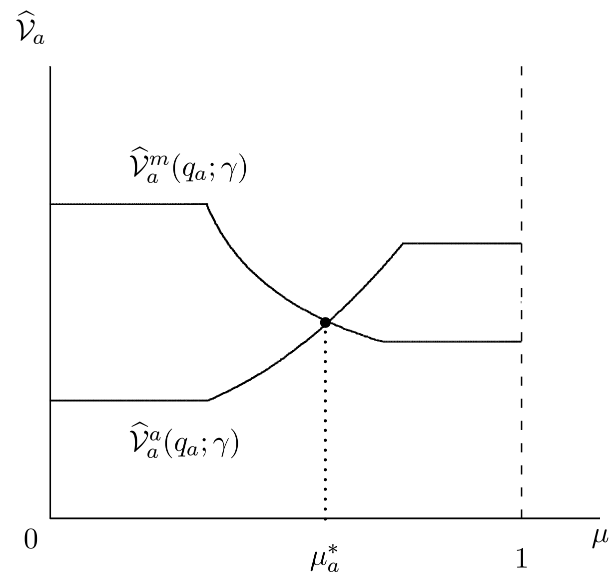

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

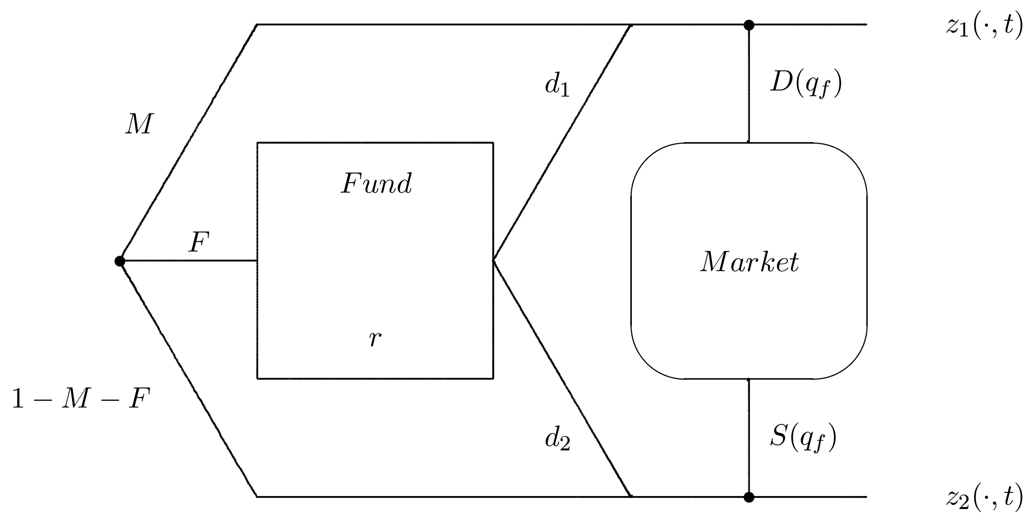

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

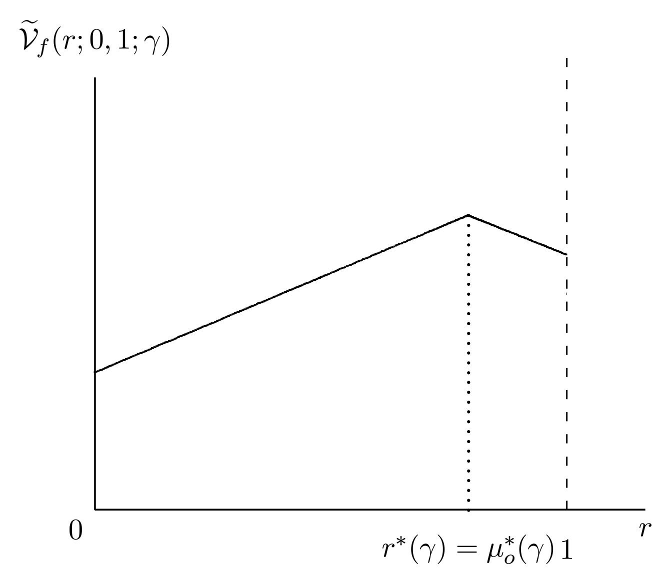

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*
For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) \,:=\, \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) \,=\, \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26]See Appendix A, Remark 10.
[27]See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

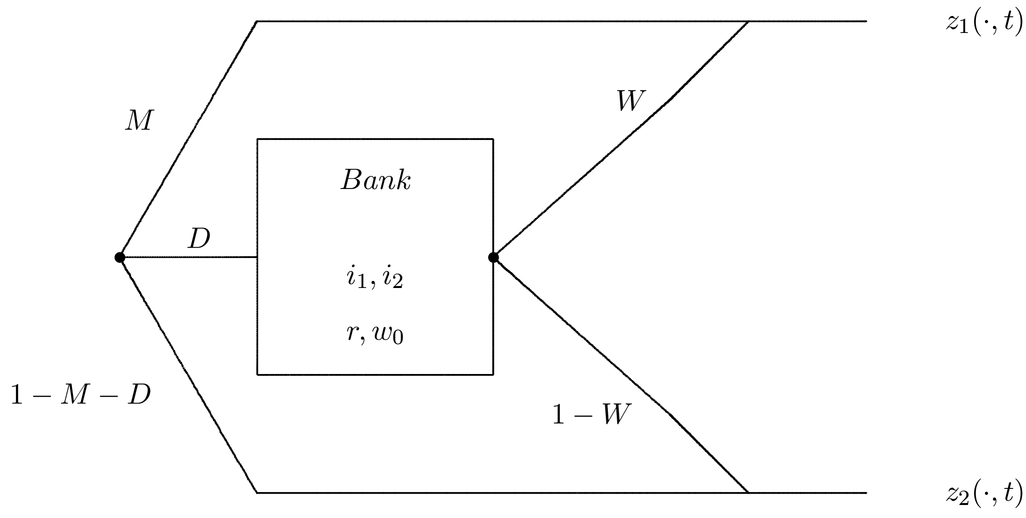

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

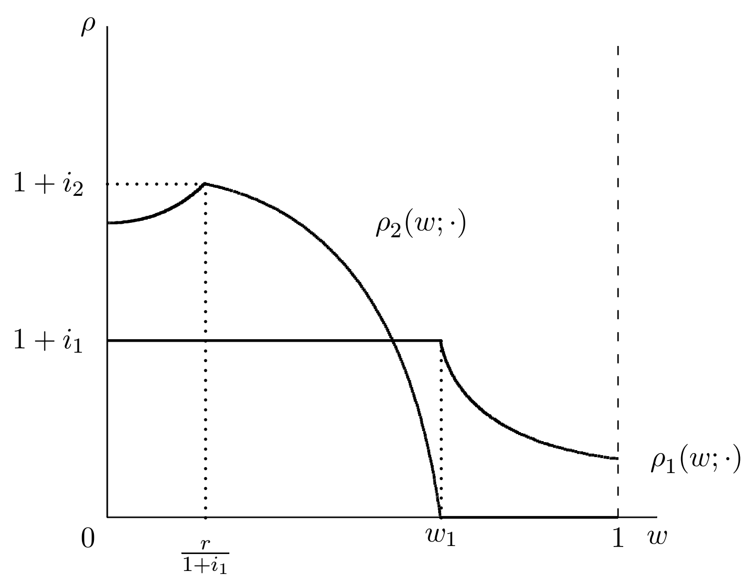

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30]See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

[1] We would like to thank seminar participants at the Norwegian School of Management (Sandvika), the University of Oslo, the University of Birmingham, the Bundesbank (Frankfurt a.M.), the Free University in Berlin, the University of Frankfurt a.M., Keele University, University of Hohenheim and Chemnitz University of Technology for helpful comments and suggestions.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$, *where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \text{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \text{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

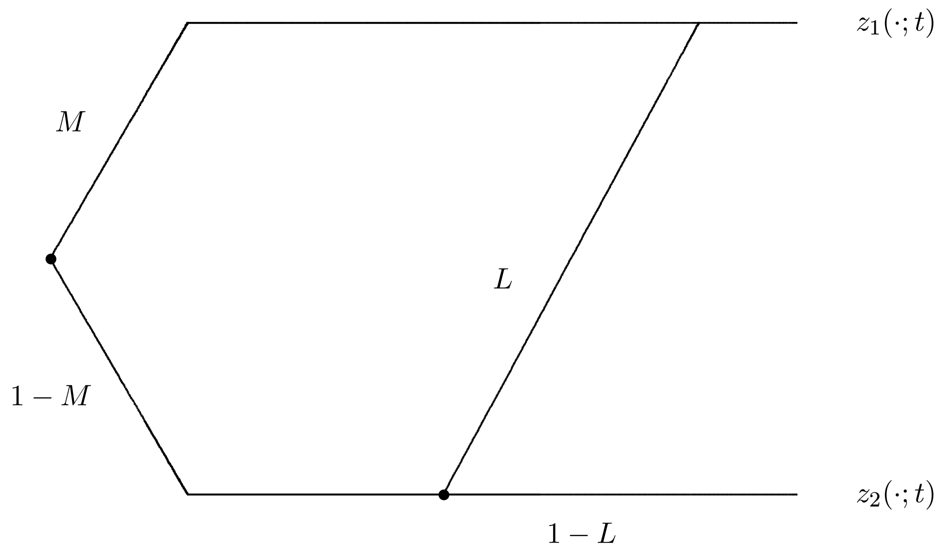

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M \cdot [\tau \cdot \beta_h + (1-\tau)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)] \cdot p(\tau)\; d\tau \\ & \qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & \qquad + (1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) \;=\; \beta_h \cdot M + (1-M)\cdot \alpha_2 \;\geq\; \beta_\ell \cdot M + (1-M)\cdot\alpha_2 \;=\; v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

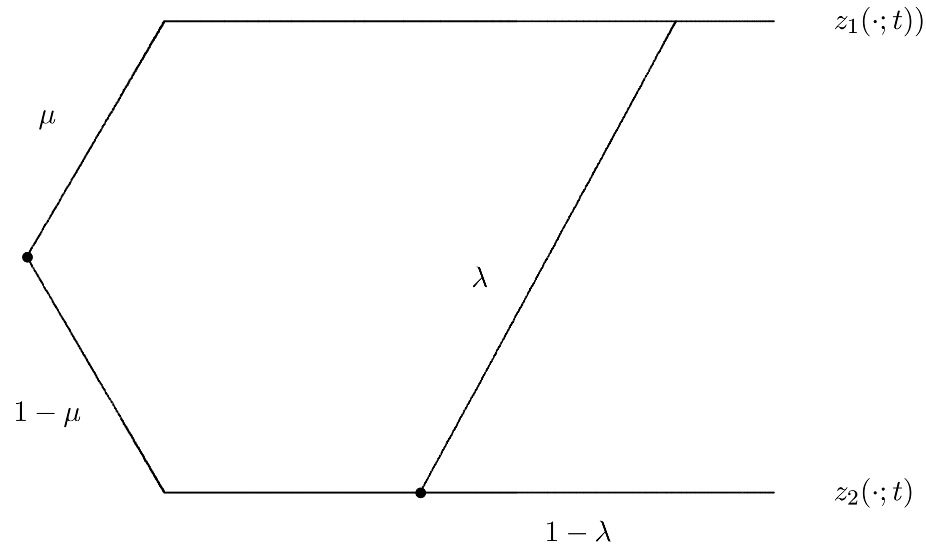

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) :=& \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ =& \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

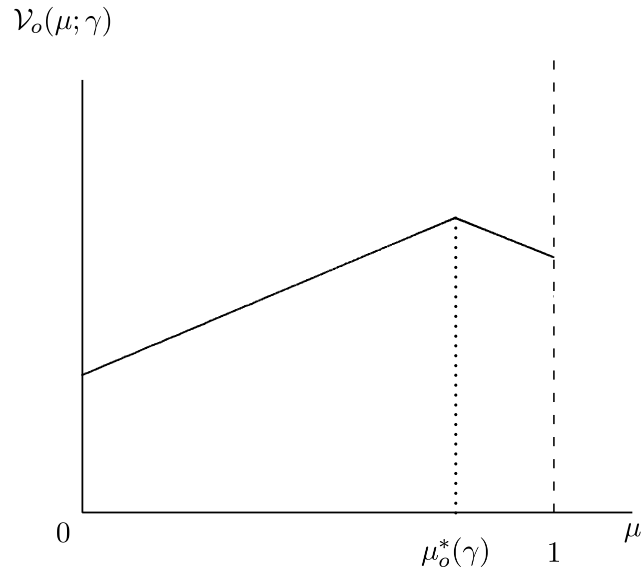

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

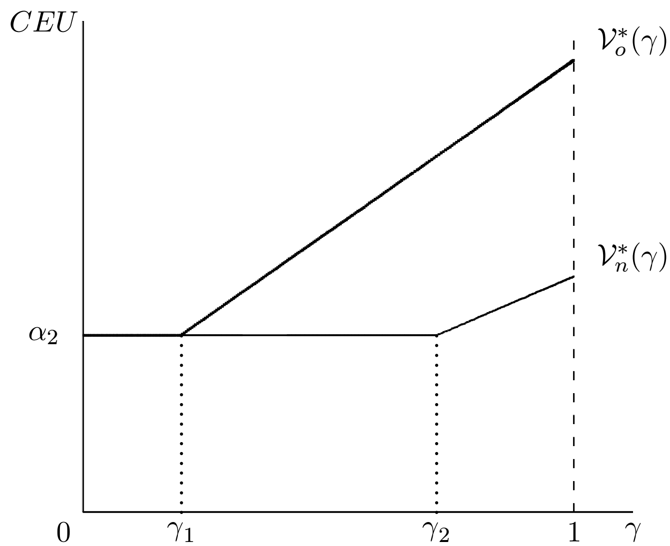

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

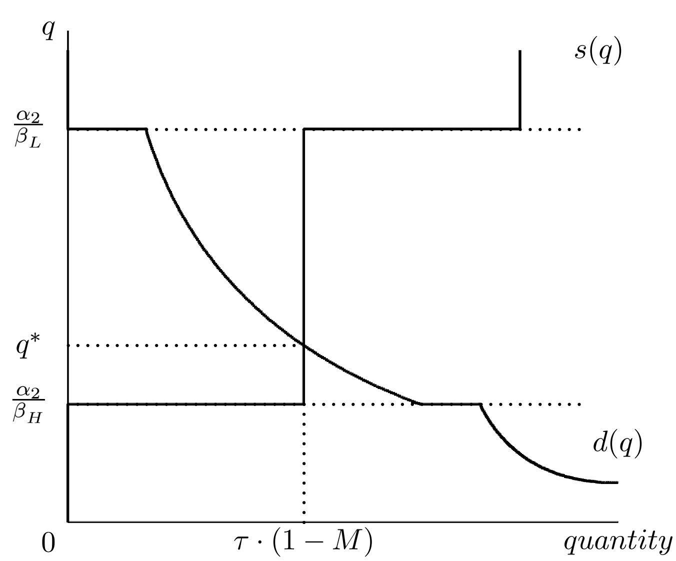

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

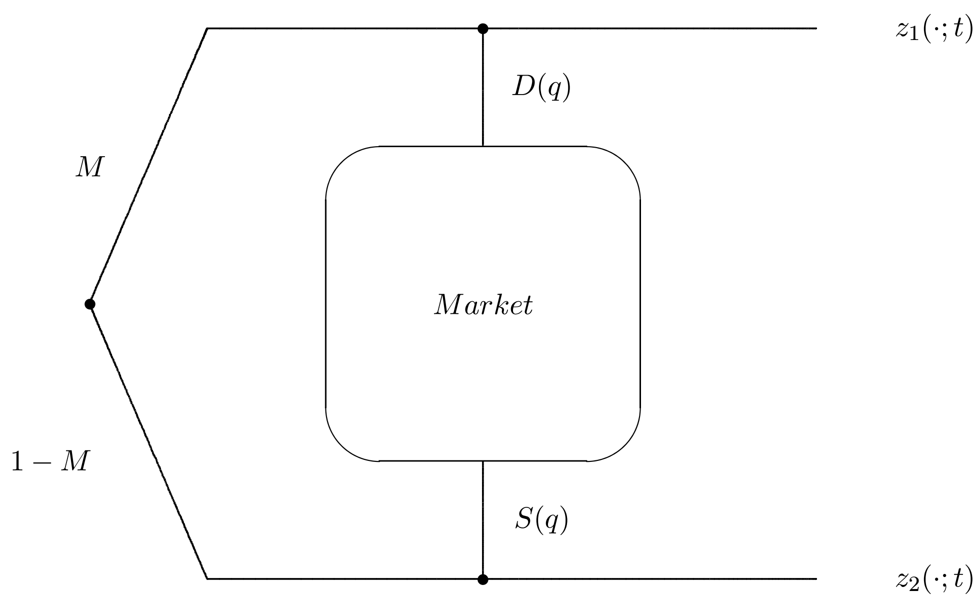

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]}[v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

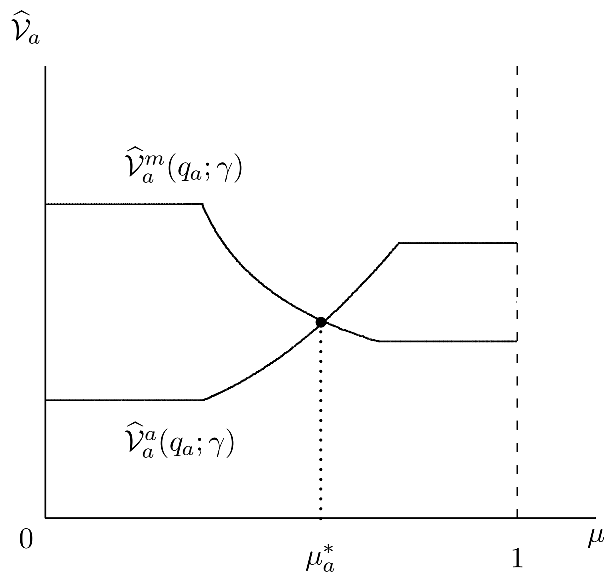

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

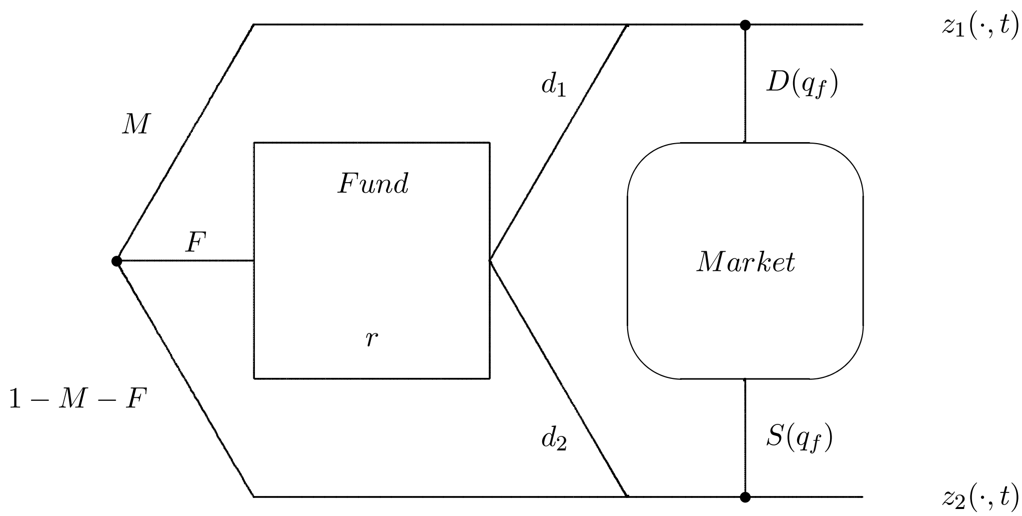

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F, M; \tau, \psi, \varphi, r; \gamma) := \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1-\pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ, φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F, M; \tau, \psi, \varphi, r; \gamma)$$

$$\text{s.t. } F + M \leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ, φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma).$$

This optimization problem is illustrated in the Figure 9.

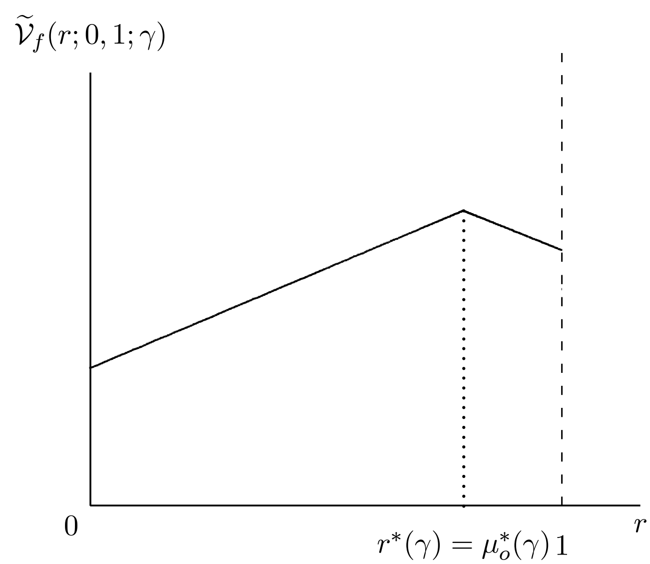

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.

[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

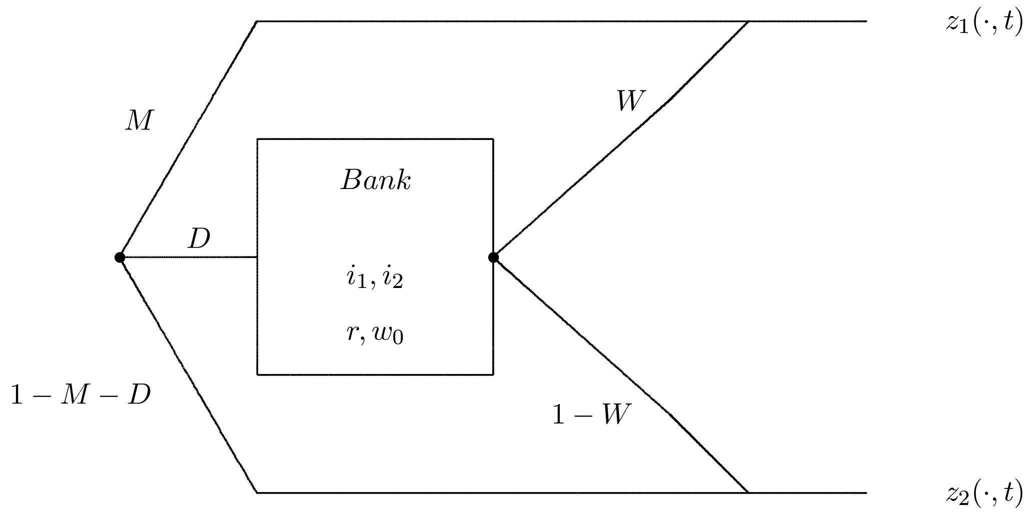

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

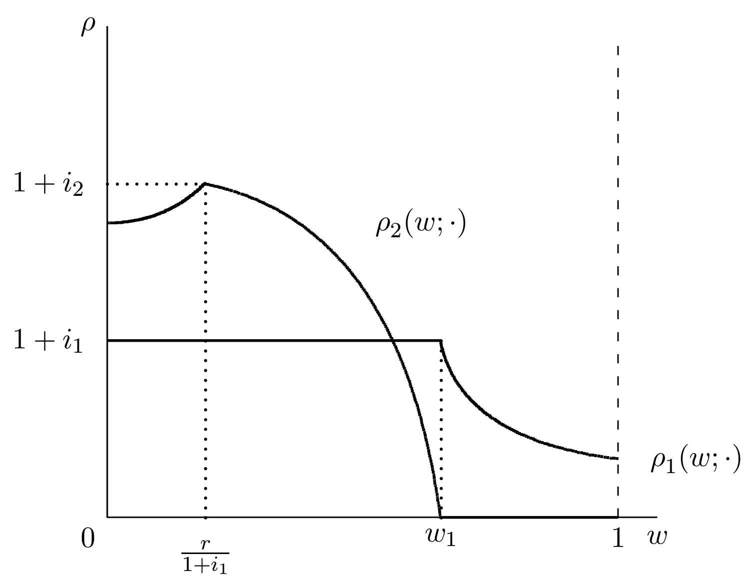

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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ABSTRACT. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. INTRODUCTION

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S. *The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

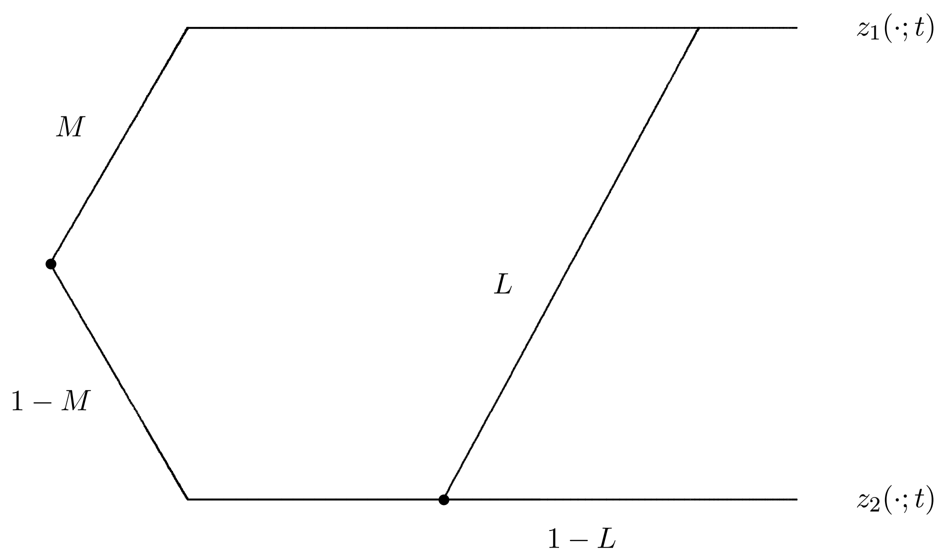

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=& \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) :=& \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

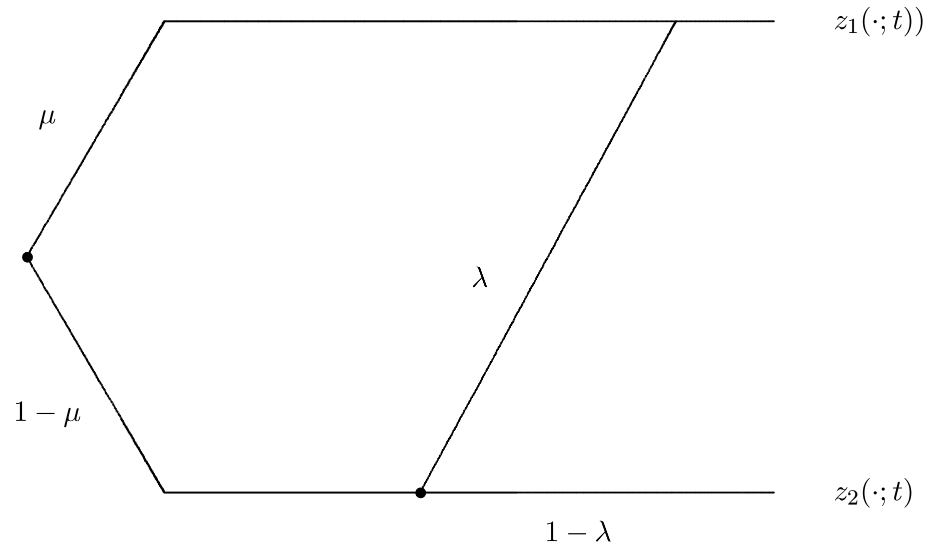

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := &\quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := &\quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{lllll} \max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\ \text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\ & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\ & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\ & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\ & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN} \end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) :=& \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ =& \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

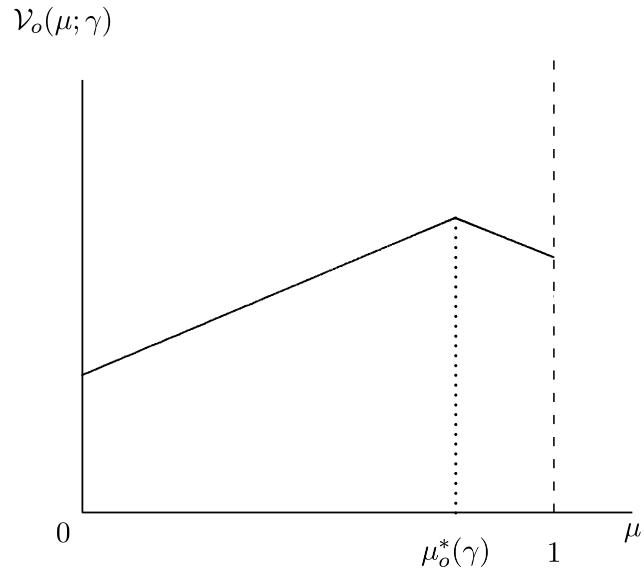

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

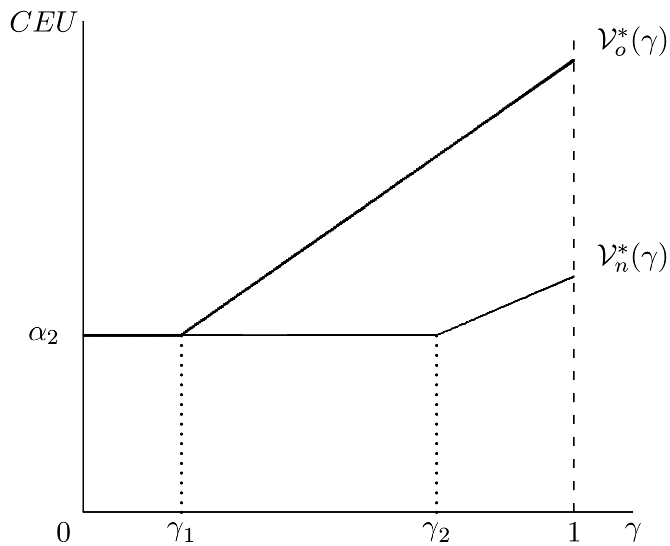

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

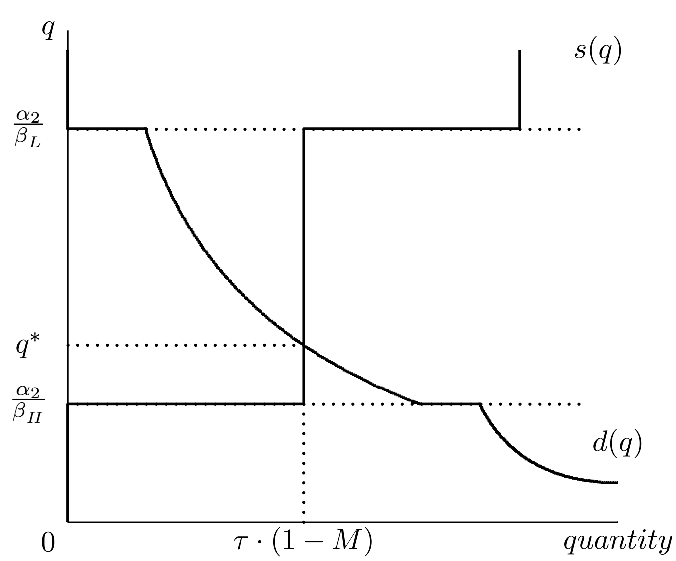

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

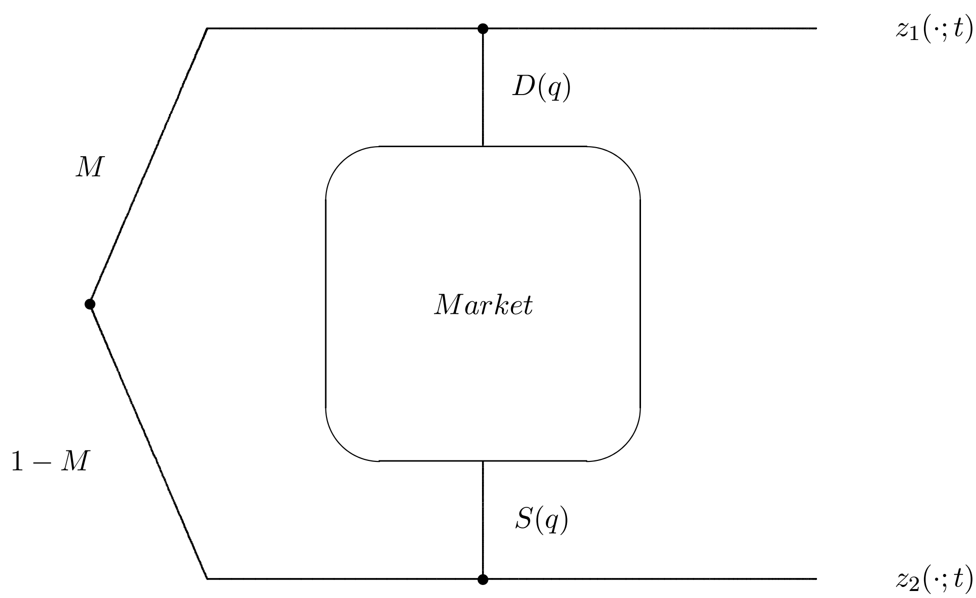

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] \,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

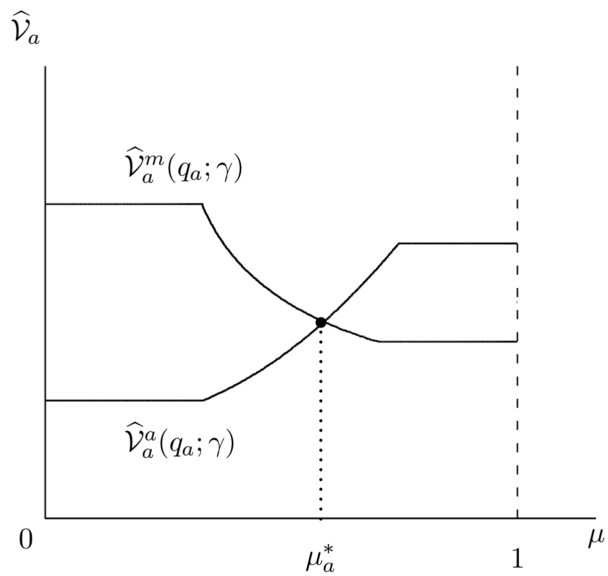

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

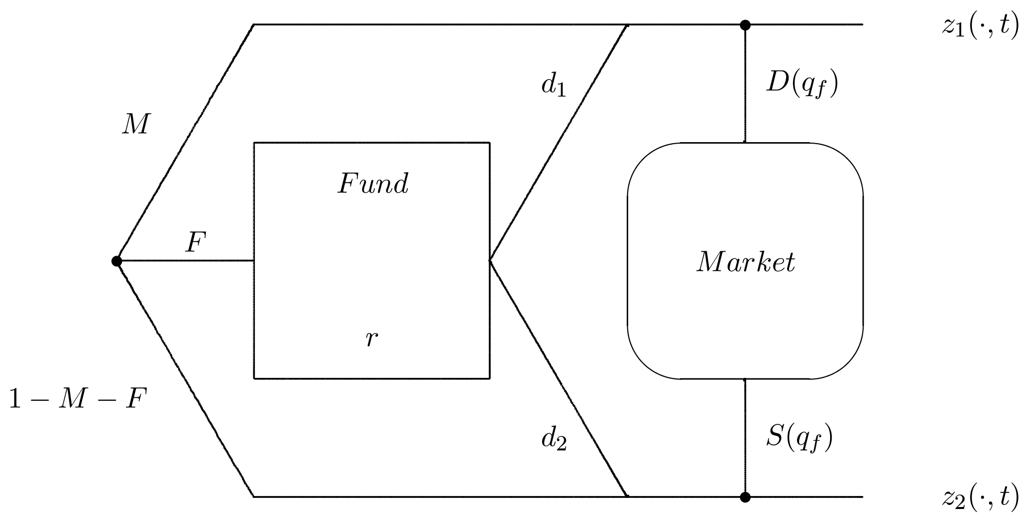

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 :=& \quad r \\ d_2 :=& \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

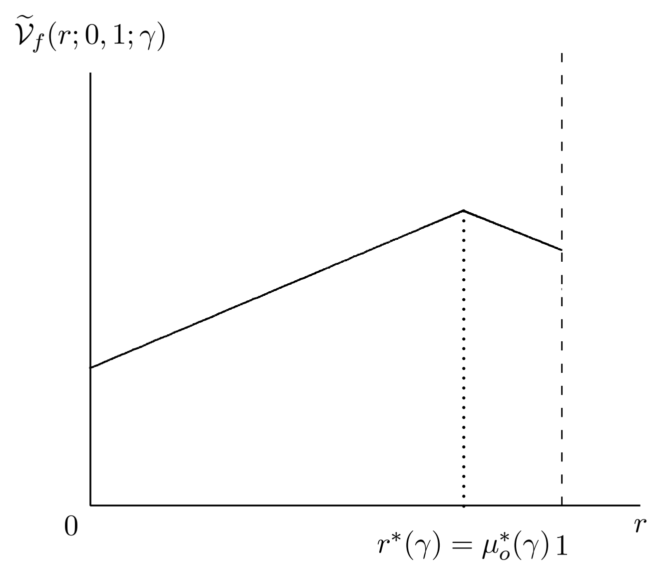

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

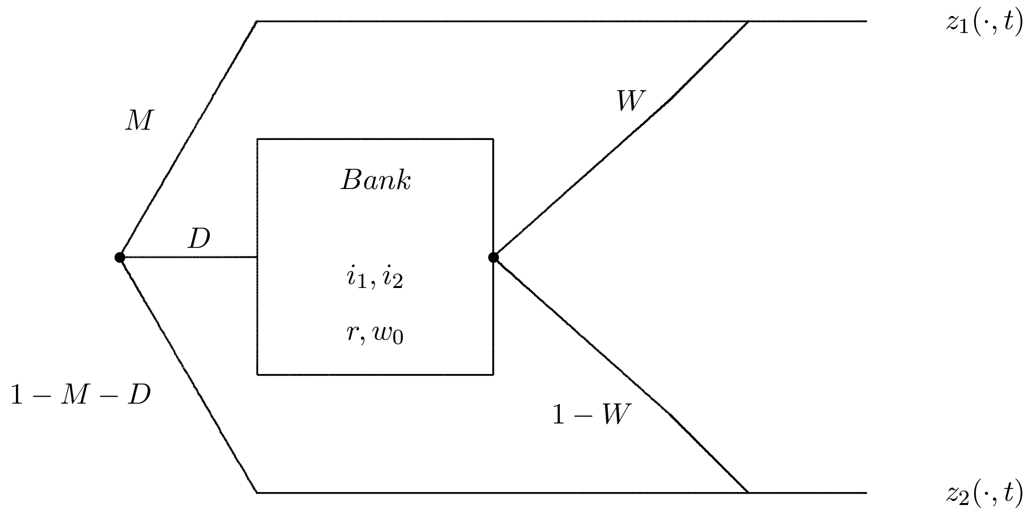

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

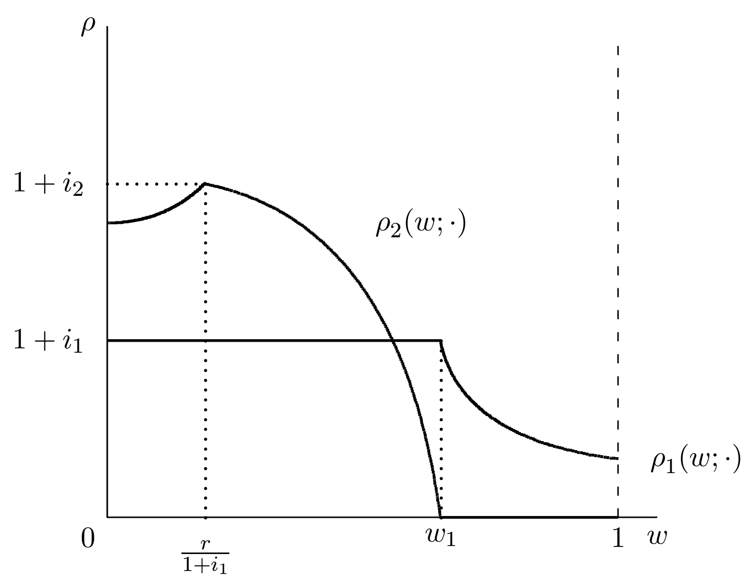

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1 - \tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

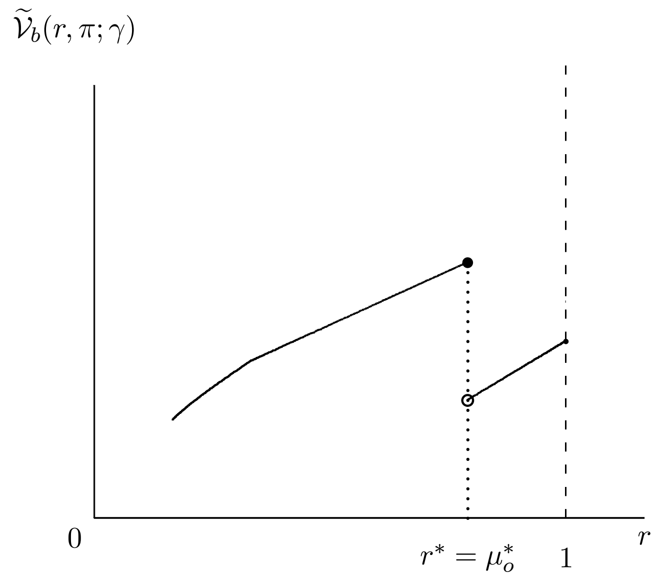

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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ABSTRACT. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. INTRODUCTION

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

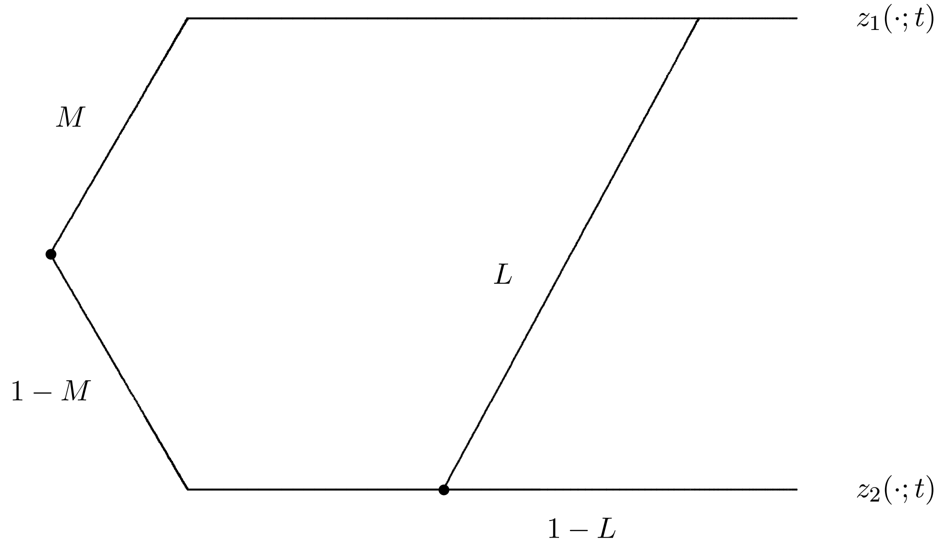

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=& \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) :=& \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

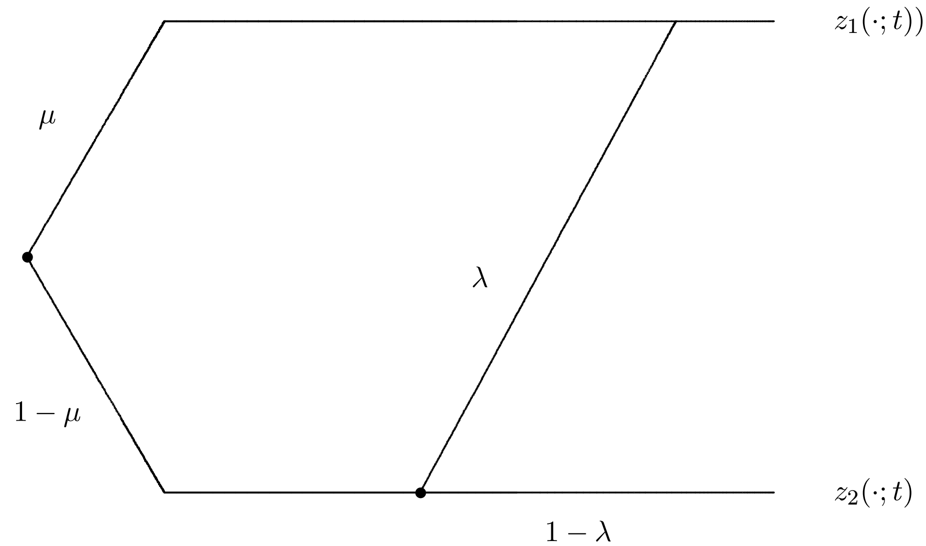

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

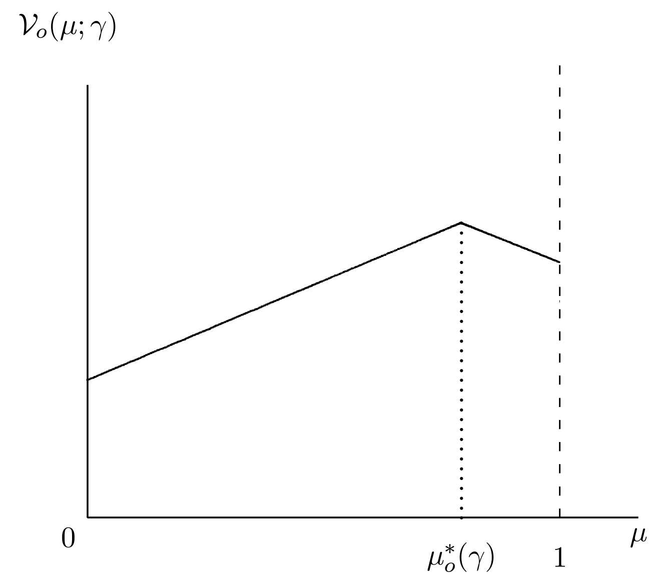

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

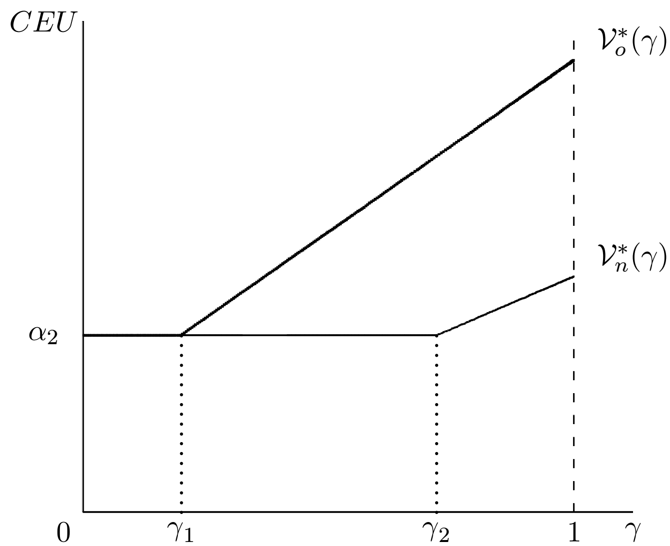

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

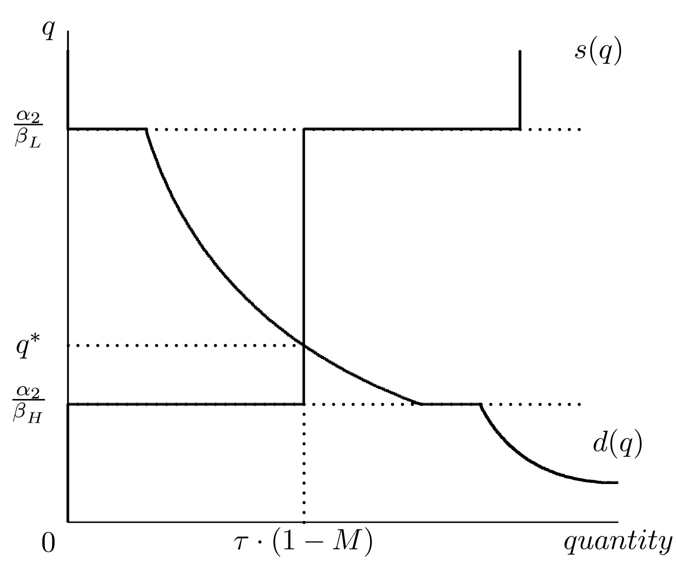

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

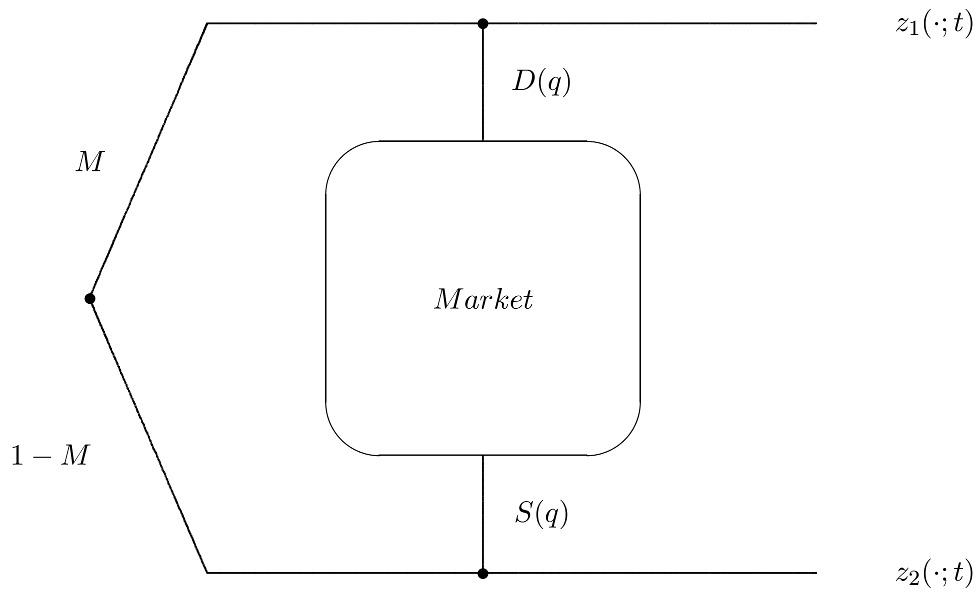

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]}[v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

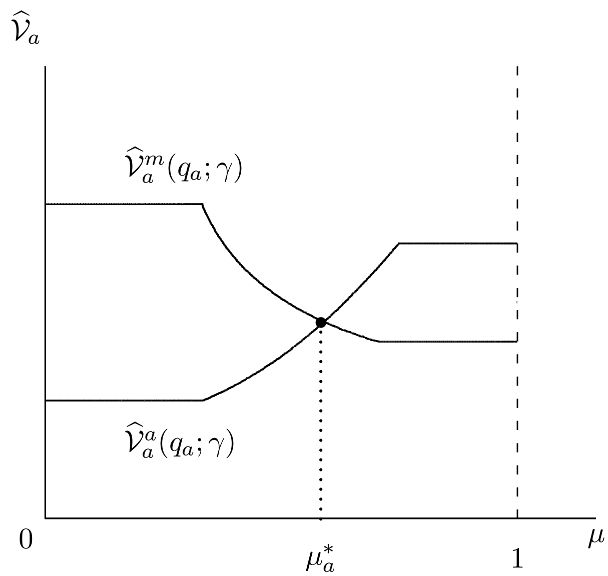

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

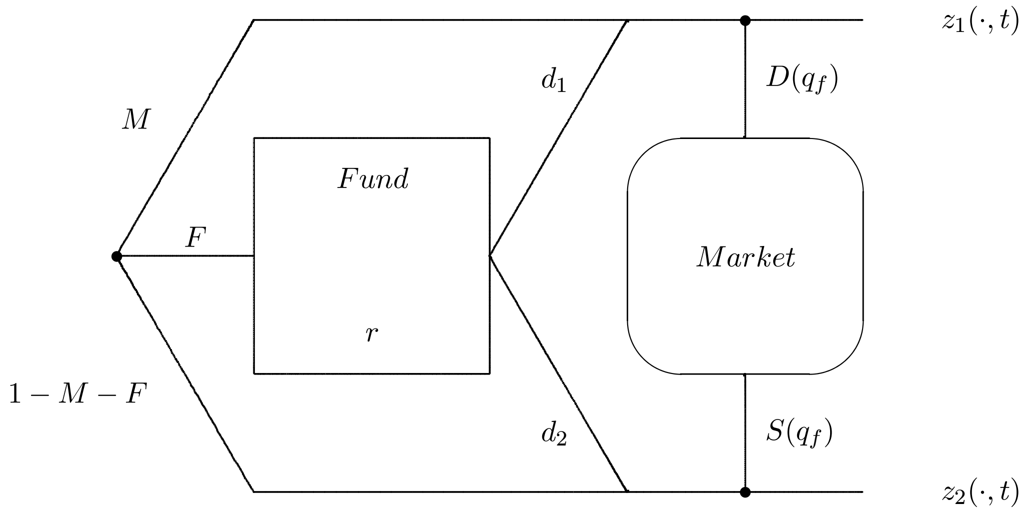

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

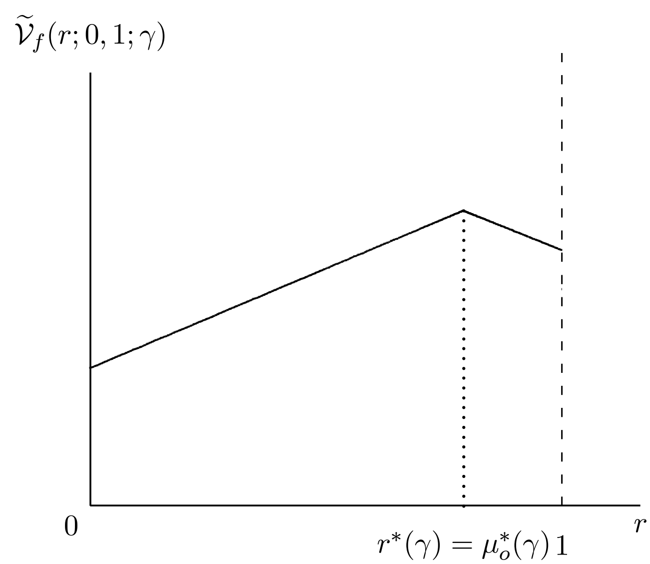

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*
For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26]See Appendix A, Remark 10.
[27]See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

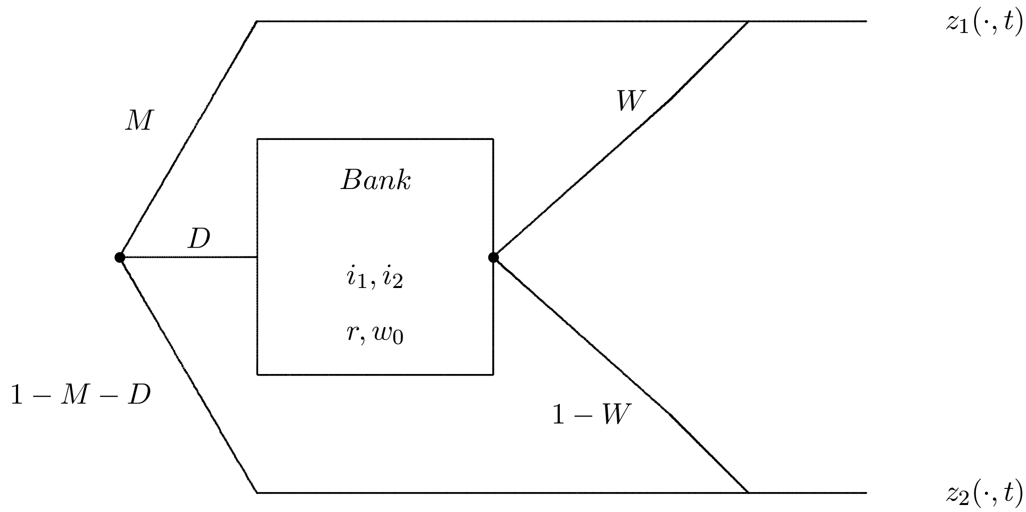

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

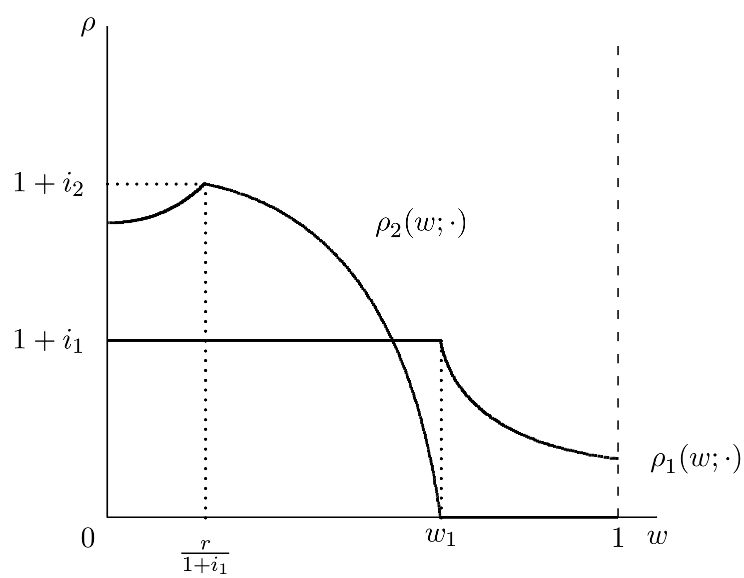

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30]See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

• held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

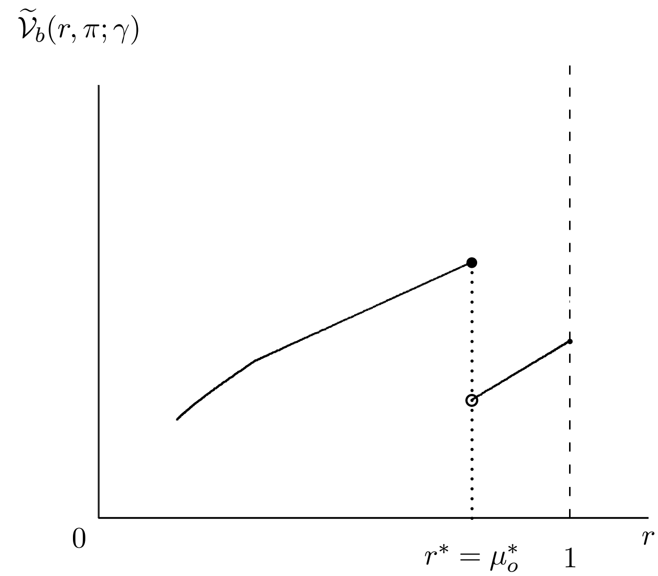

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ =& \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. BANKS, MUTUAL FUNDS, OR ASSET MARKETS?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$*, where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

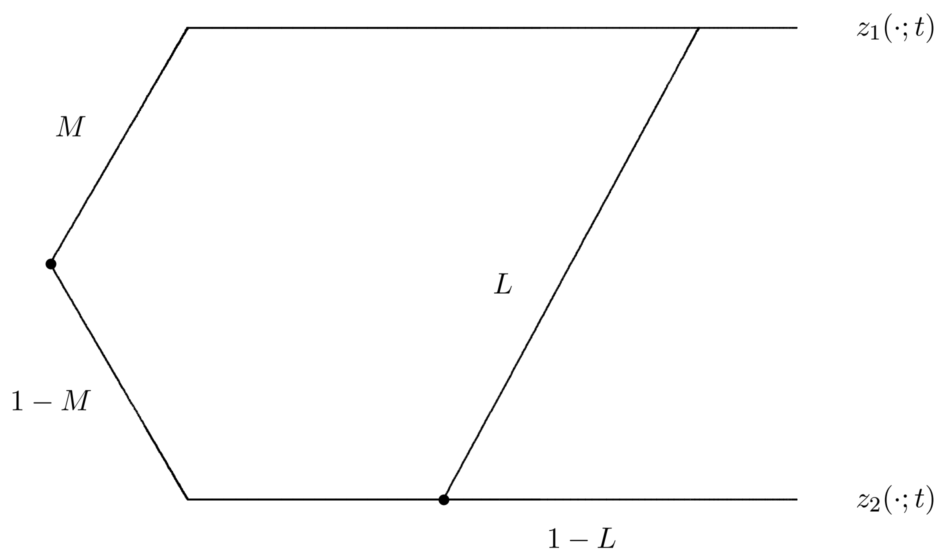

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\; d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

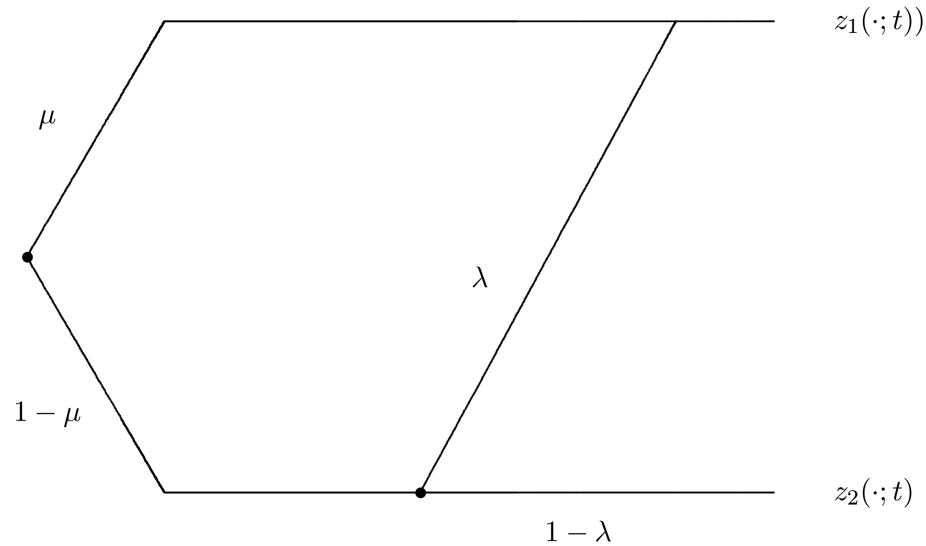

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll} \max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\ \text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \text{S}_h \\ & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \text{S}_\ell \\ & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \text{F}_1 \\ & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \text{F}_2 \\ & \multicolumn{3}{l}{\lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0.} & \text{NN} \end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) :=& \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ =& \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

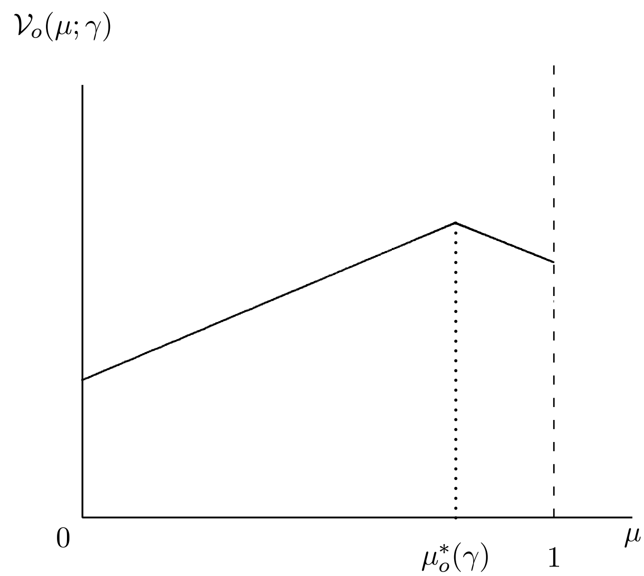

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

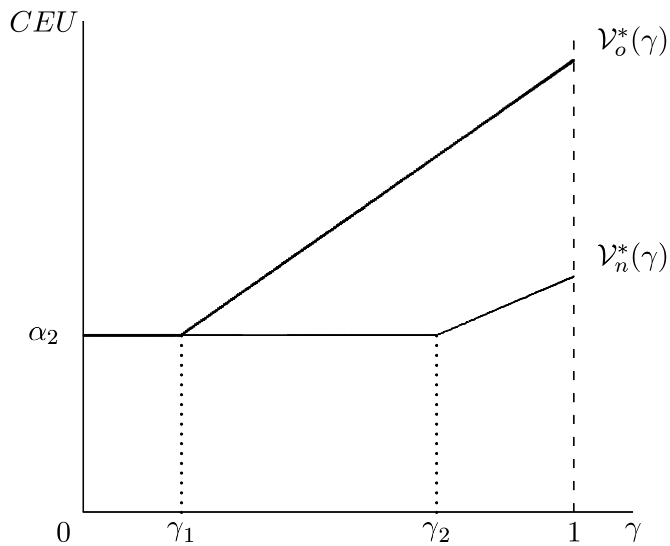

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

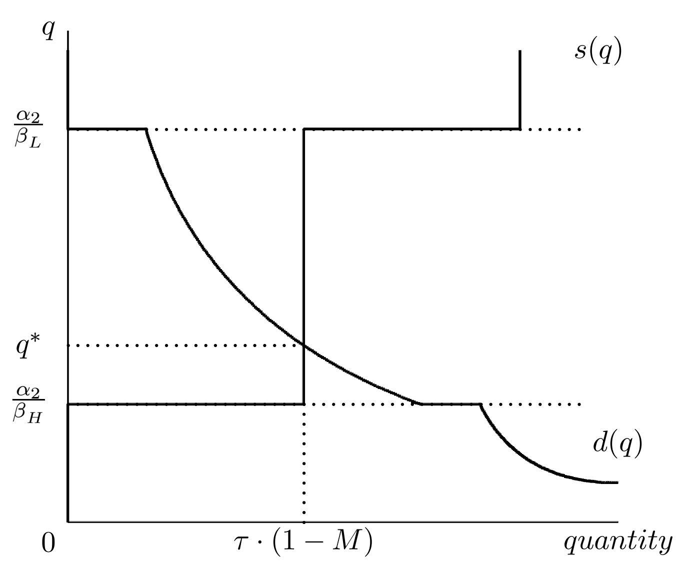

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

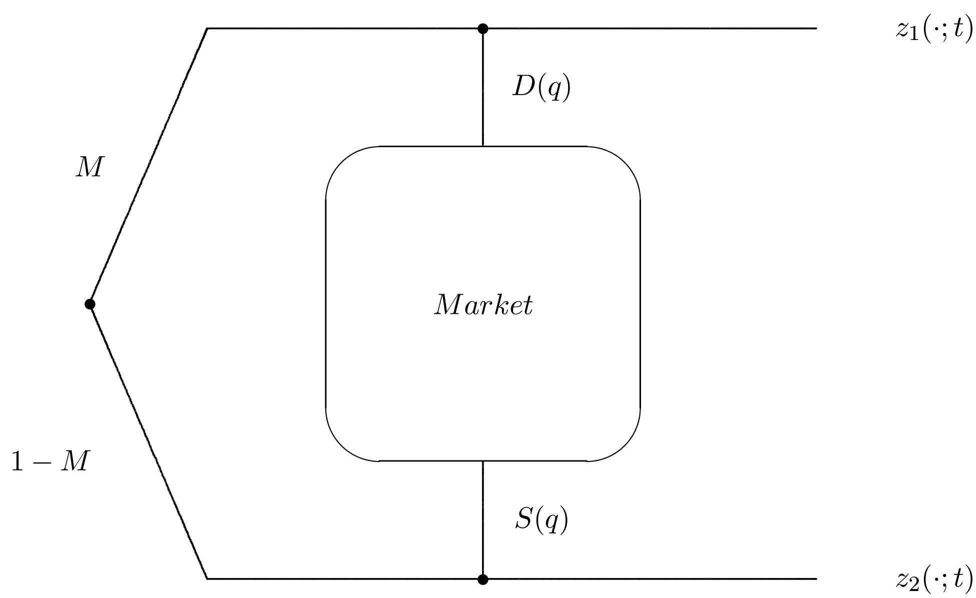

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned} \mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] \,. \end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0,1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi,$
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi),$
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}].$

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

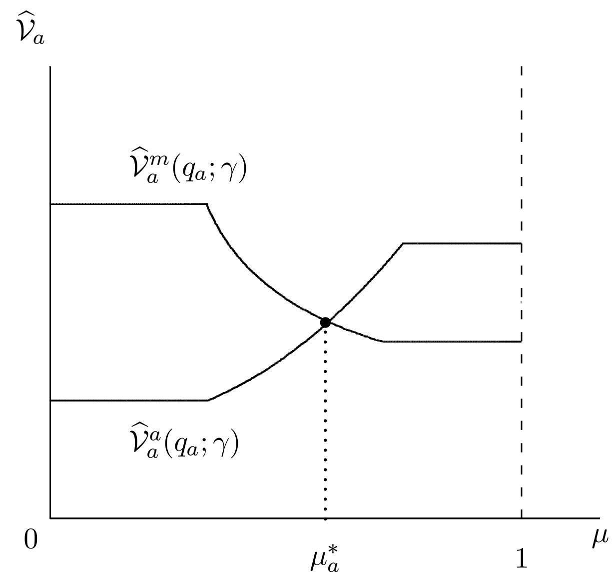

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

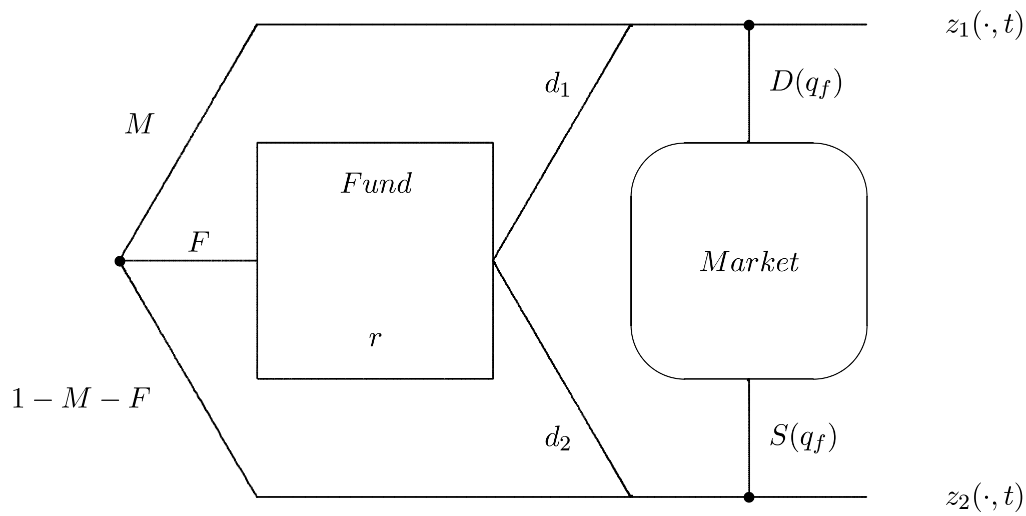

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

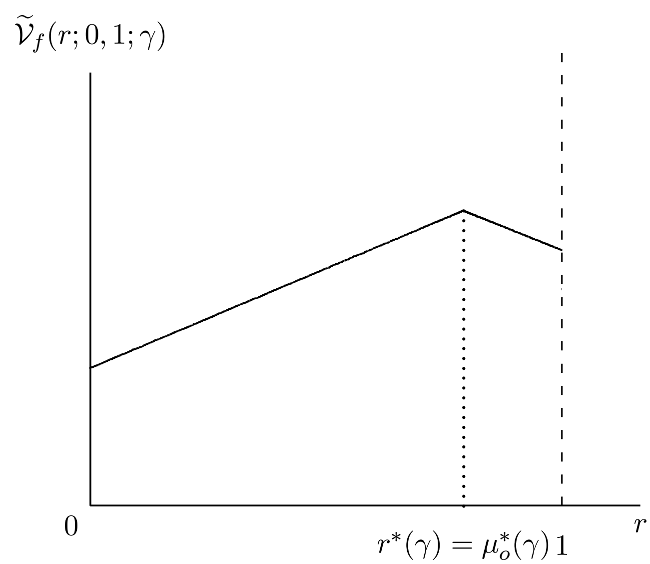

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

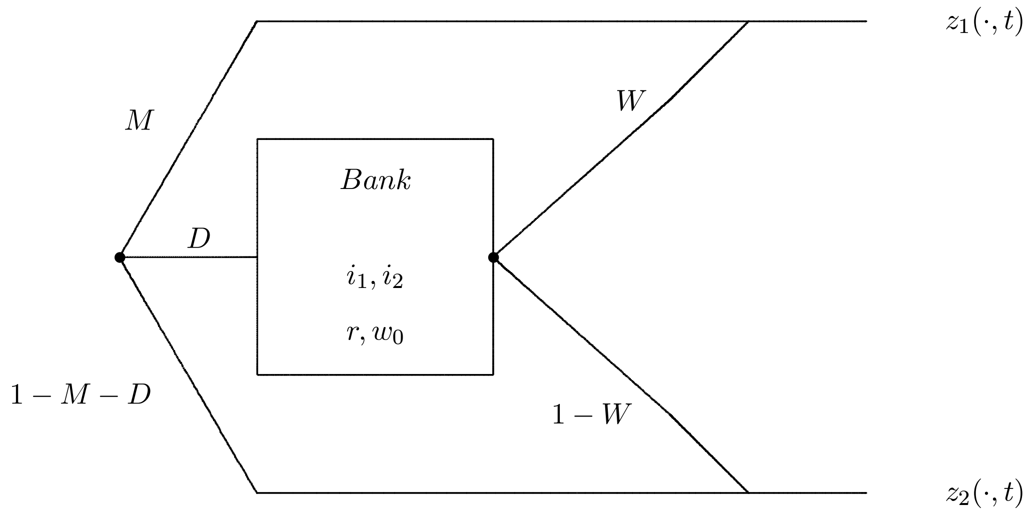

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

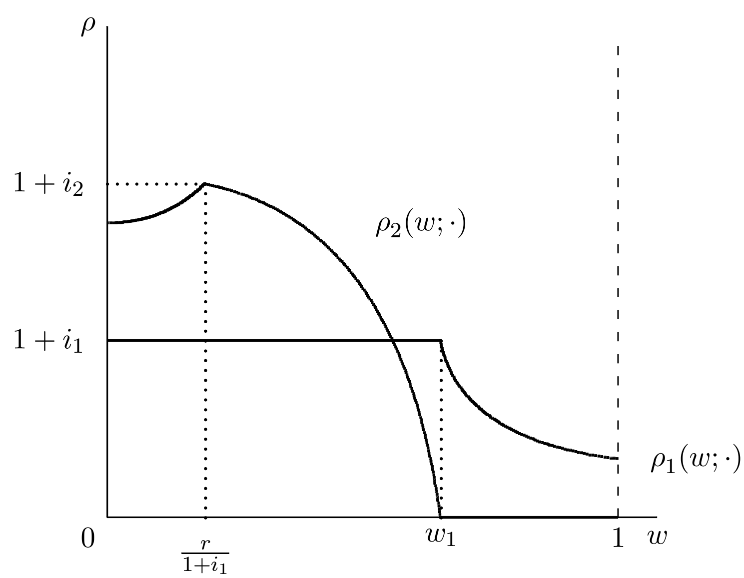

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}&\mathcal{V}_b(D, M; r, w_0; \gamma)\\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \mathcal{V}_b(D, M; r, w_0; \gamma) \quad \text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

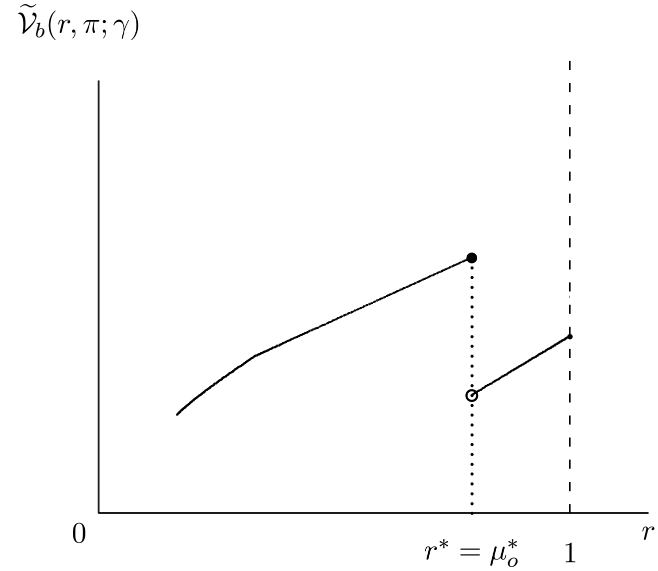

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \; & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \; & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

[1] We would like to thank seminar participants at the Norwegian School of Management (Sandvika), the University of Oslo, the University of Birmingham, the Bundesbank (Frankfurt a.M.), the Free University in Berlin, the University of Frankfurt a.M., Keele University, University of Hohenheim and Chemnitz University of Technology for helpful comments and suggestions.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

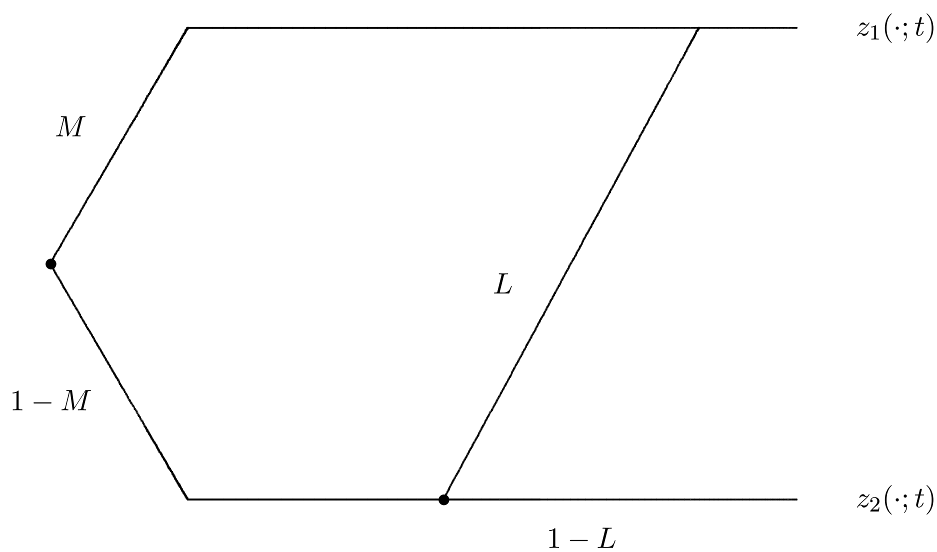

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) &:= \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) &:= \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & \qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & \qquad + (1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

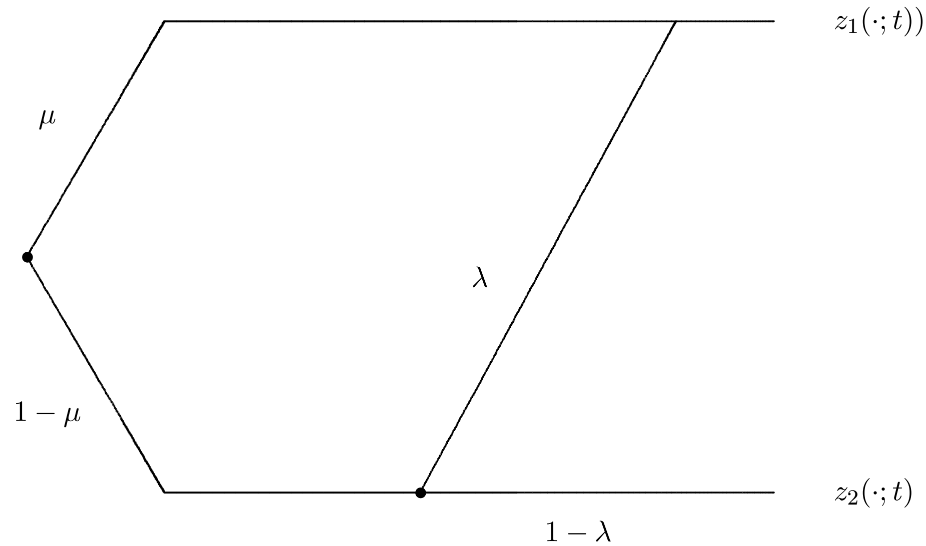

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := & \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := & \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{lllll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

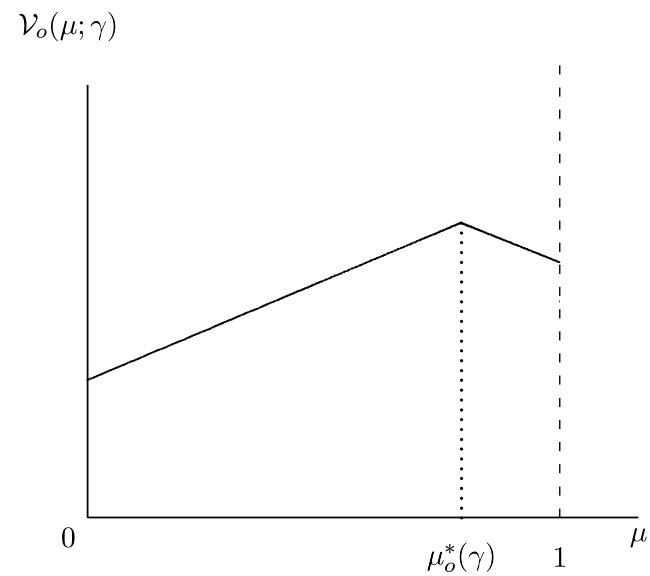

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

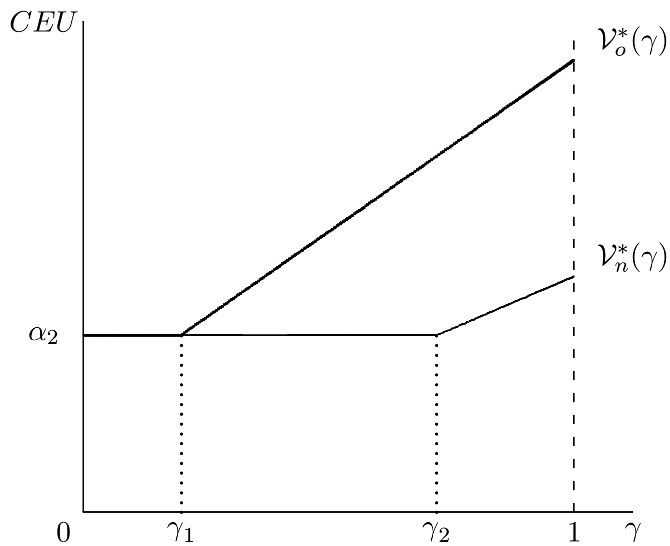

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

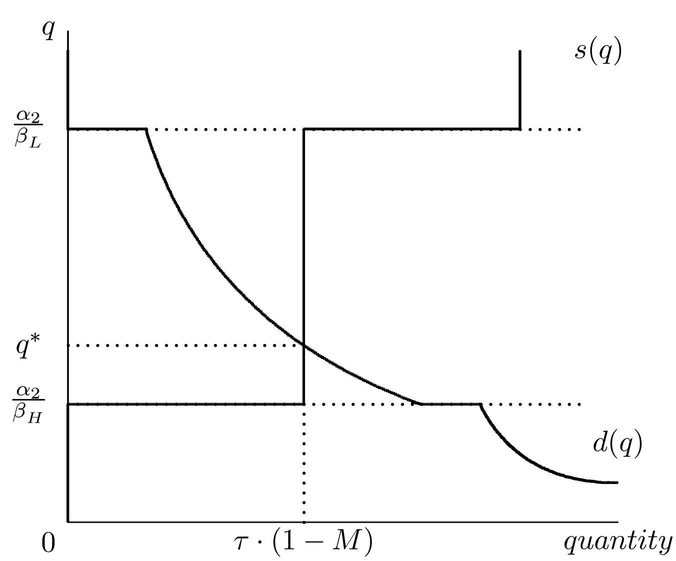

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

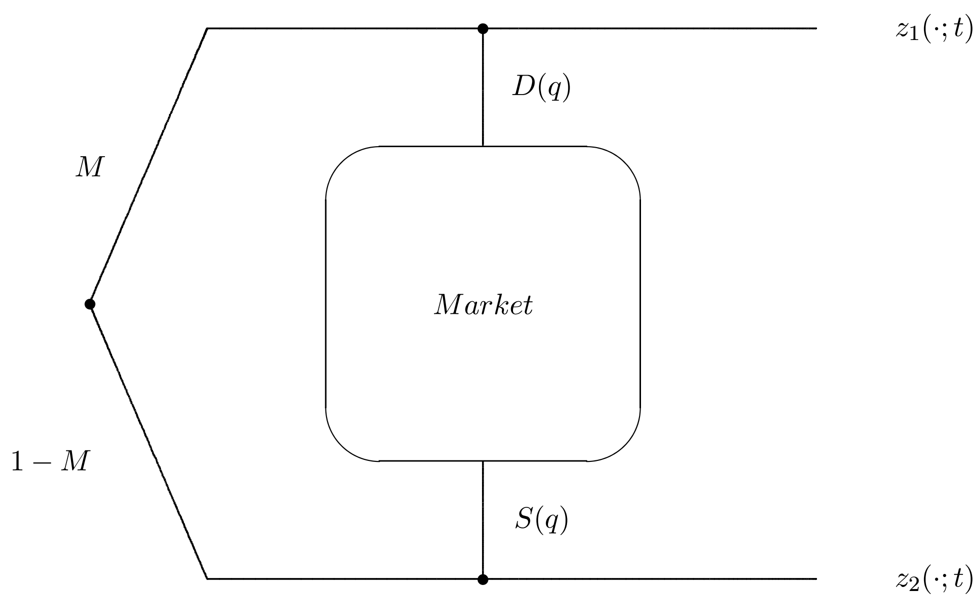

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

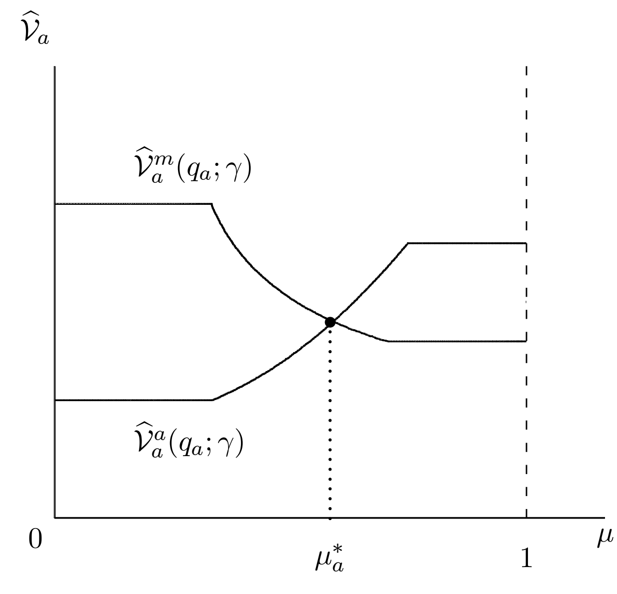

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

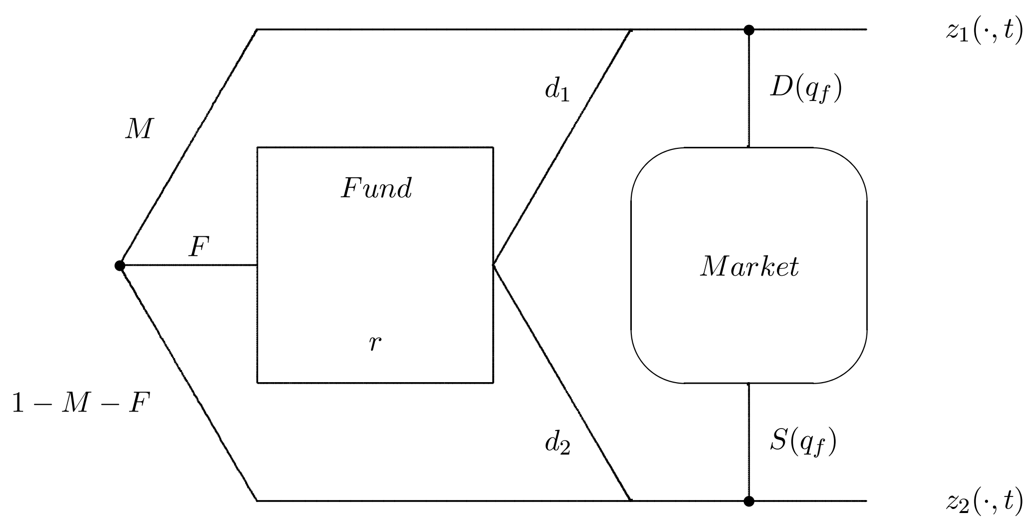

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

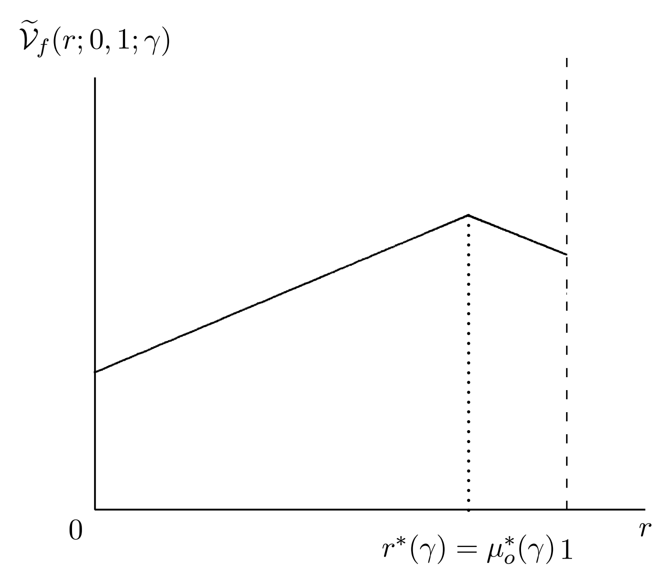

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.
For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

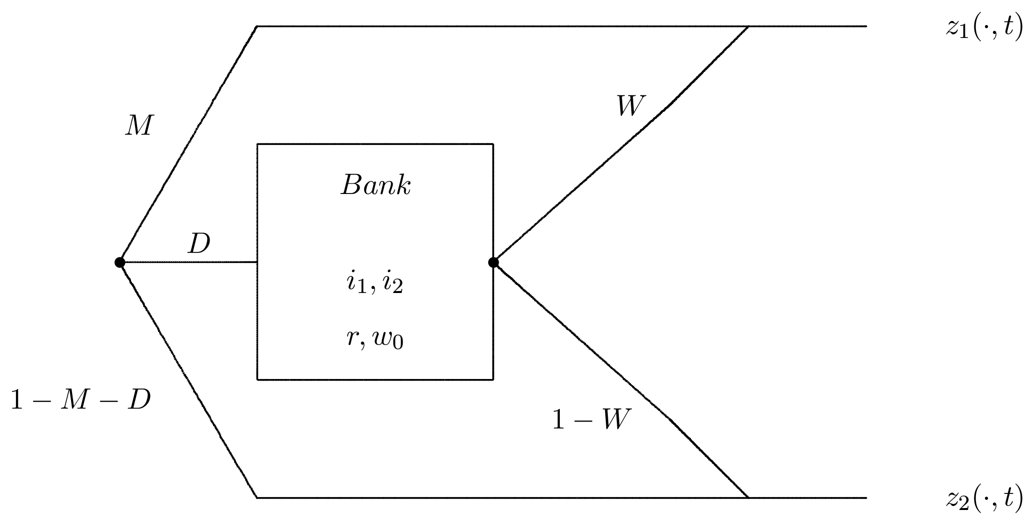

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

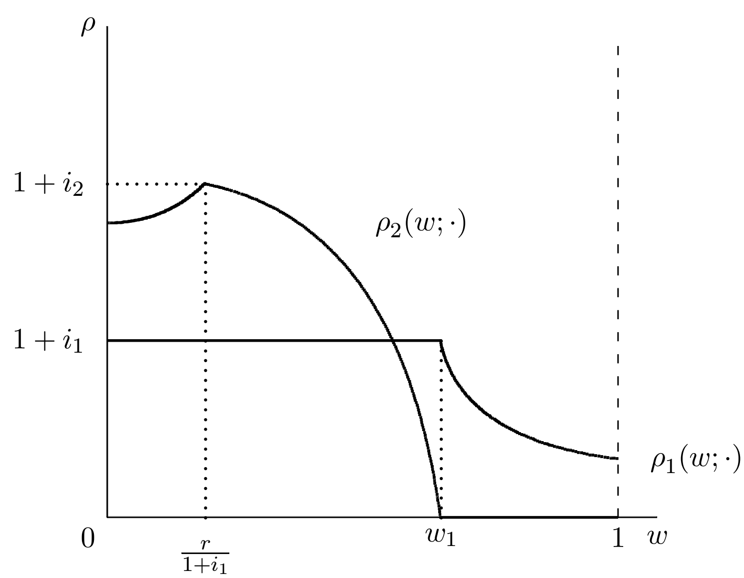

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

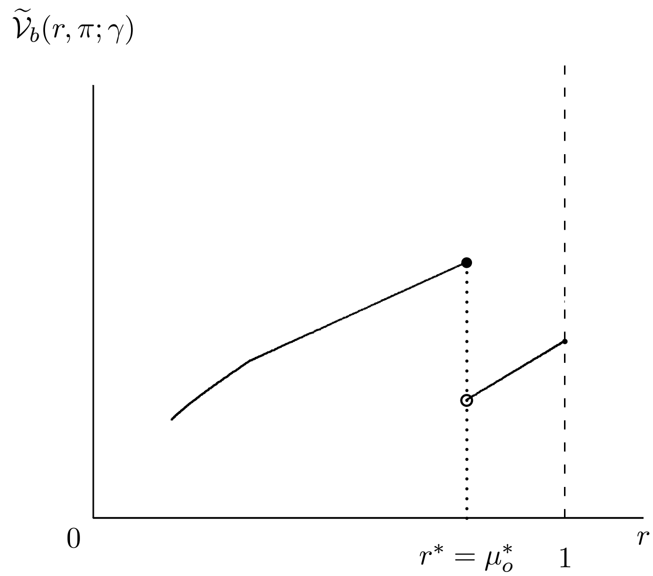

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

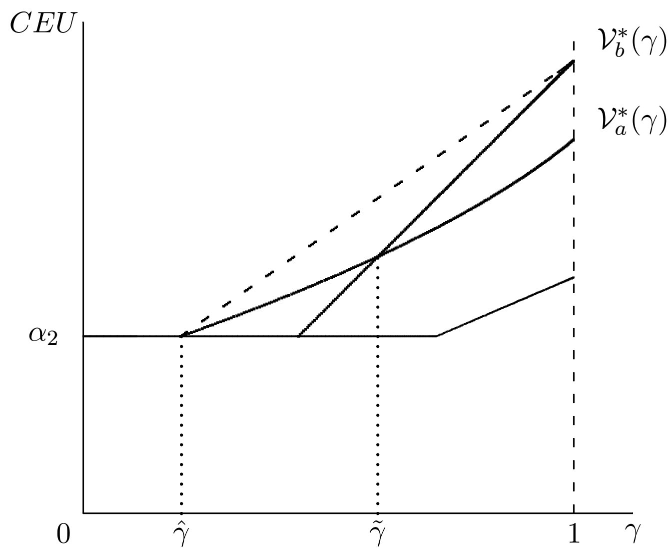

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1 - \pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1 - \pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1 - r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1 - \alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1 - \mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1 - \mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$*, where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

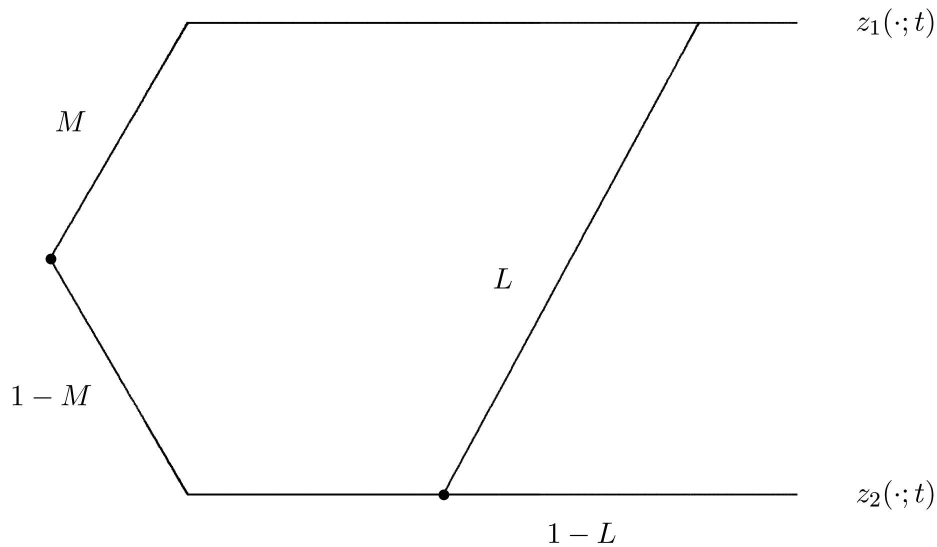

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) &:= \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) &:= \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) :=\ & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ =\ & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau=\pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) \;:=\; \mathcal{V}_n(M_n^*(\gamma);\gamma) \;=\; \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

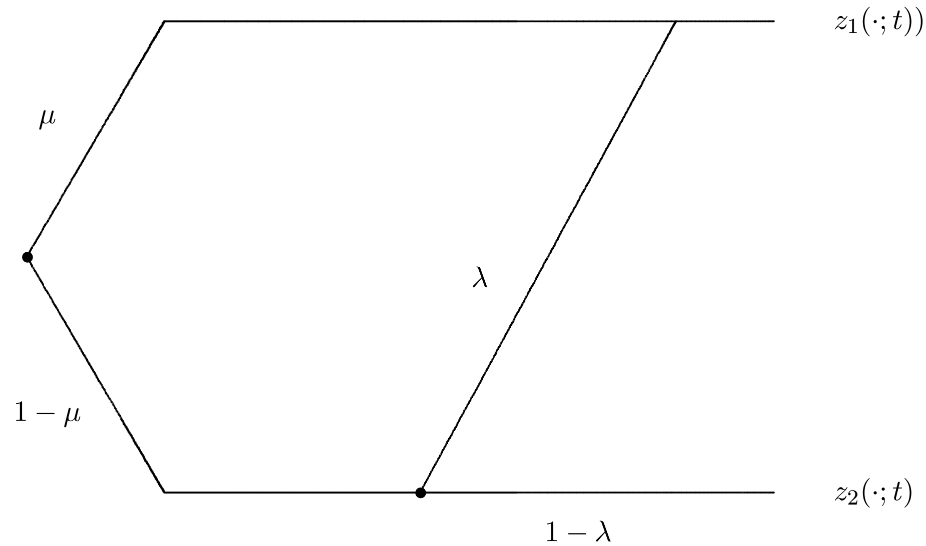

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \text{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \text{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \text{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \text{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \text{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

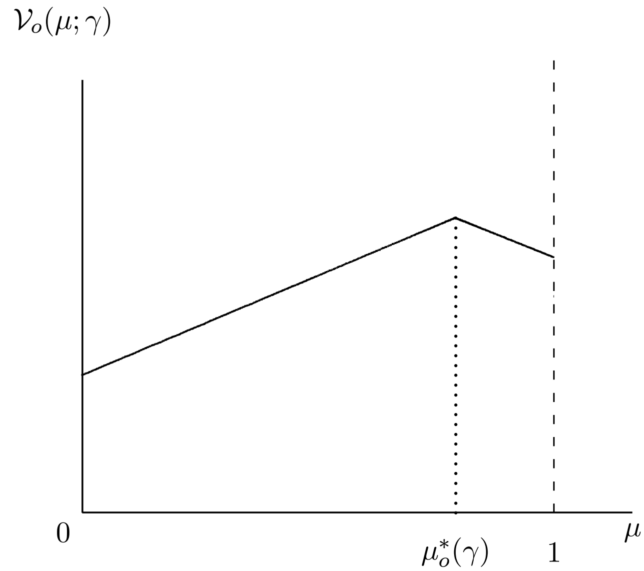

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

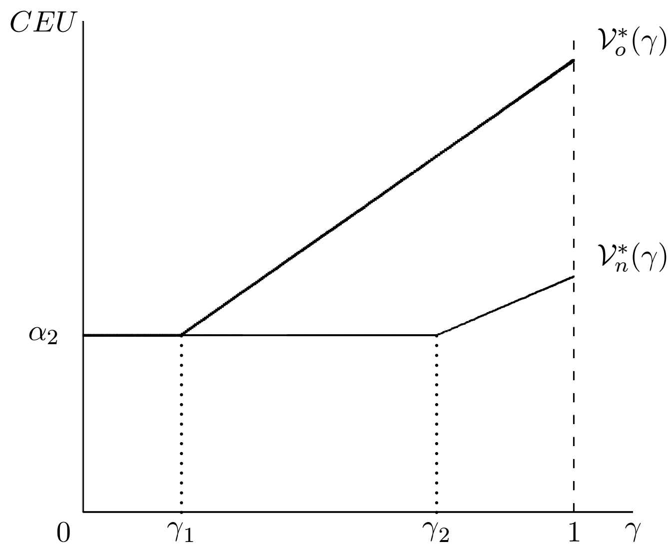

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

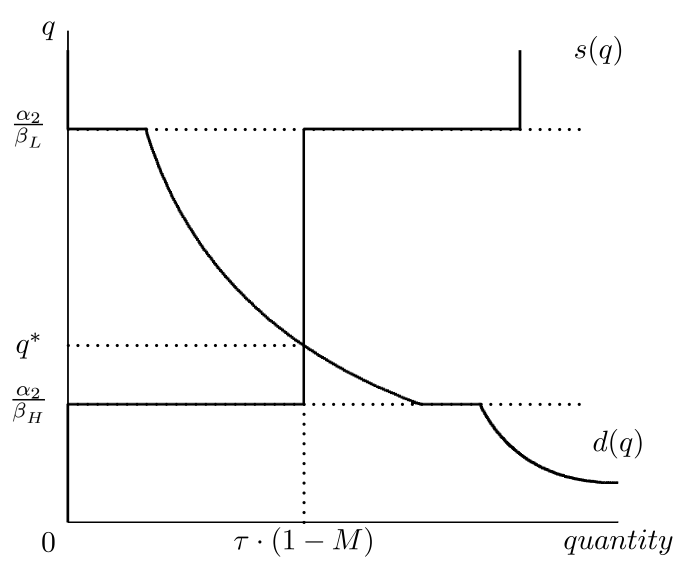

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

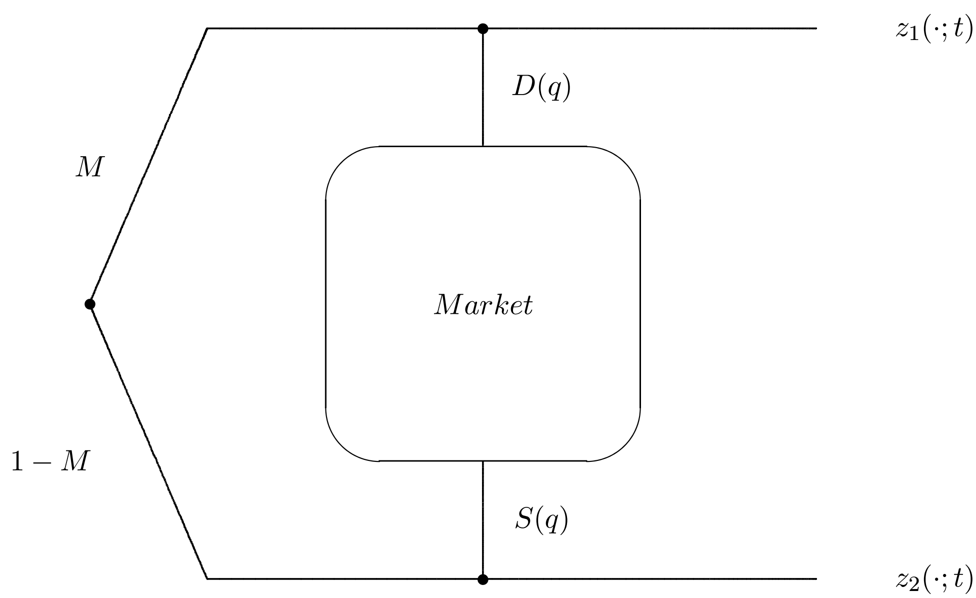

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M) \cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\mathcal{V}_a(M,\mu;\gamma) := \quad \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]}[v_a(M;\tau,\mu;t)].$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

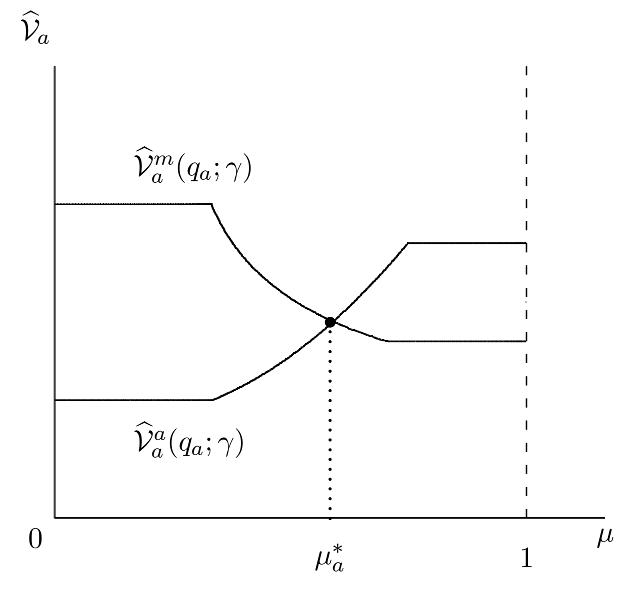

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

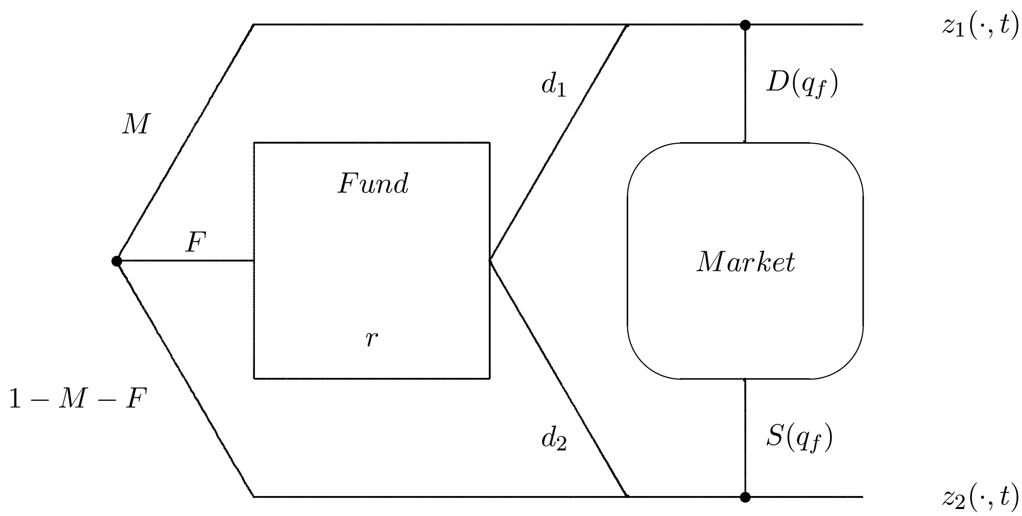

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

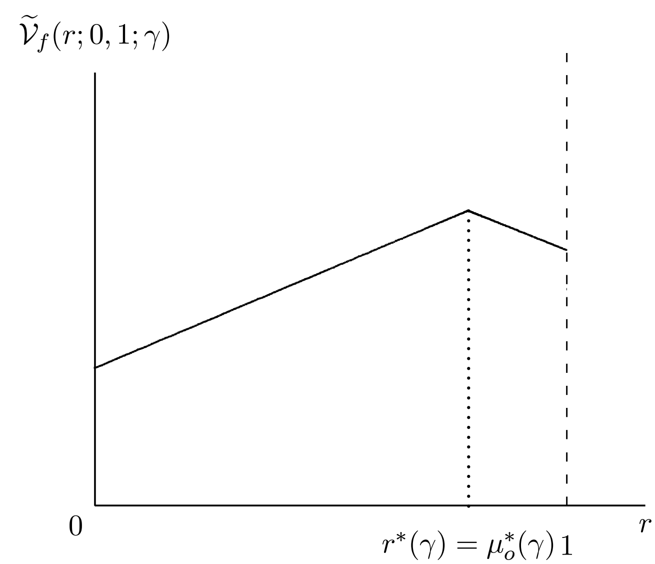

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0, 1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

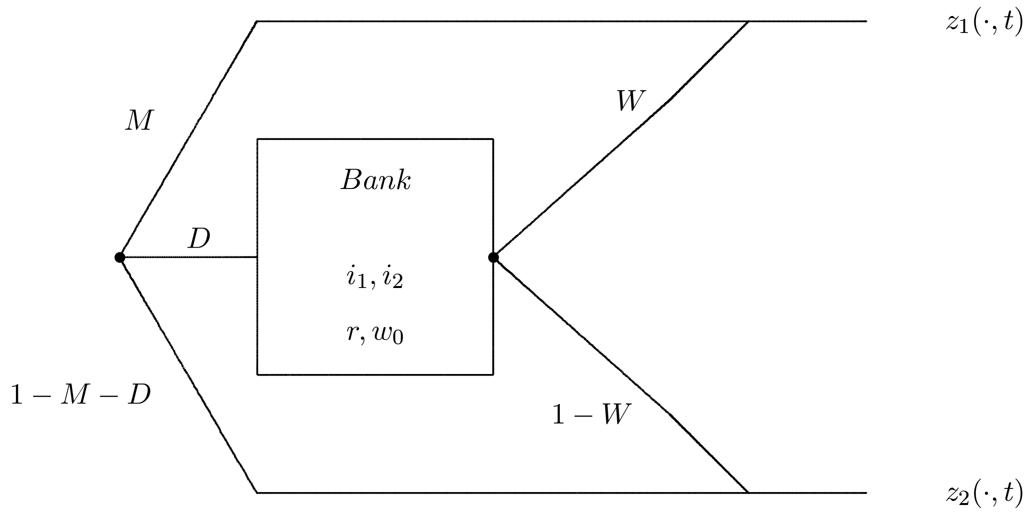

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

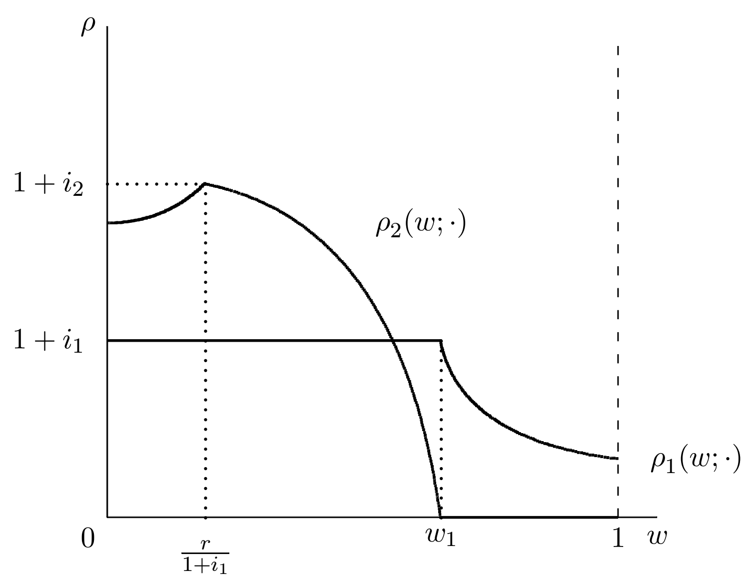

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

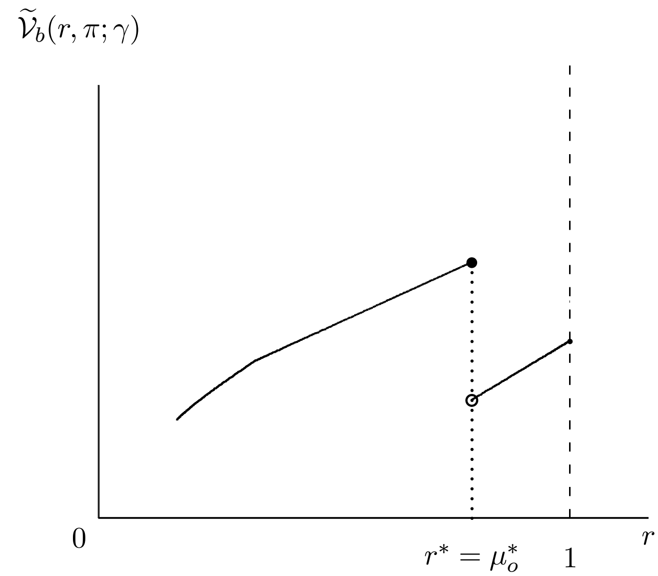

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \ & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \ & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. BANKS, MUTUAL FUNDS, OR ASSET MARKETS?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

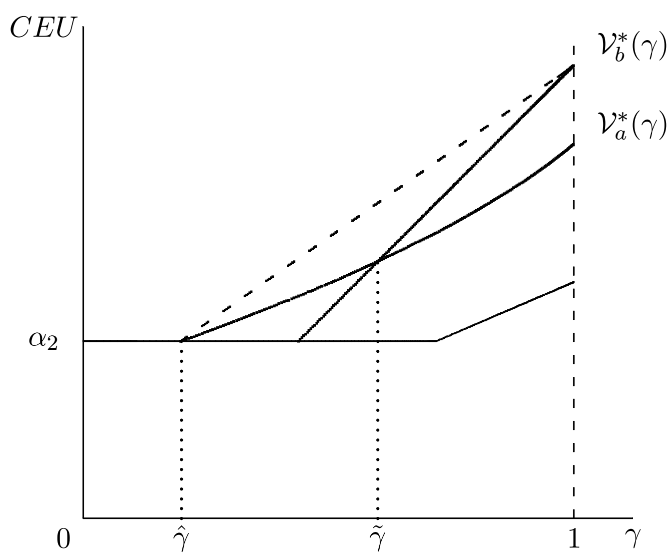

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

First Version: November 1997
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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}_+^2 \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

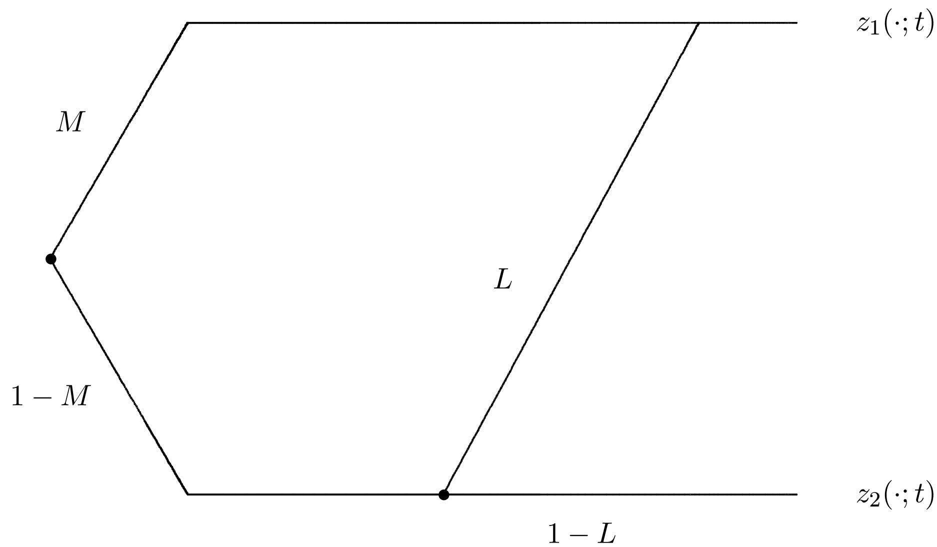

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1-M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) \;:=\; \mathcal{V}_n(M_n^*(\gamma);\gamma) \;=\; \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

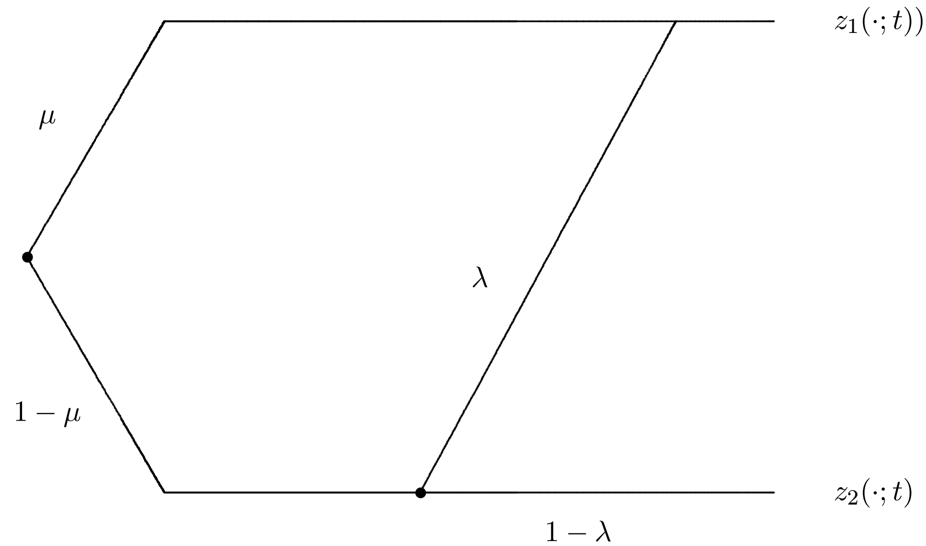

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := & \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := & \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau)\cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu)\cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau)\cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu)\cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) :=&\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ =&\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

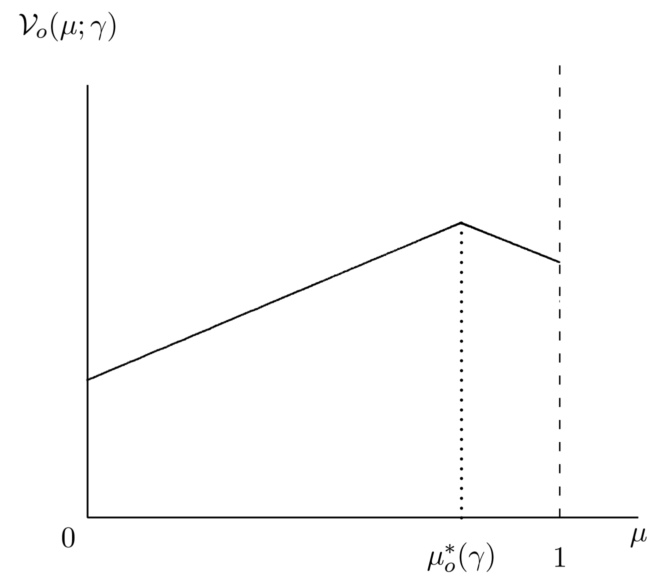

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

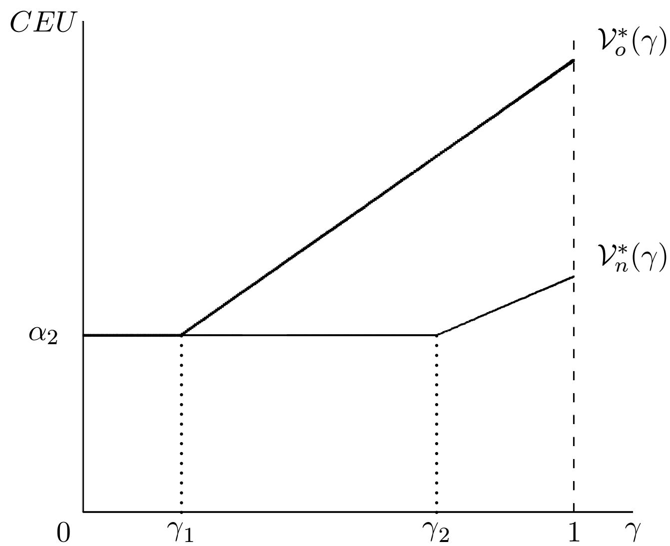

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

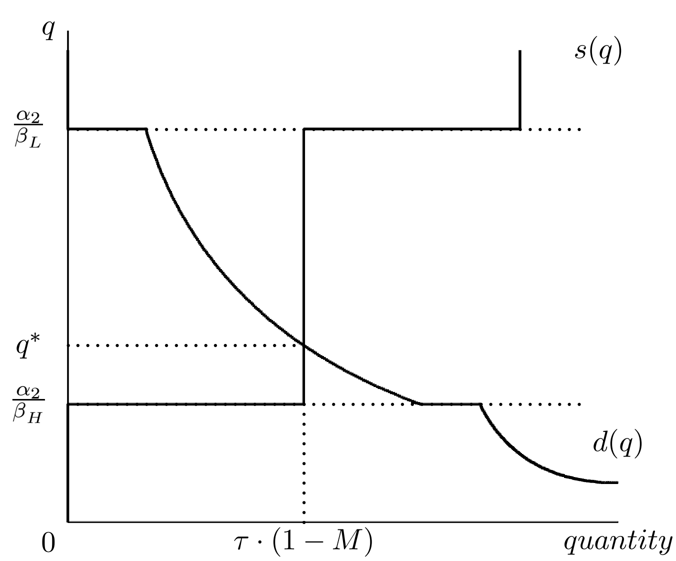

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau, \mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau, \mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

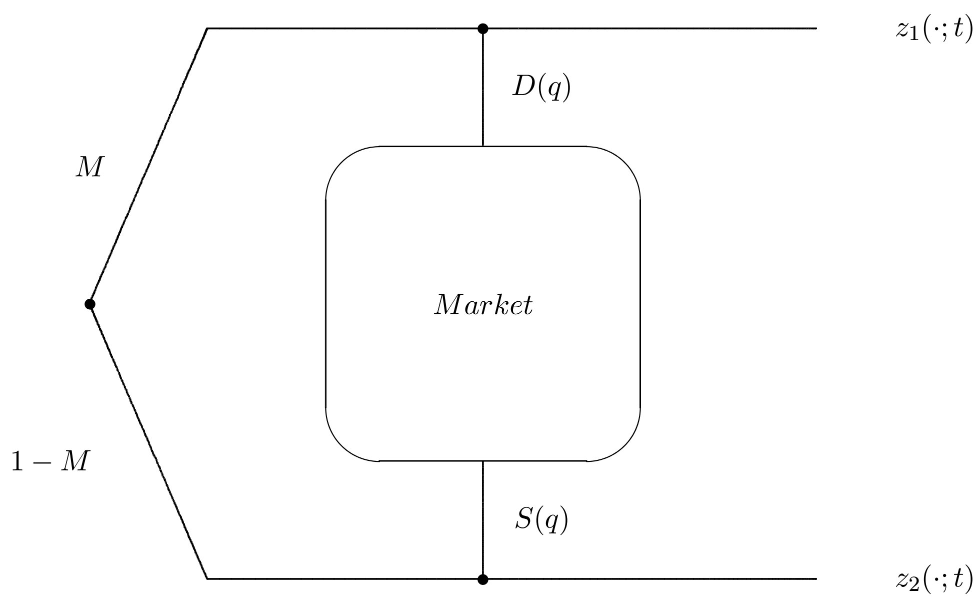

Figure 6: Asset market

Denote by $R^m(q_a; t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a; t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M; \tau, \mu; t)$, from the investment decision M is easily computed:

$$v_a(M; \tau, \mu; t) := M \cdot R^m(q_a^E(\tau, \mu); t) + (1 - M) \cdot R^a(q_a^E(\tau, \mu); t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma = 1,$ *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi,$
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1),$ *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi),$
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

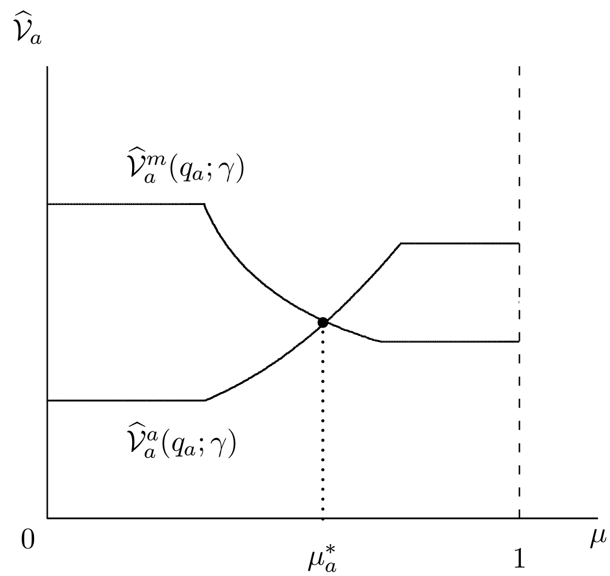

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

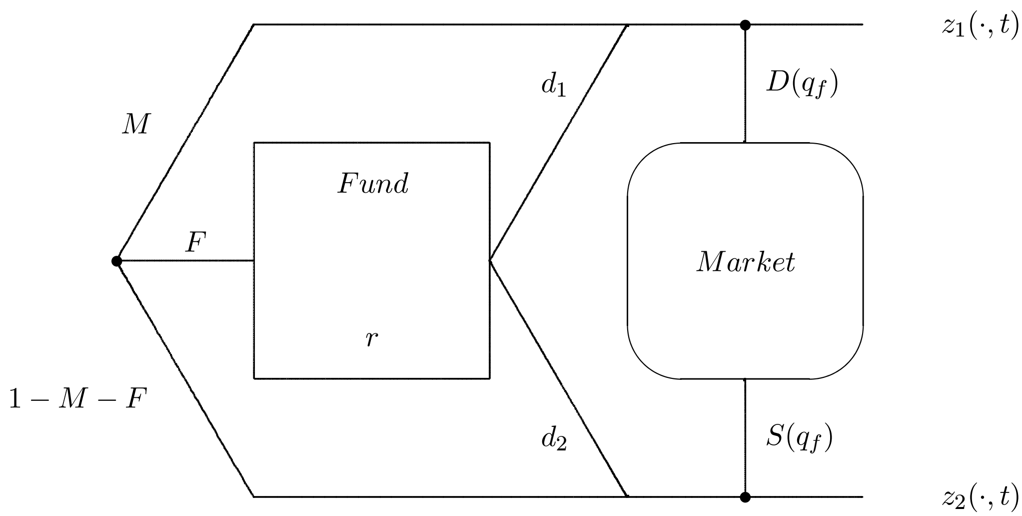

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by[24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

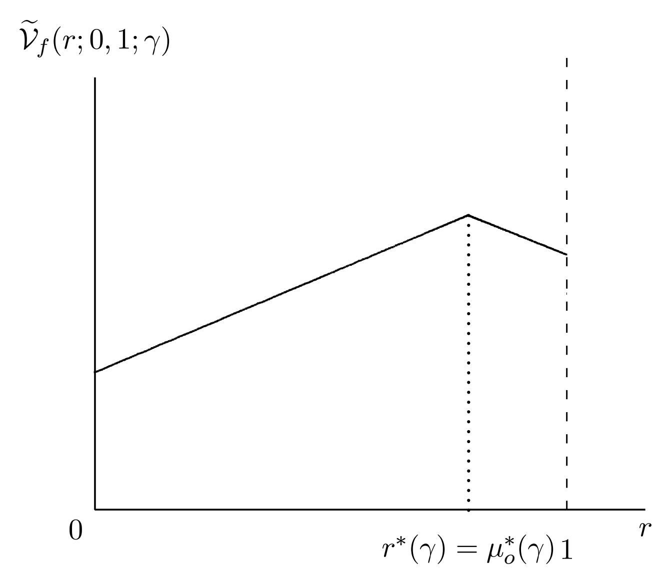

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*
For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

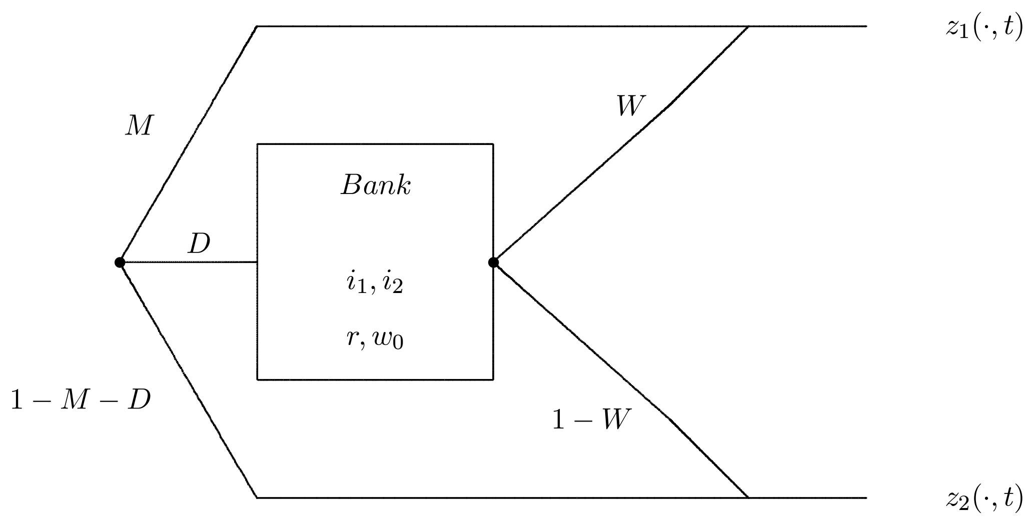

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

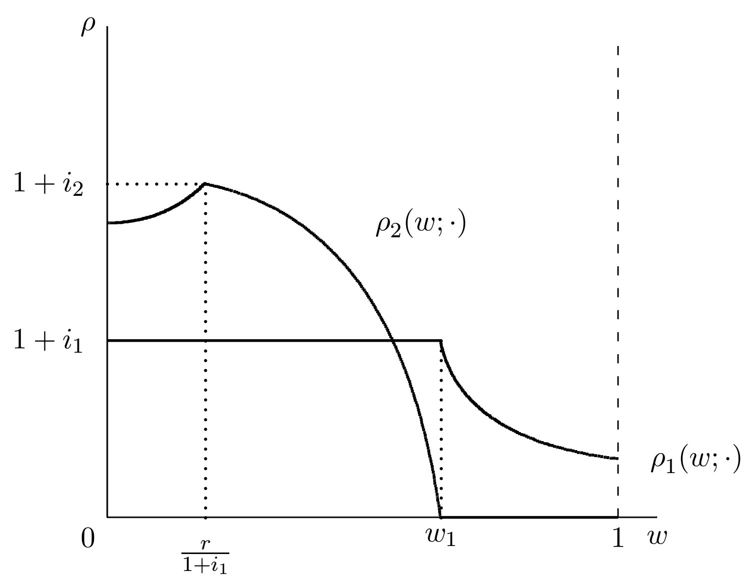

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) :=& \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ =& \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

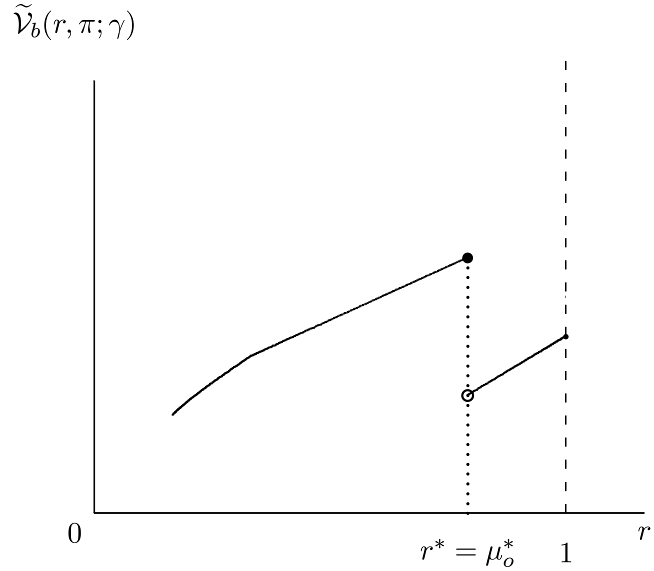

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

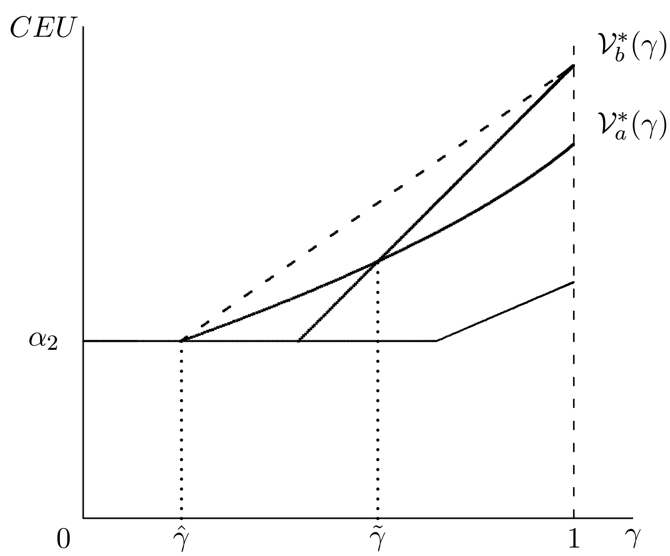

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.
[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}_+^2 \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

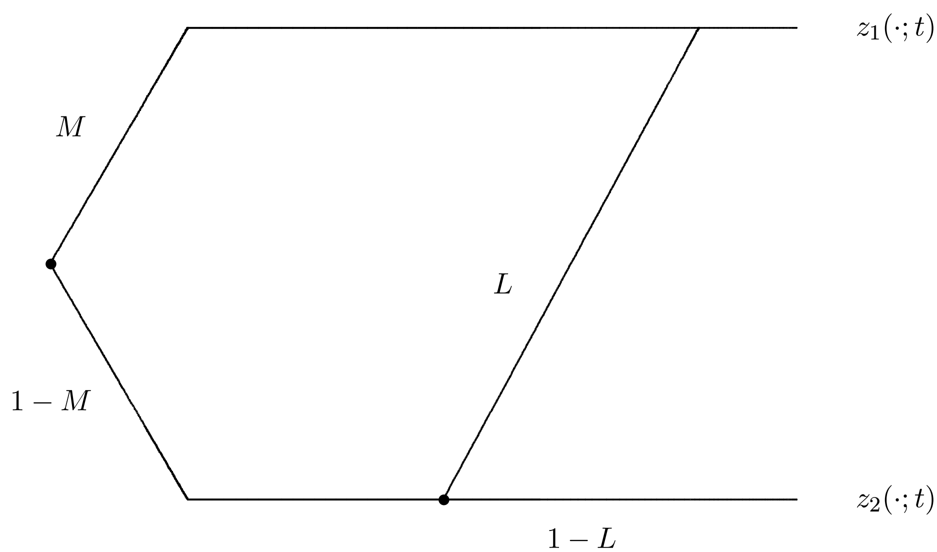

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{array}{rl} z_1(M, L) := & M + \alpha_1 \cdot L \cdot (1 - M) \quad \text{and} \\ z_2(M, L) := & \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & \qquad +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & \qquad +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

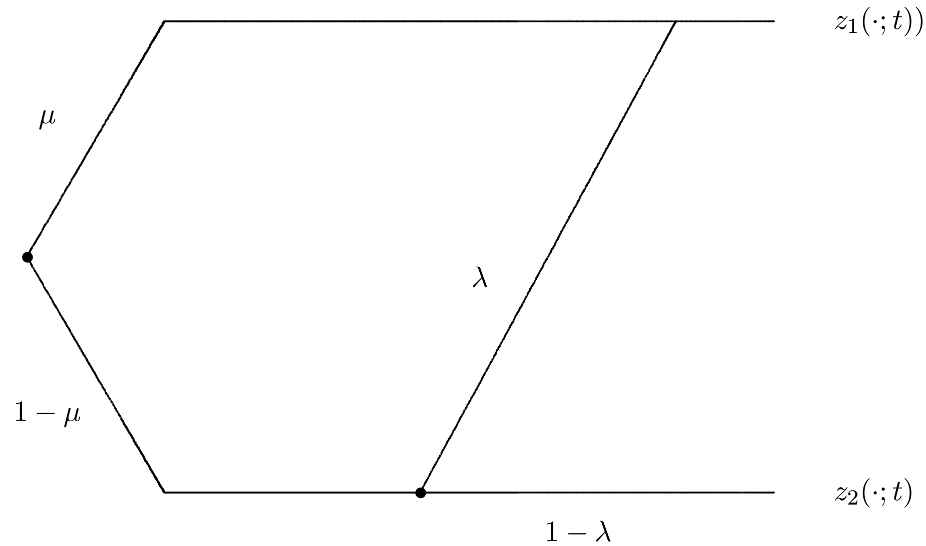

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := &\quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := &\quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \multicolumn{3}{l}{\lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0.} & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := &\quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = &\quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

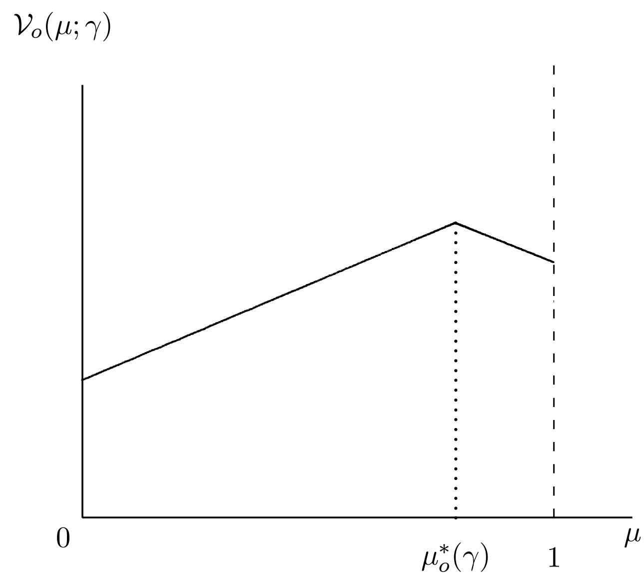

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

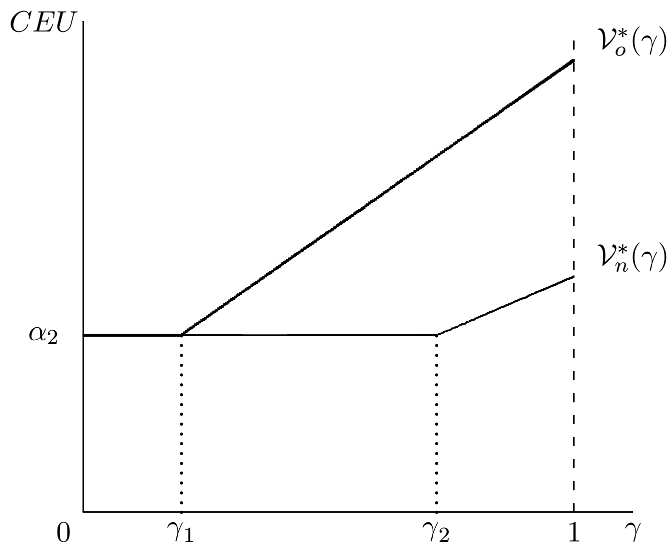

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

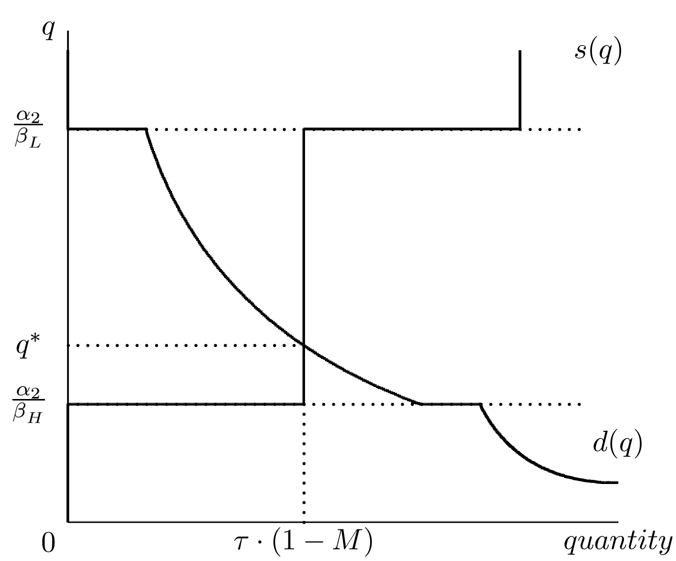

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

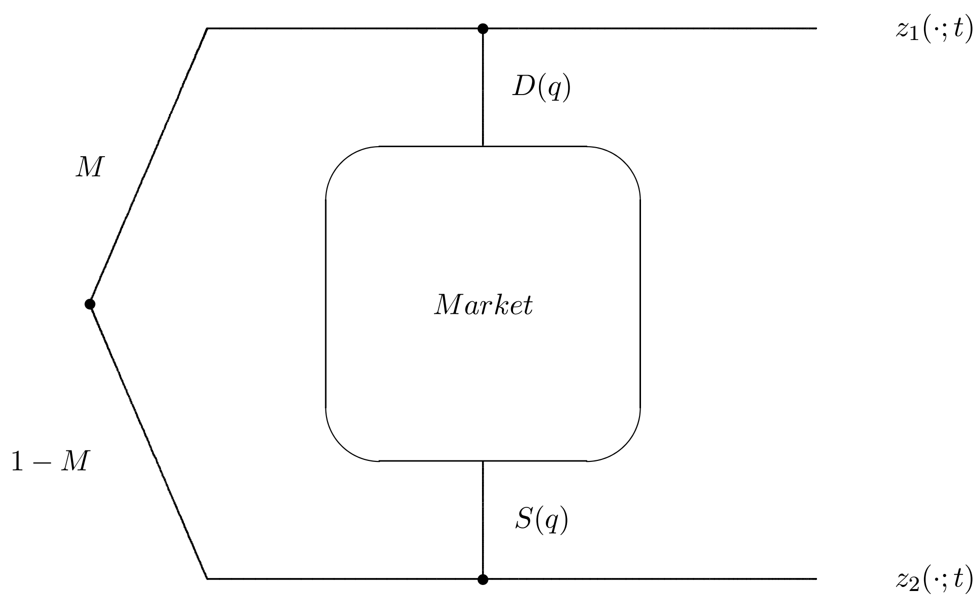

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

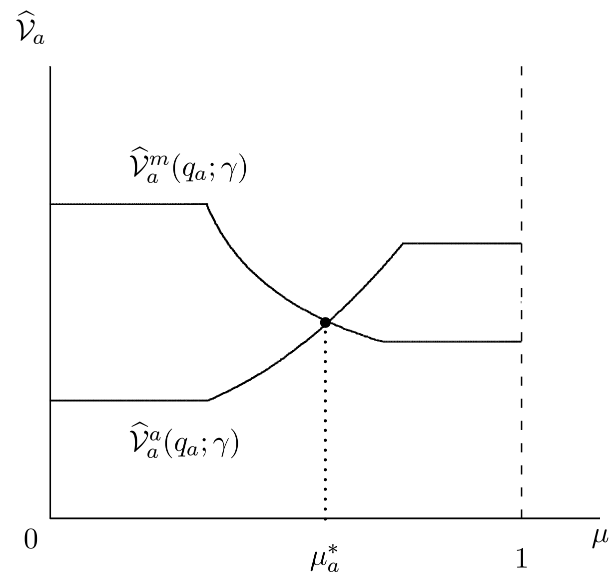

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

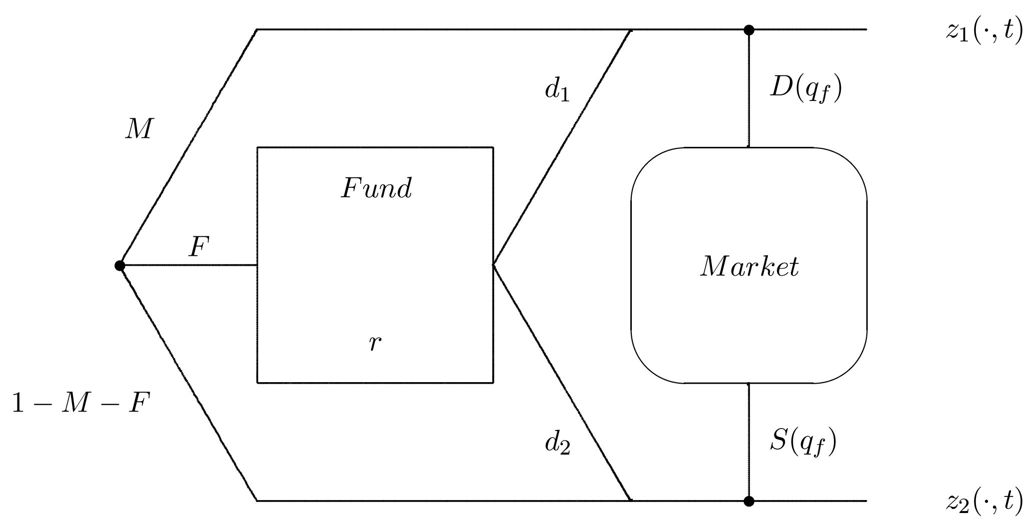

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

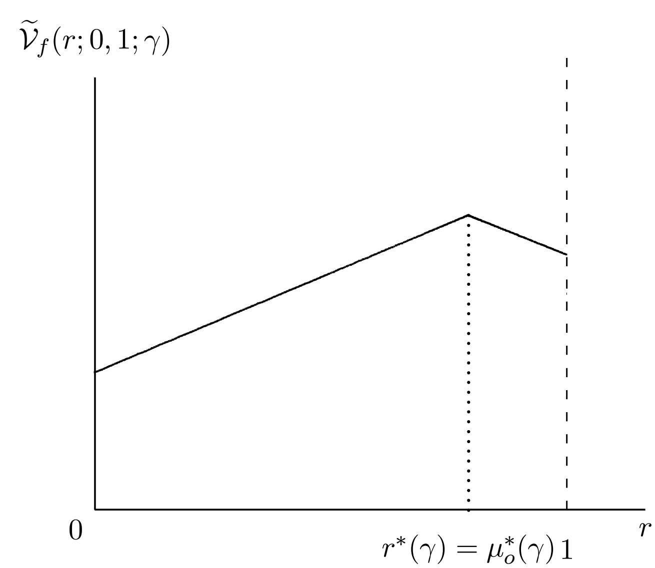

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

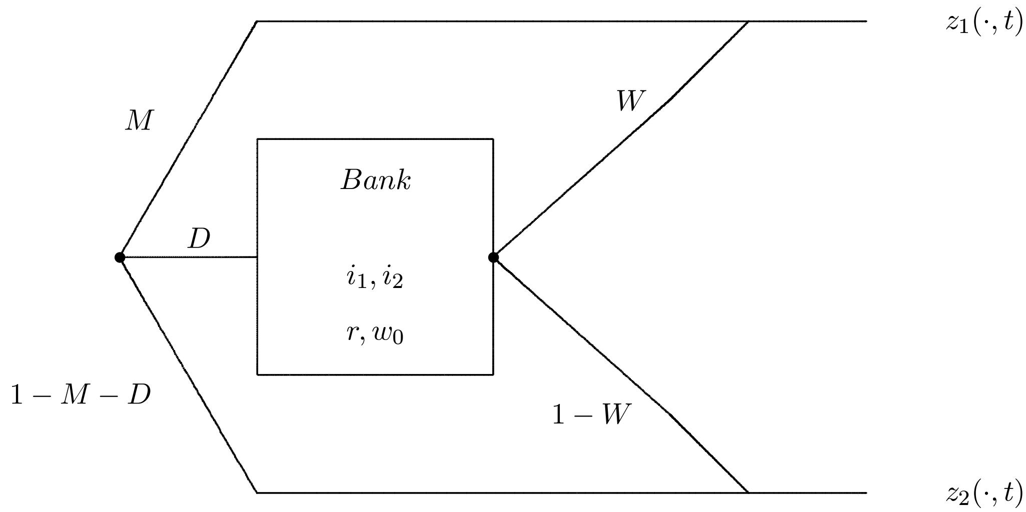

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

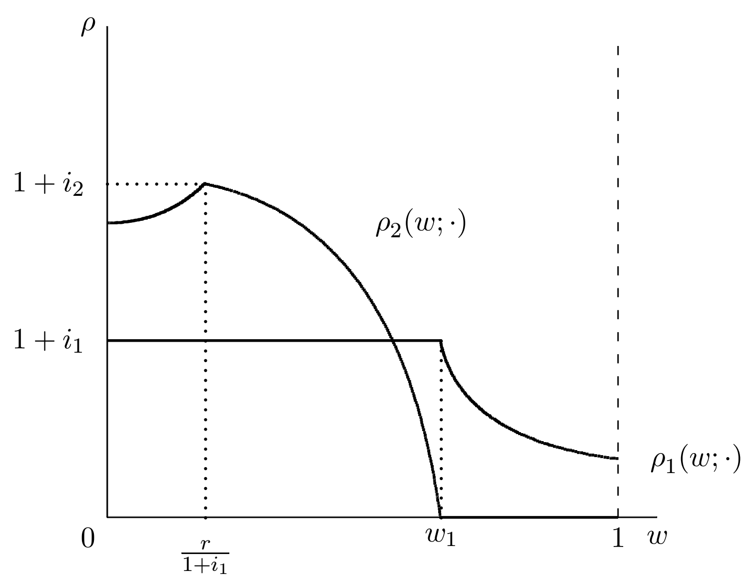

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$
$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

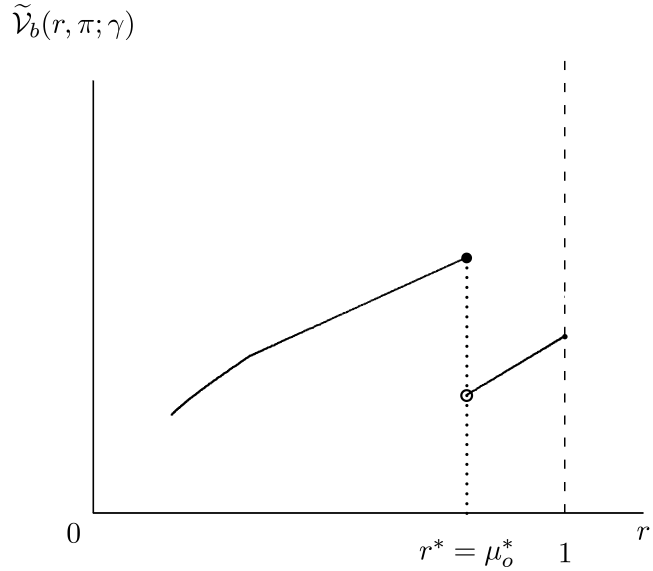

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*)*, with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

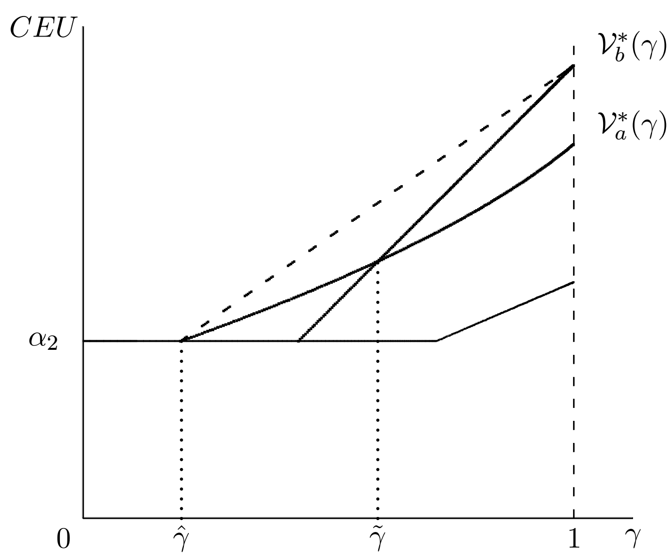

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1 - \pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1 - \pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1 - r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1 - \alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1 - \mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1 - \mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

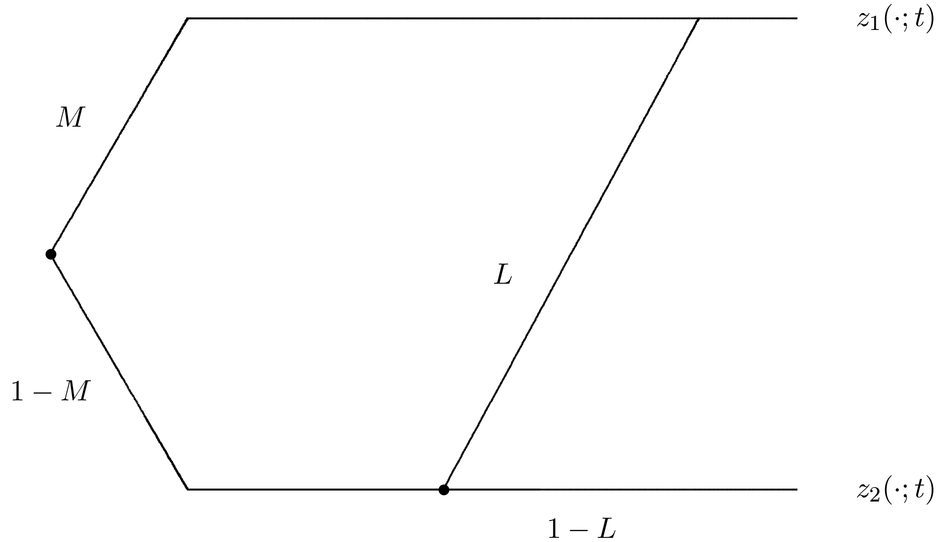

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=& \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) :=& \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M; t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) :=\ & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ =\ & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2 \, , \, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

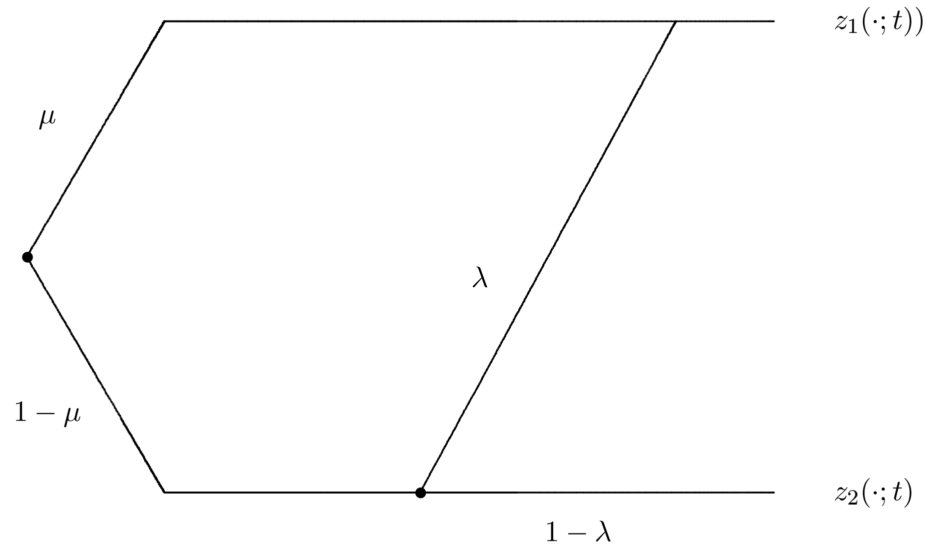

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \multicolumn{3}{l}{\lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0.} & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

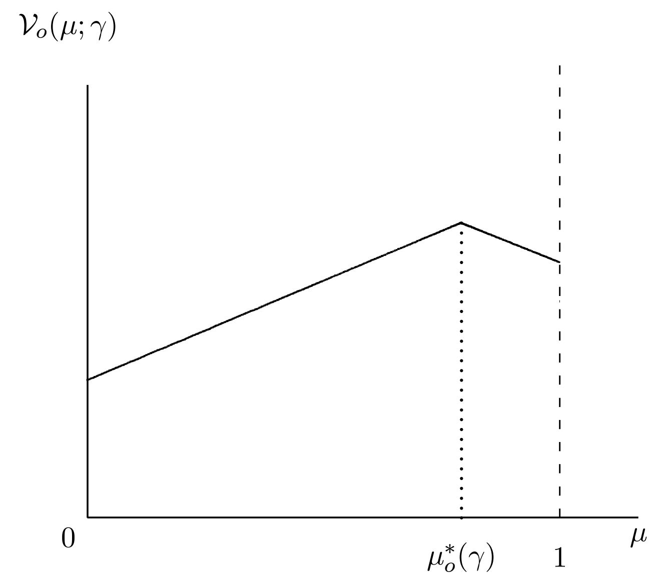

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

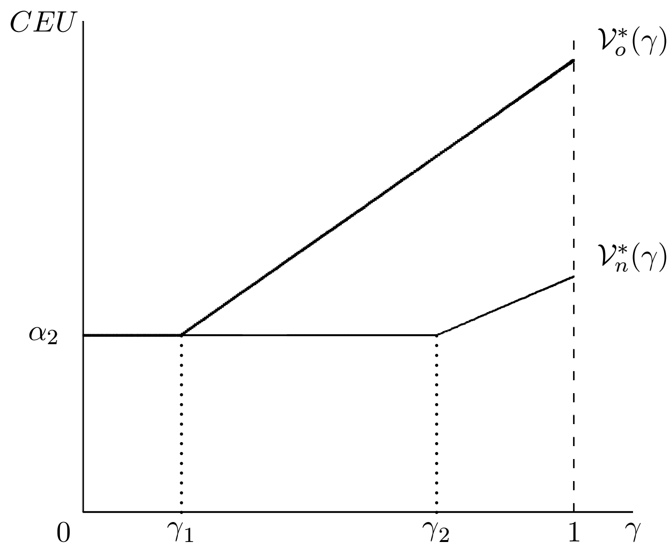

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

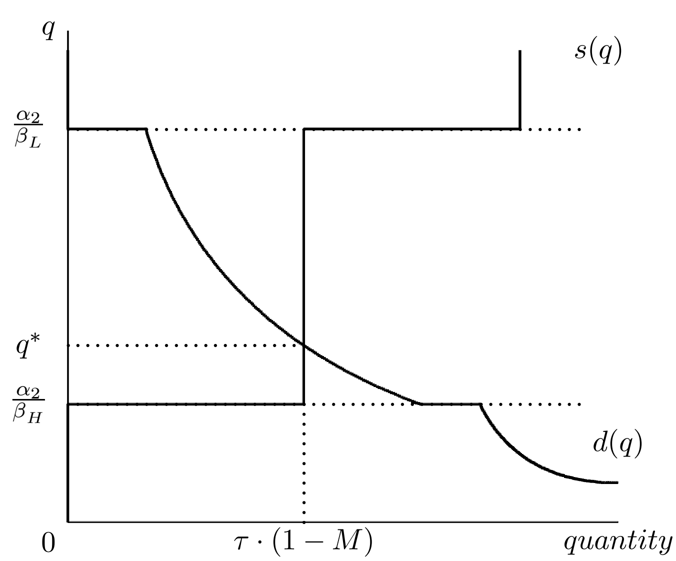

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

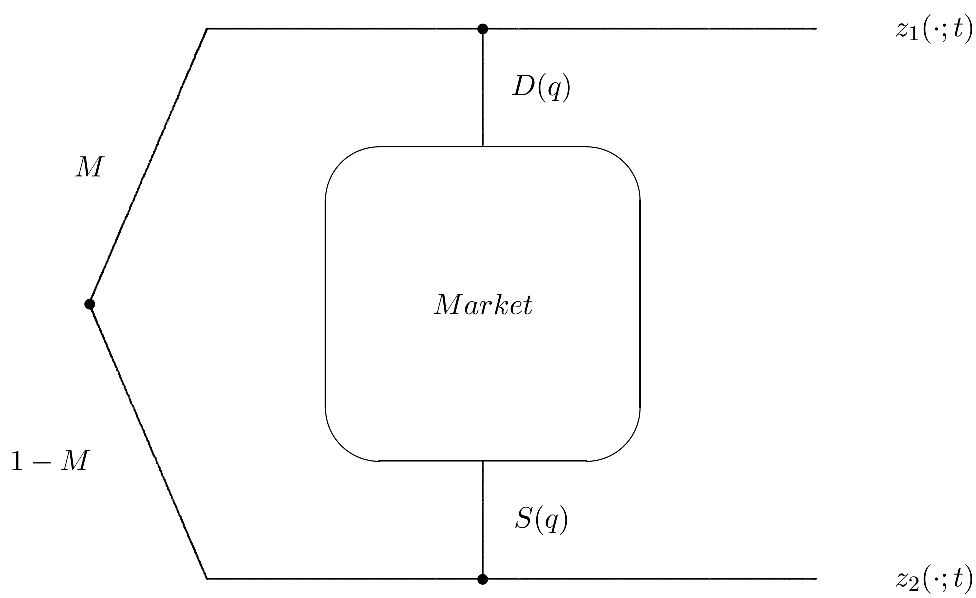

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

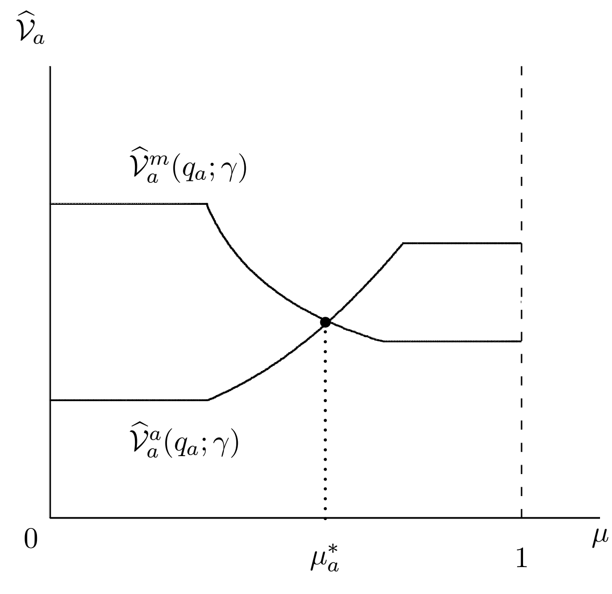

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

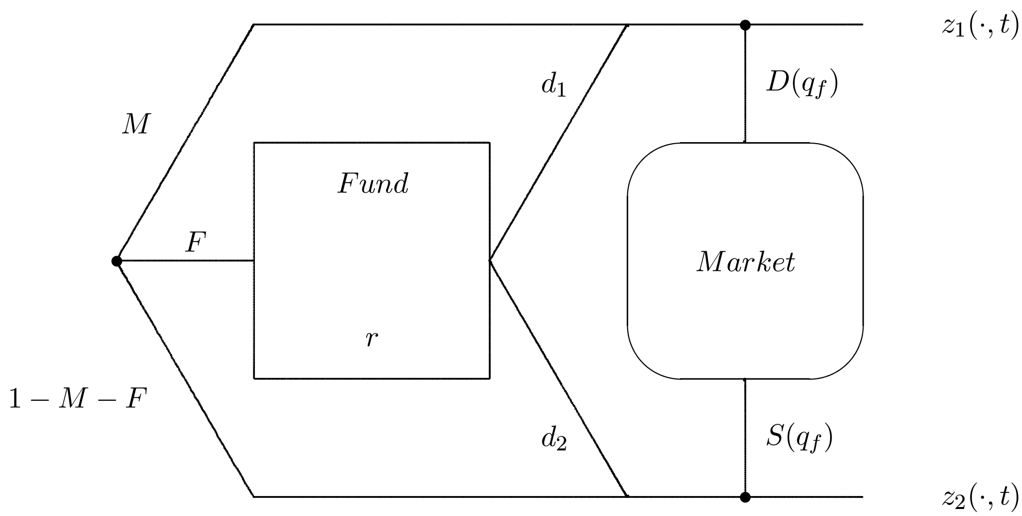

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 :=& \quad r \\ d_2 :=& \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

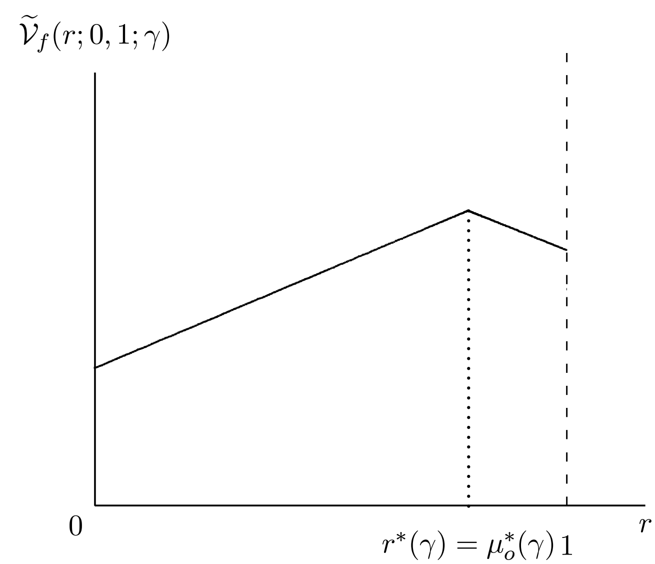

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0, 1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

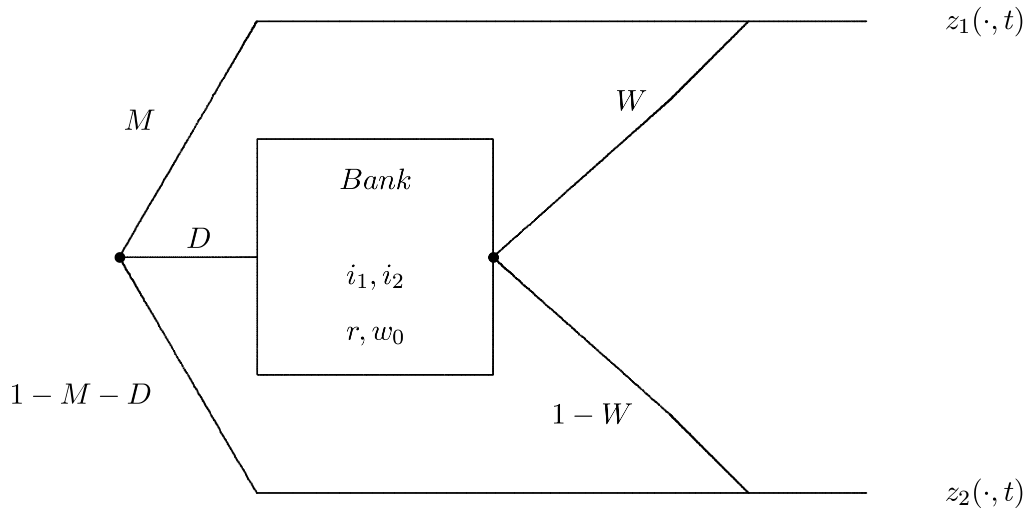

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

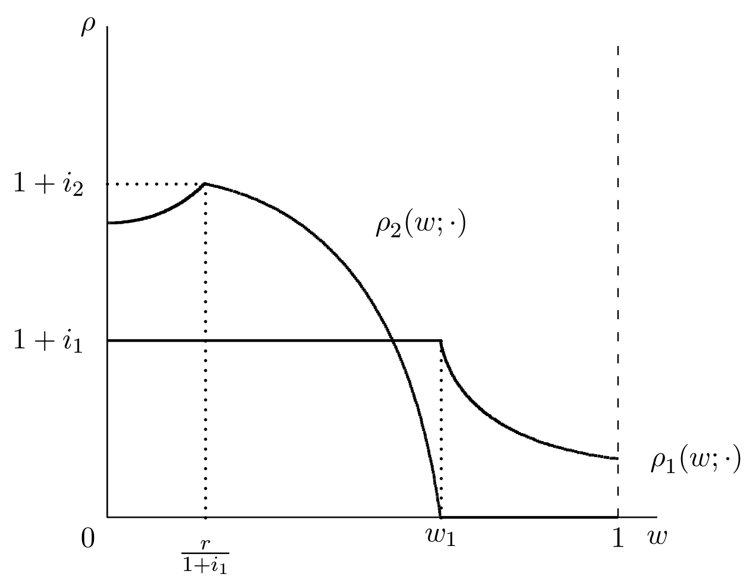

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

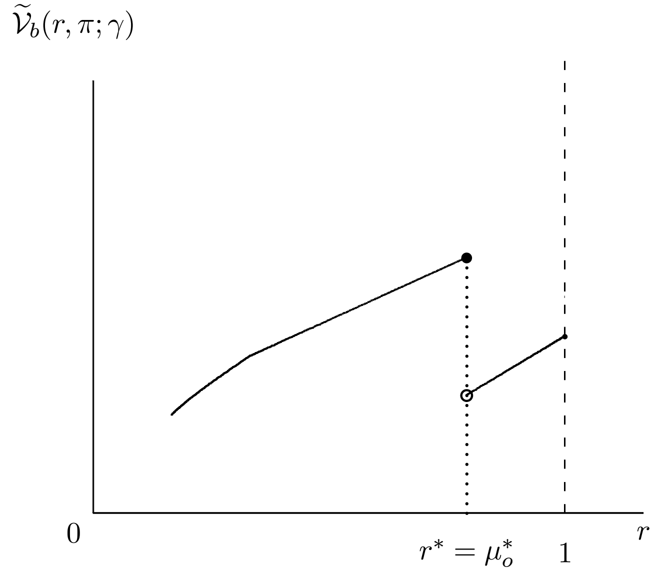

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \ \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \ \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

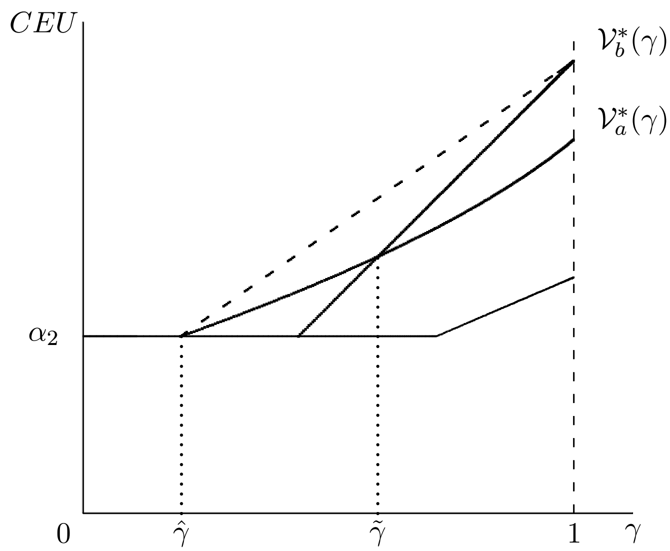

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t,\tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

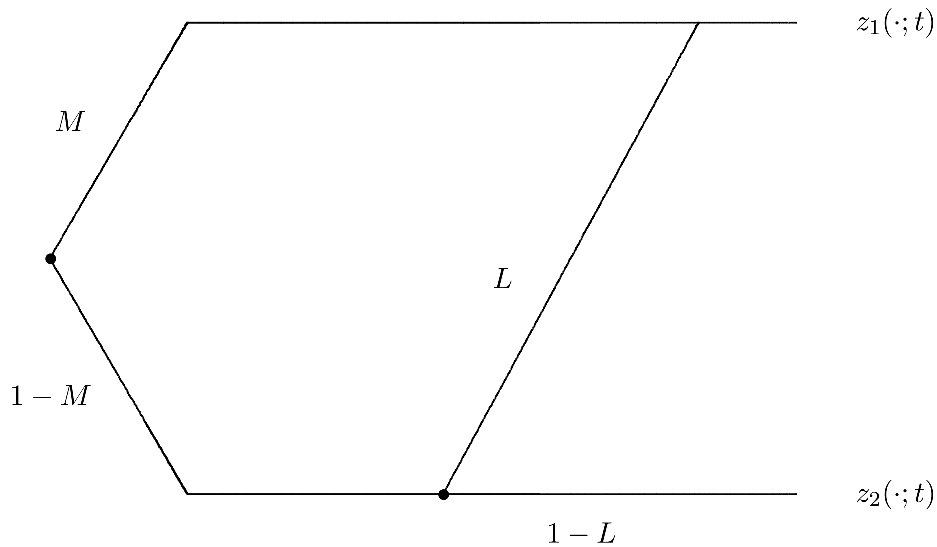

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell \cdot M + (1-M)\cdot \alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

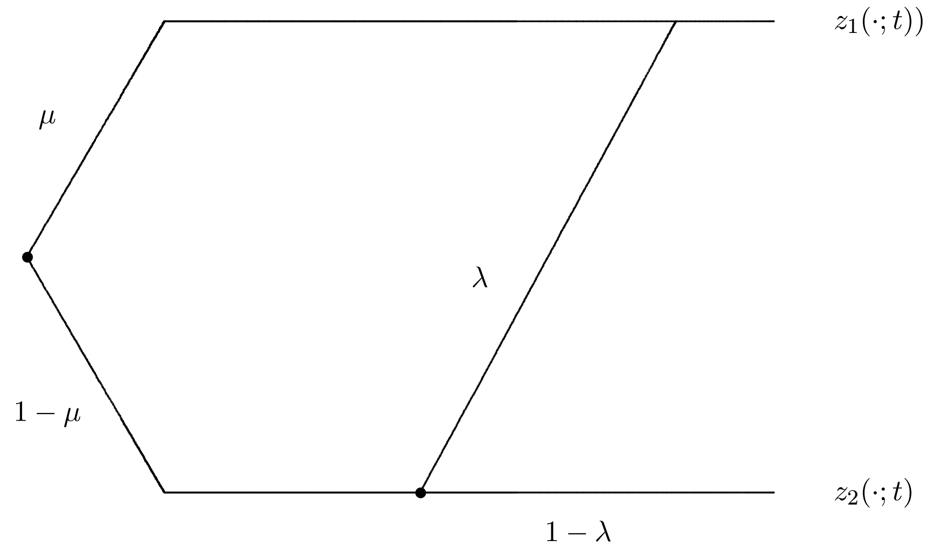

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := \quad & \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := \quad & \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \text{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \text{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \text{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \text{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \text{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

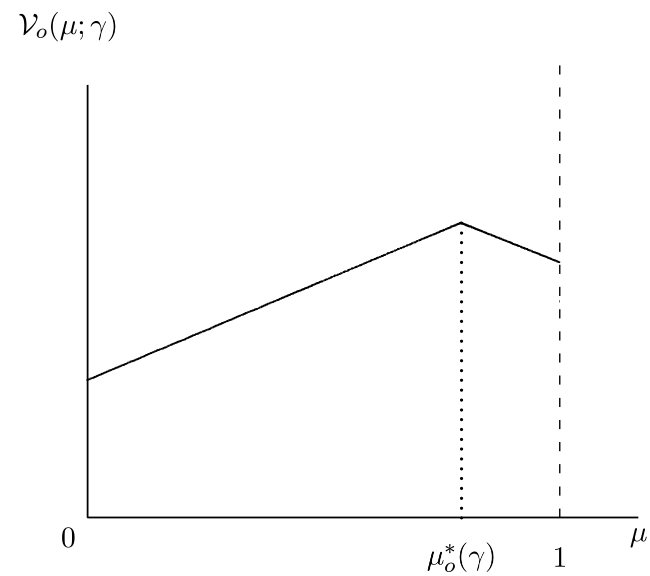

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

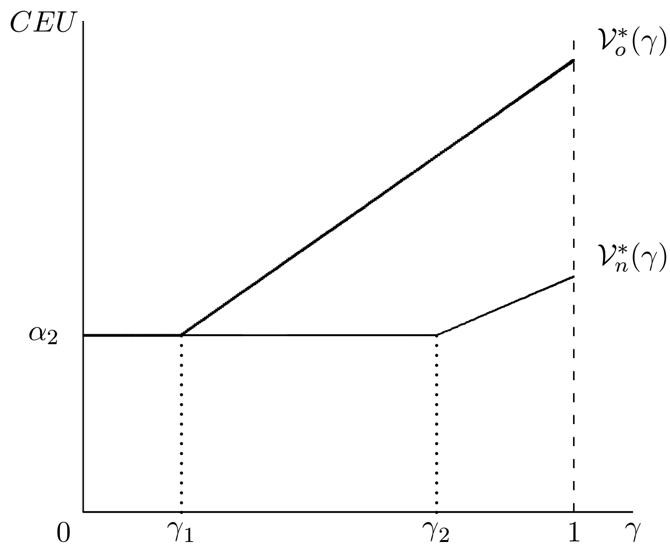

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

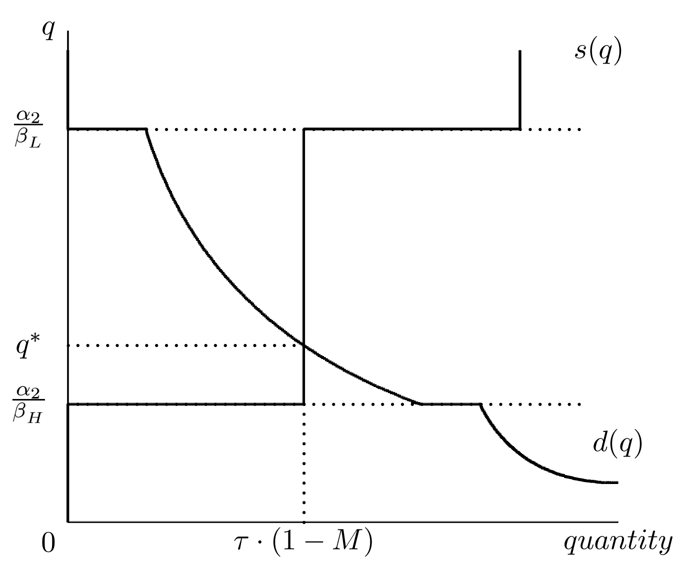

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu)\cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu)\cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu)\cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu)\cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

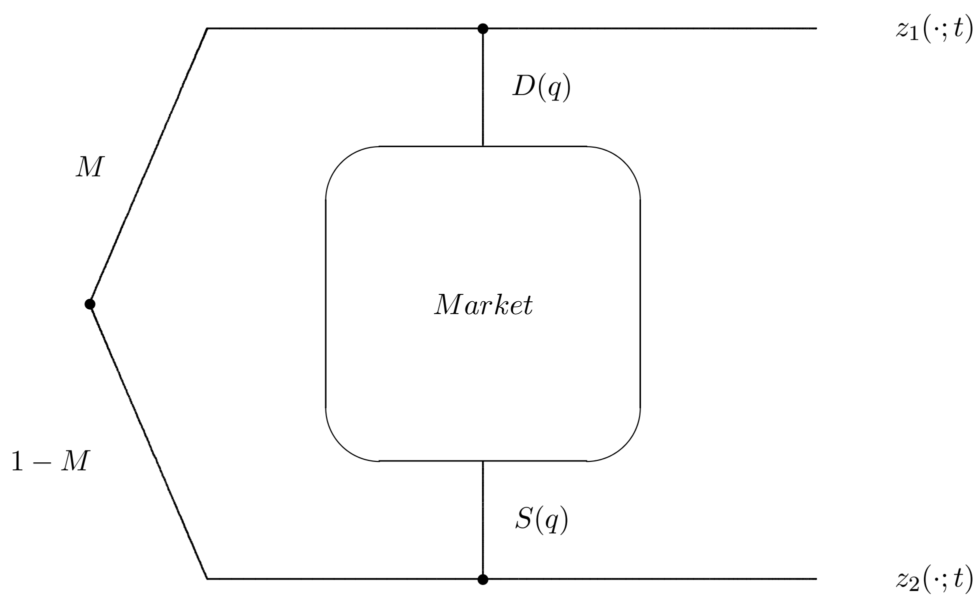

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M) \cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)] \,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi,$
- *if* $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi),$
- *if* $\gamma \in [0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

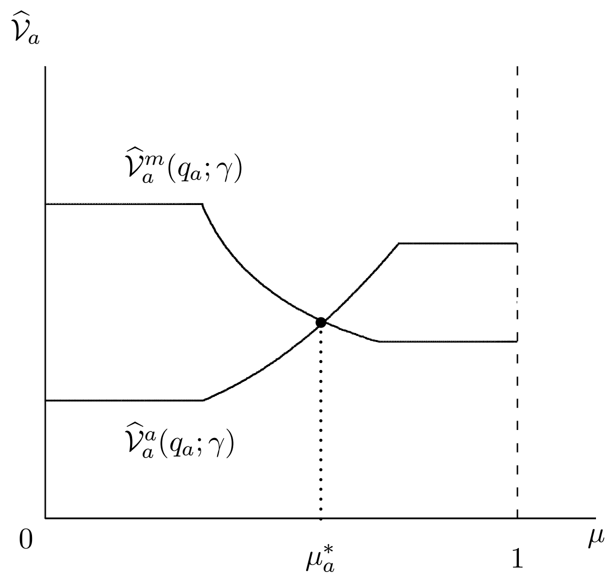

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

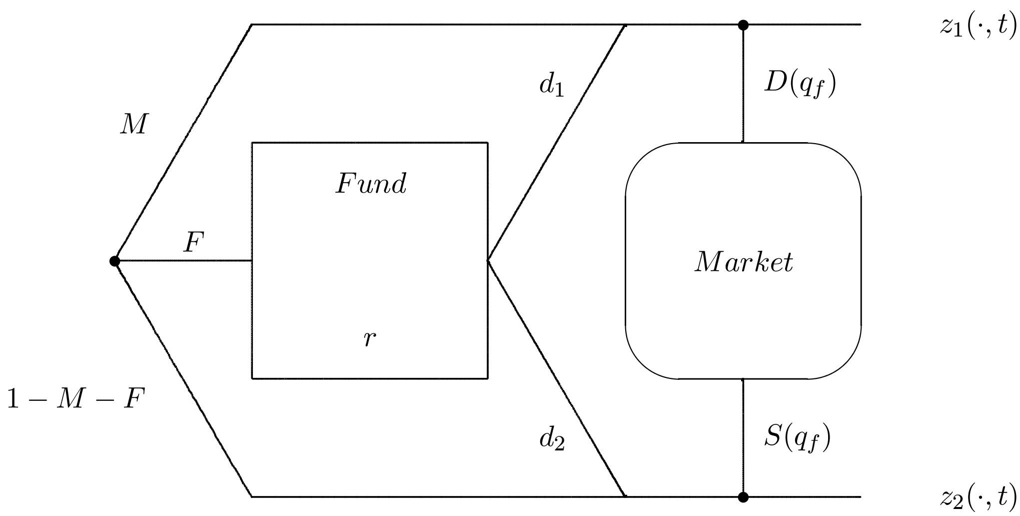

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau)\cdot\mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r)\cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

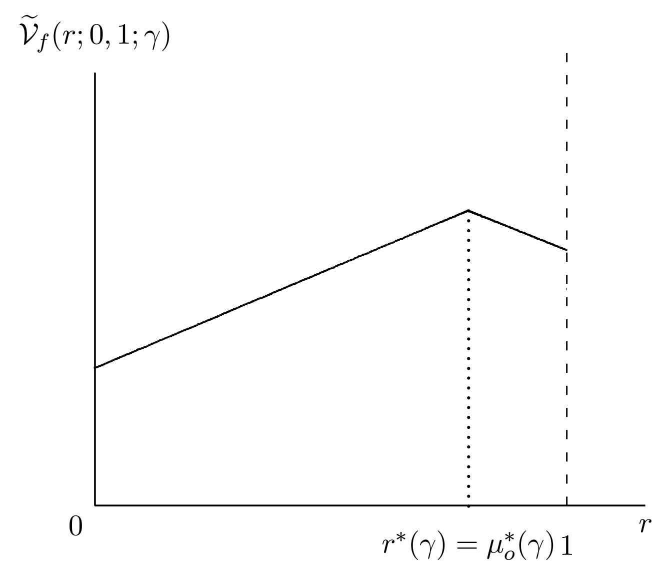

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.
For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) \;:=\; \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) \;=\; \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26]See Appendix A, Remark 10.
[27]See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

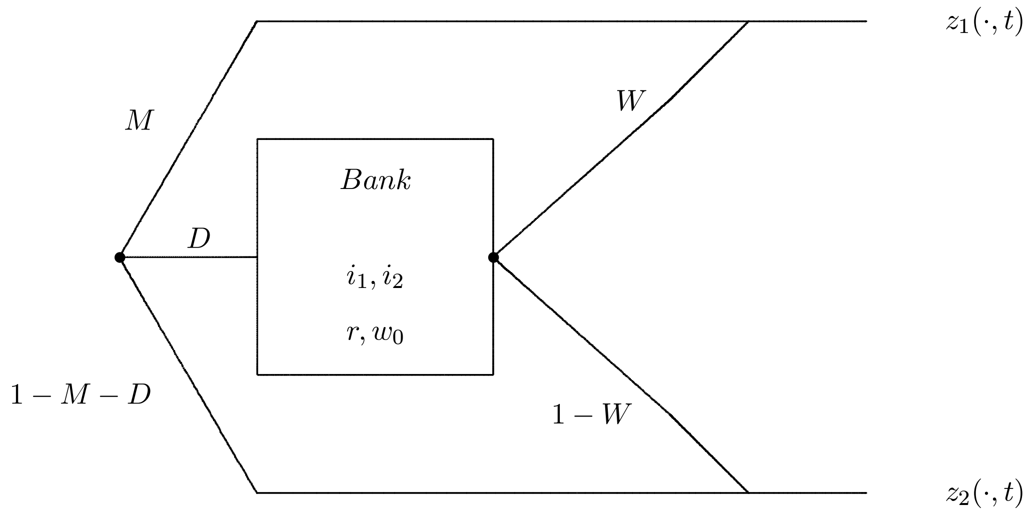

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1+i_1)\cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1+i_1)\cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1-r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1+i_2)\cdot(1-w_0) = \alpha_2 \cdot (1-r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1-r)}{1-w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1+i_1)\cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

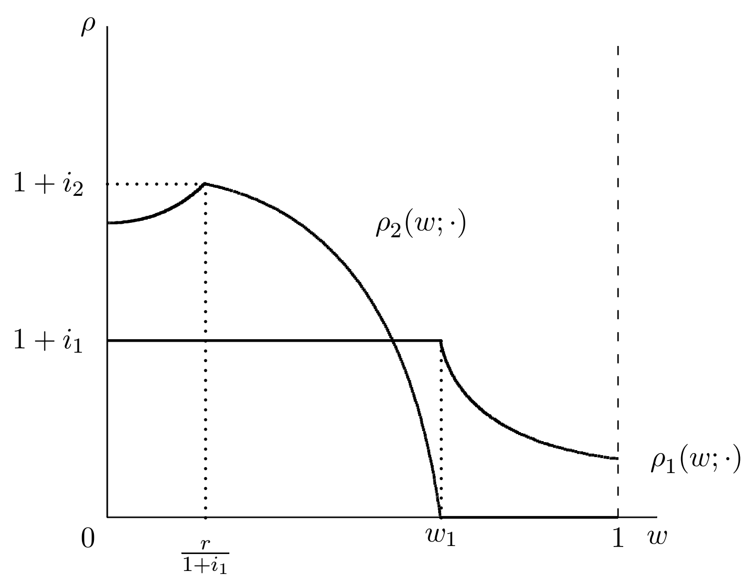

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}&\mathcal{V}_b(D,M;r,w_0;\gamma)\\ :=\ &\gamma\cdot\int[\tau\cdot v_b(D,M;\tau,r,w_0;h)\ +\ (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)]\cdot p(\tau)\ d\tau\\ &\qquad +(1-\gamma)\cdot\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t).\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D,M;r,w_0;\gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D,M;r,w_0;\gamma)$$

$$\text{s.t.} \quad D+M\leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$.

It remains to determine the deposit contract parameters (r,w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau=\pi$ and $\tau=1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r<1$. As a consequence, banks offer deposit contracts on the basis of $w_0=\pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r,\pi;\gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0=0$ and $w_0=1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

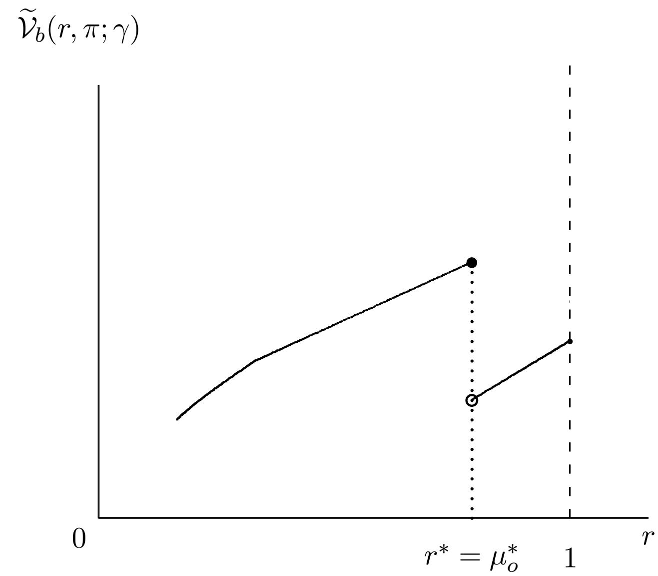

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The “worst-case” pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

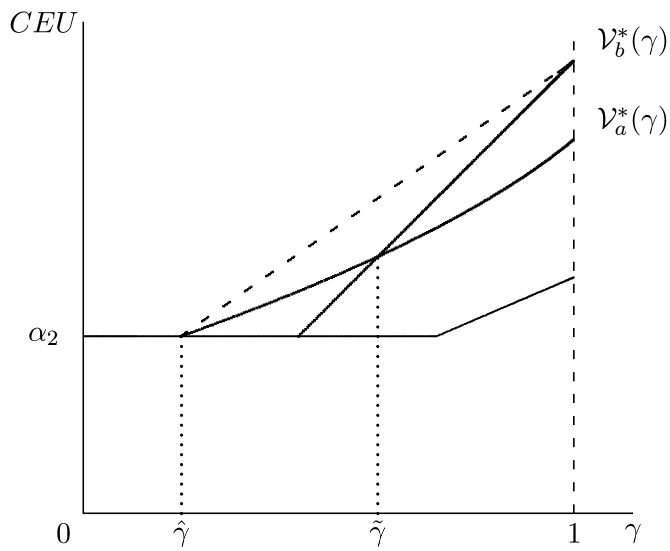

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S|\ f(s) \geq t\}$ and $\{s \in S|\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S|\ f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S|\ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0,1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

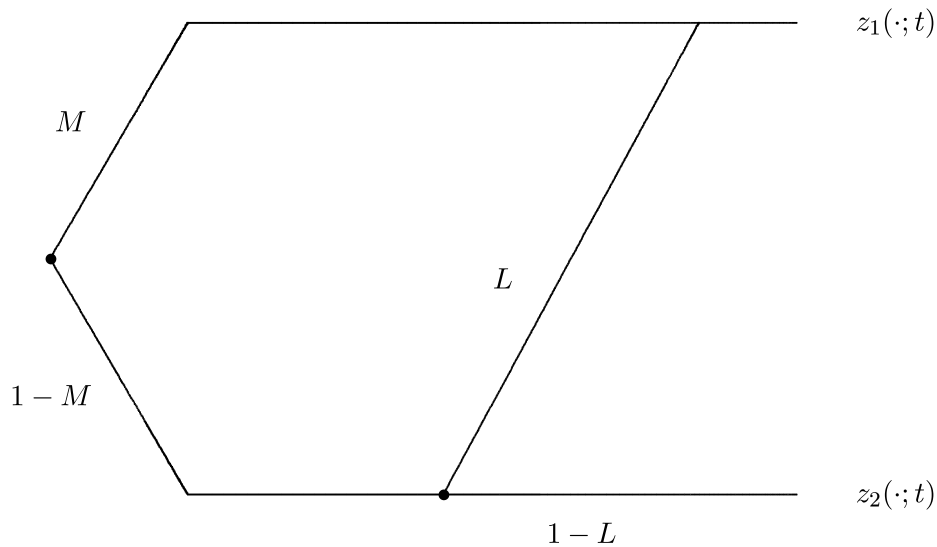

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1-M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1-\tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & \qquad +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & \qquad +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau=\pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

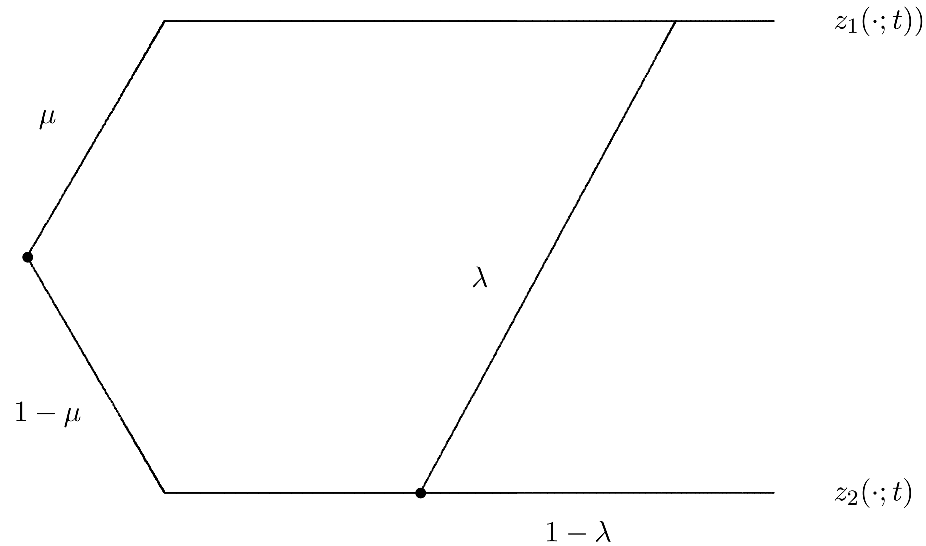

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := & \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := & \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll} \max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\ \text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \text{S}_h \\ & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \text{S}_\ell \\ & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \text{F}_1 \\ & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \text{F}_2 \\ & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \text{NN} \end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) &:= u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ &= \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

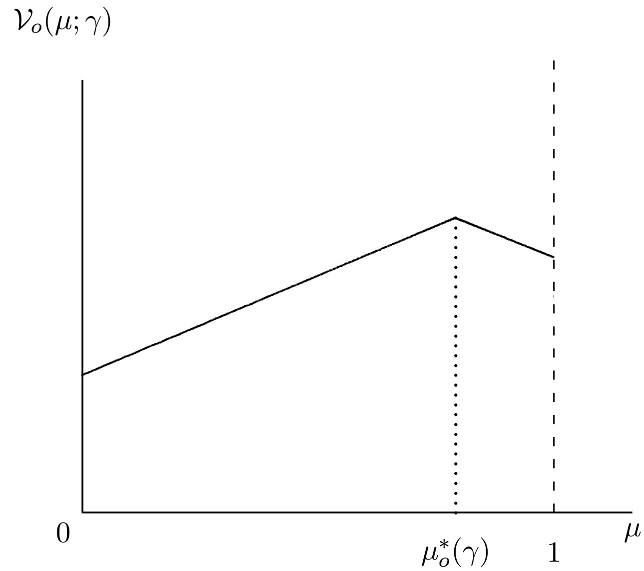

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

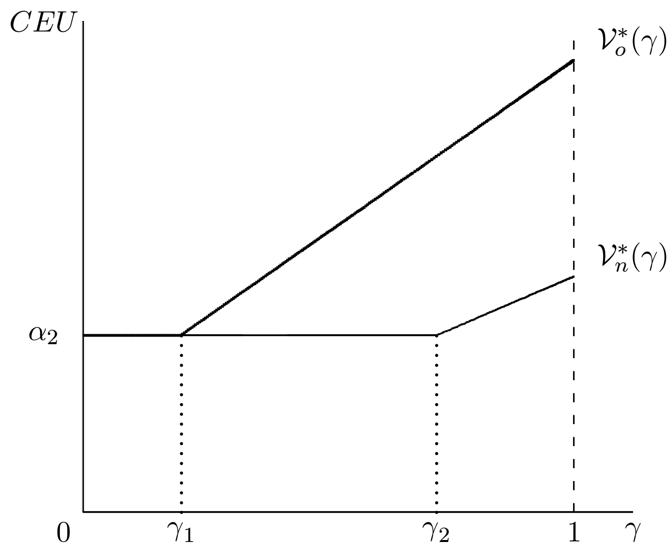

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

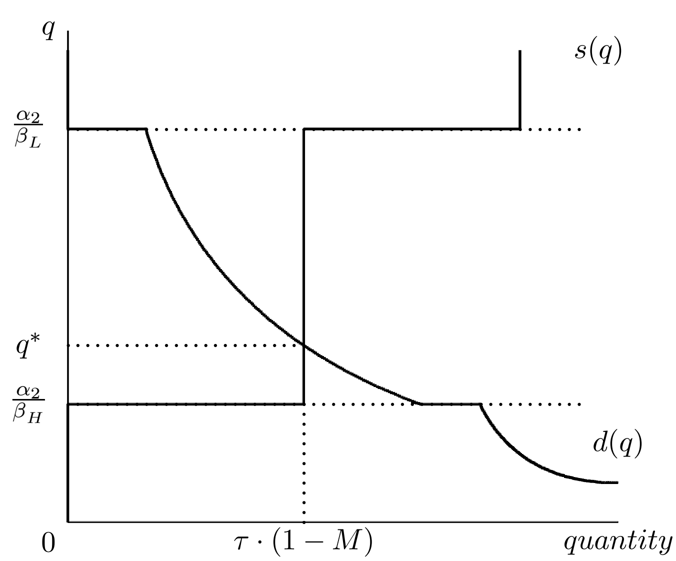

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

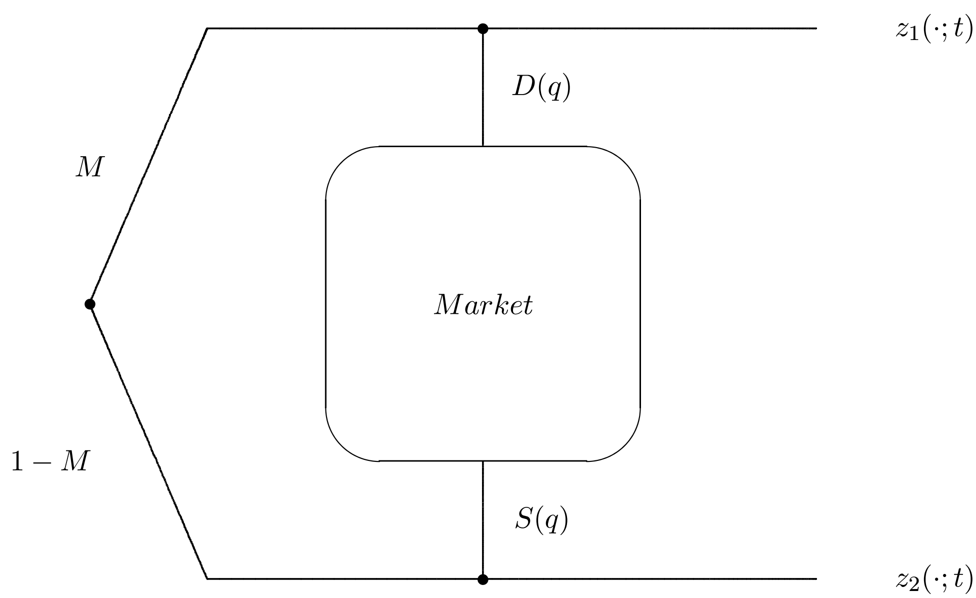

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned} \mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] \,. \end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma), \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

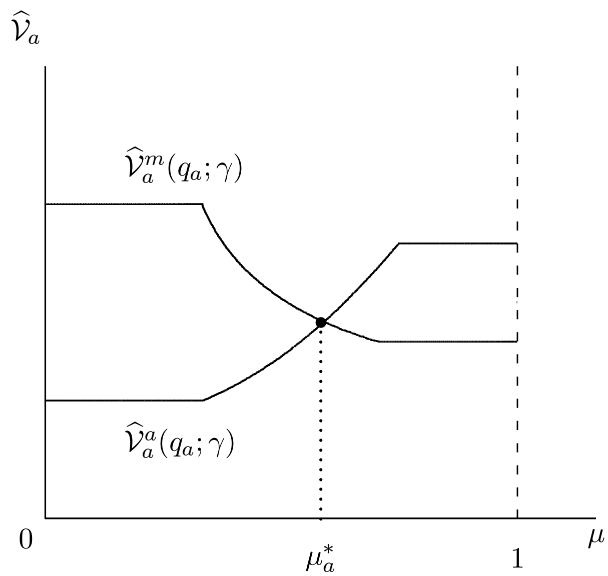

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

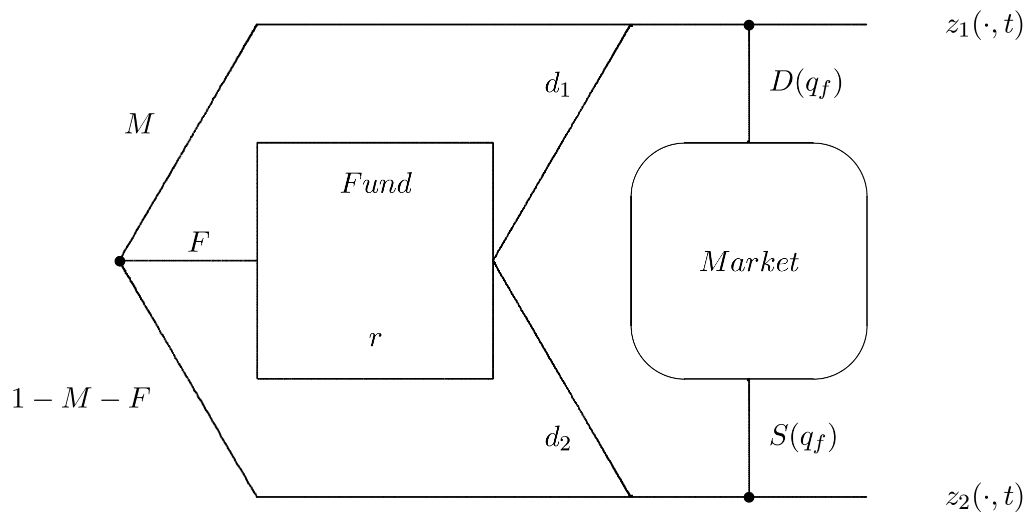

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by[24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

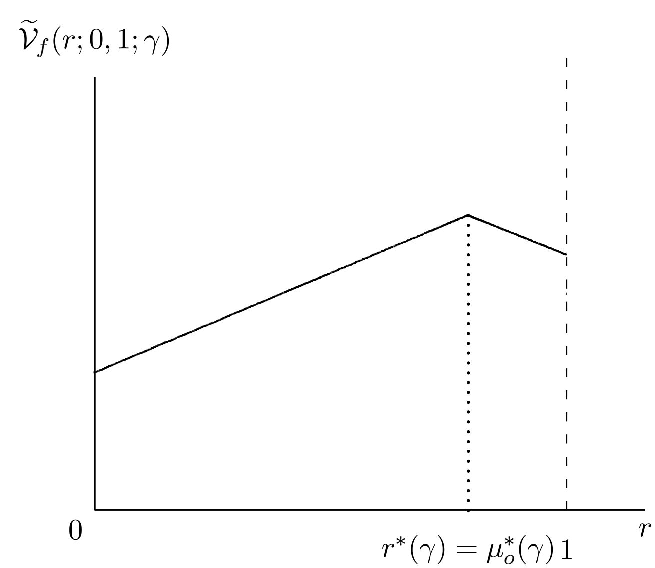

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

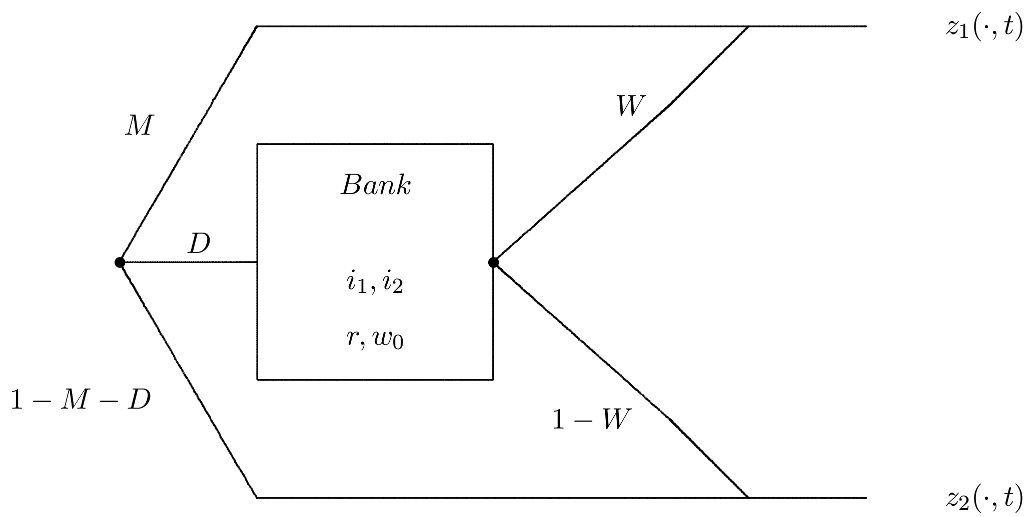

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

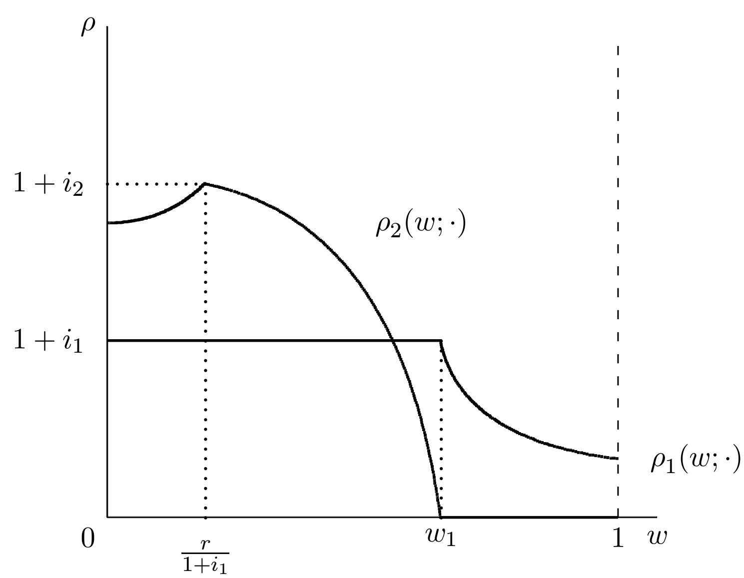

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30]See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

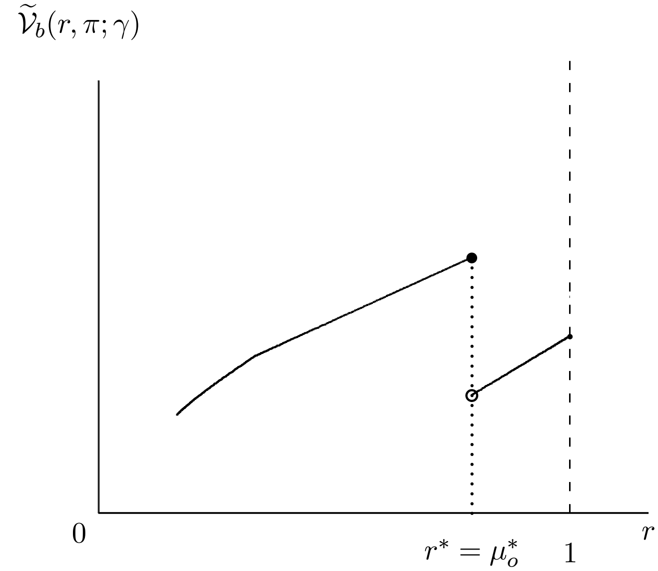

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

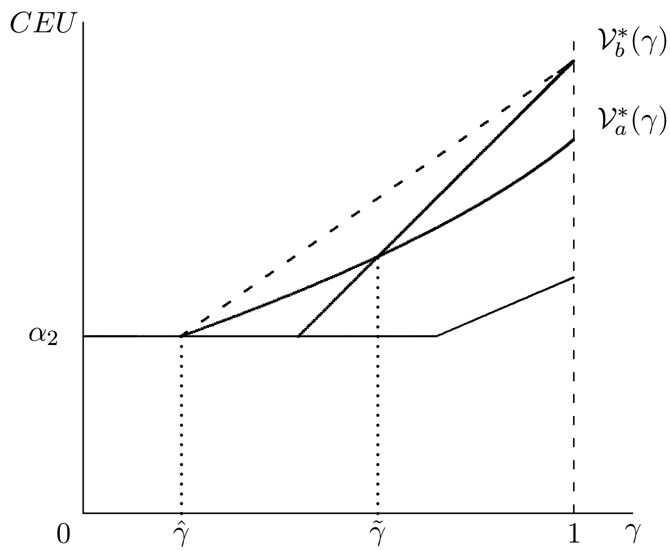

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \; f(s) \geq t\}$ and $\{s \in S | \; f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S | \; f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S | \; f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)}, \\ \overline{\tau}(\mu) &:= \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

First Version: November 1997
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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

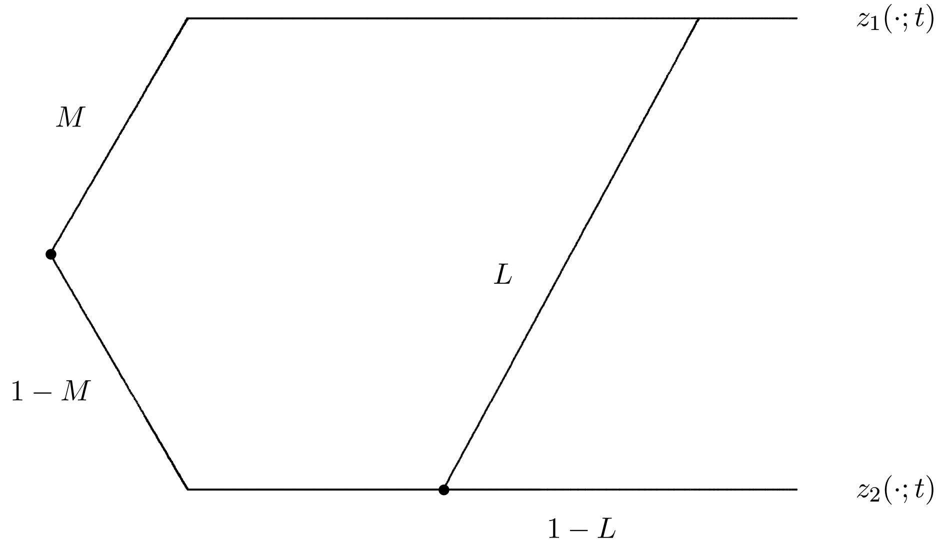

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau=\pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2 \,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

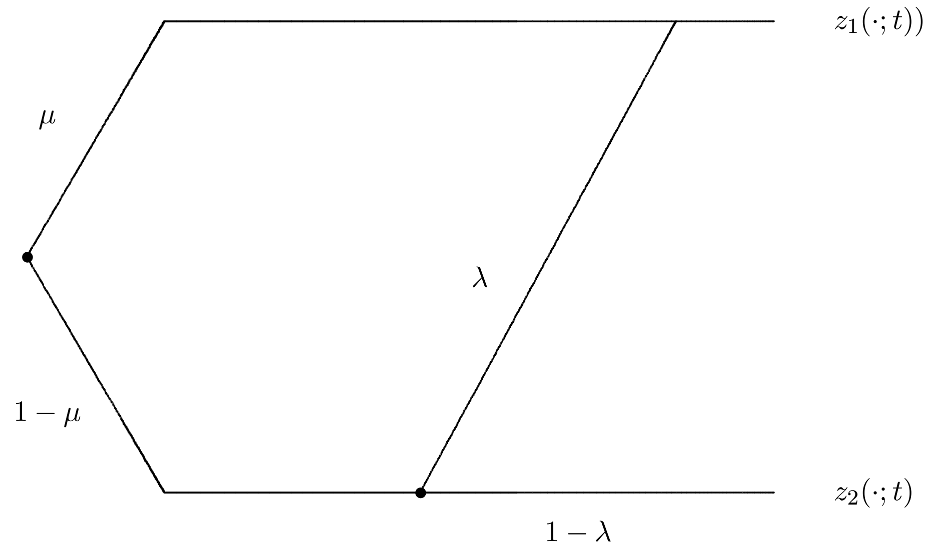

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \text{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \text{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \text{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \text{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \text{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) :=& \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ =& \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

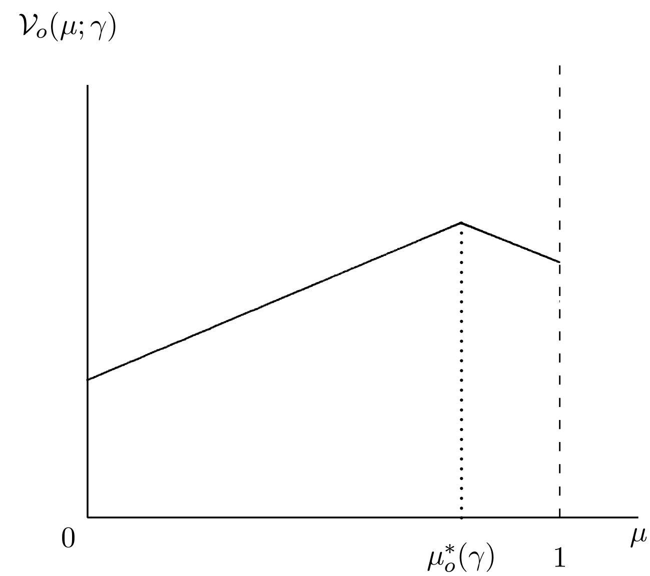

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

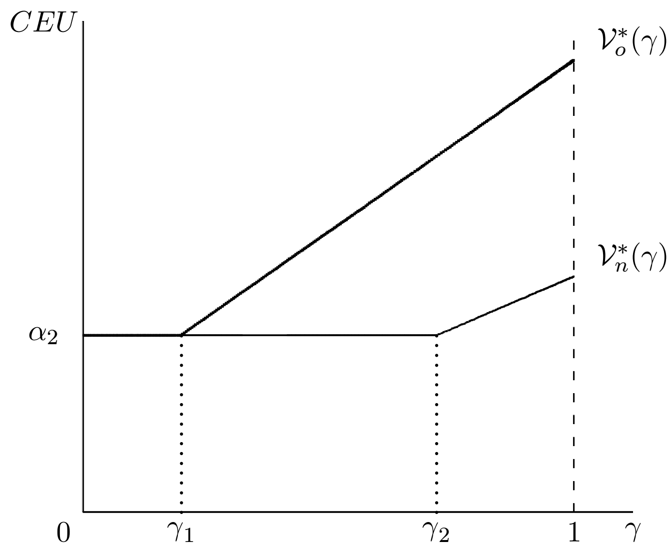

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

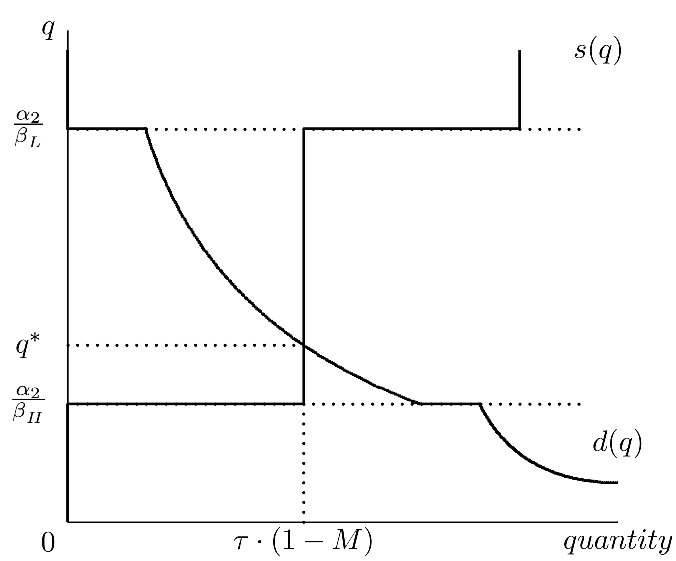

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

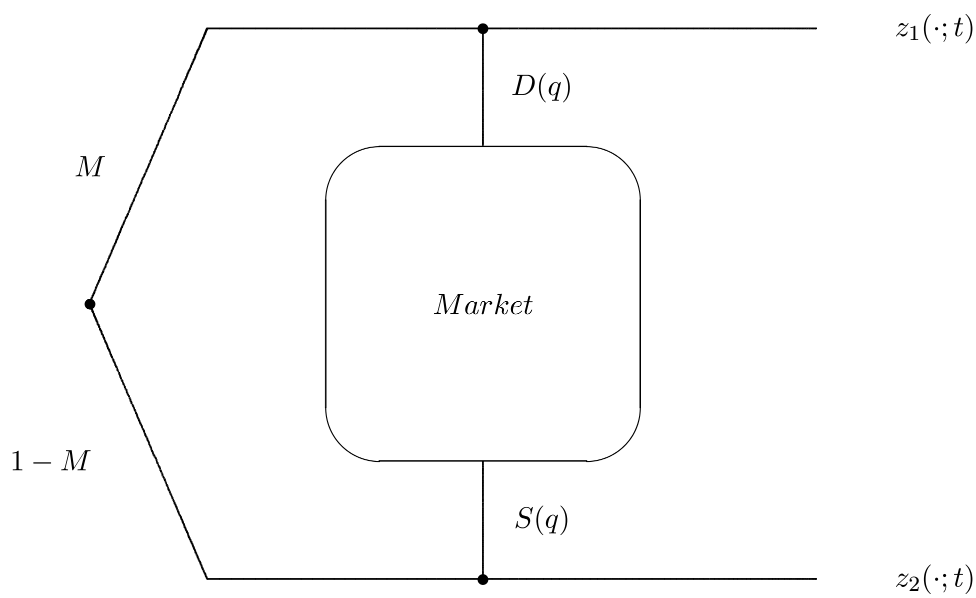

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned} \mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] \, . \end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0,1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

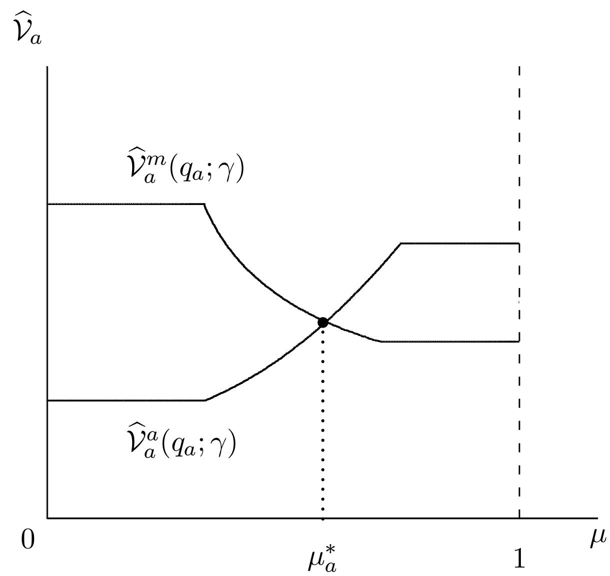

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

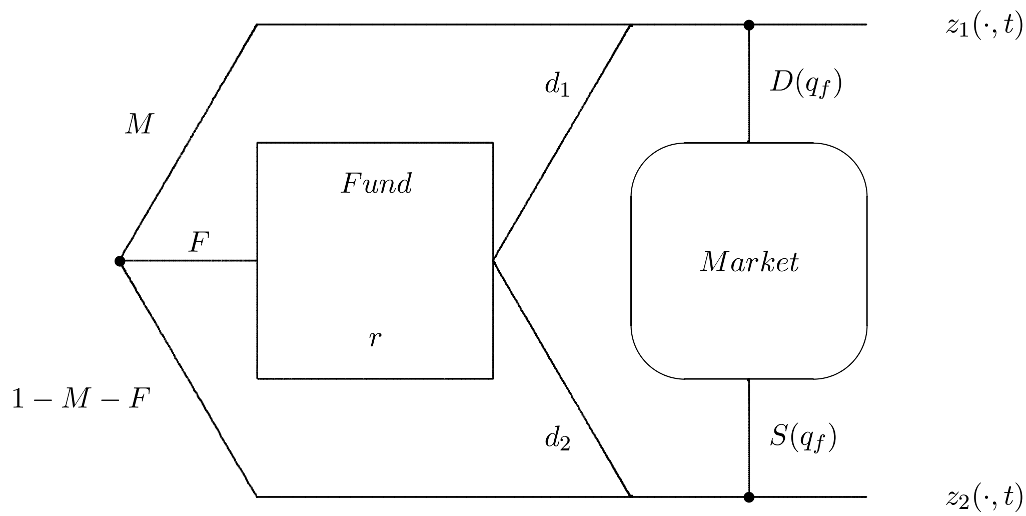

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

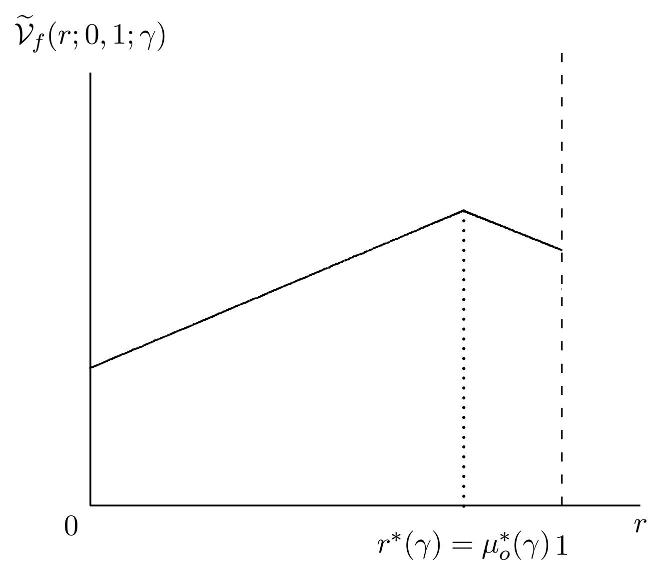

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0, 1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

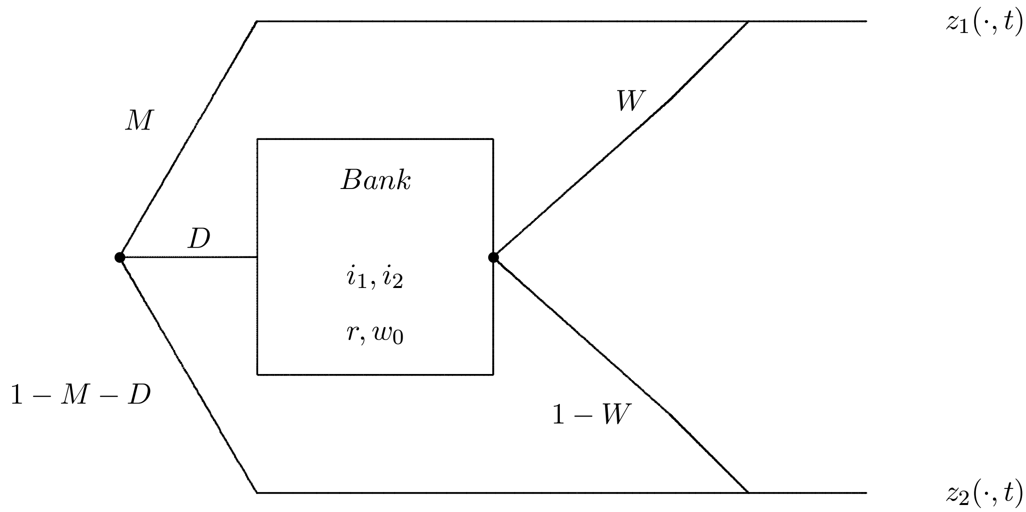

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

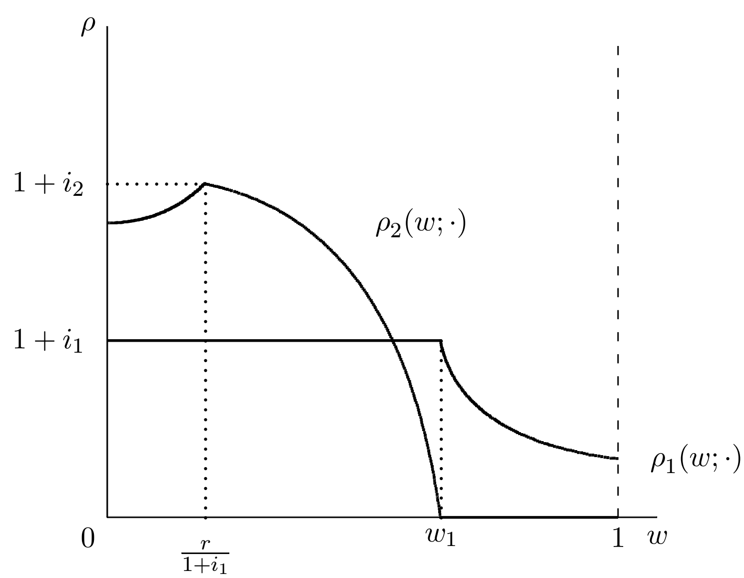

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

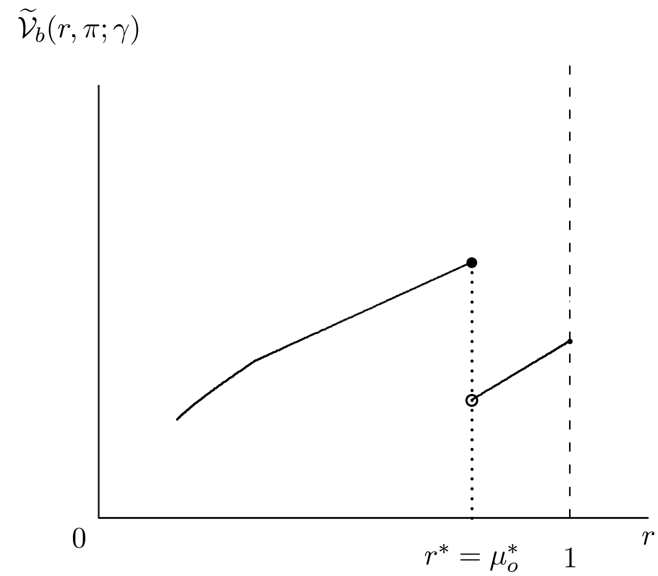

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \ \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \ \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. BANKS, MUTUAL FUNDS, OR ASSET MARKETS?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

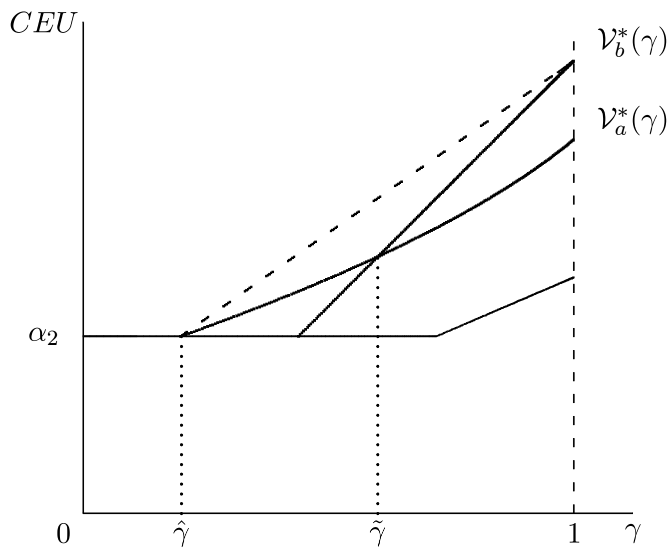

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \ \text{ for all } \ E \subsetneq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \ f(s) \geq t\}$ and $\{s \in S | \ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\int^{C} f \, d\nu = \int_0^{\infty} \nu(\{s \in S | \ f(s) \geq t\}) \, dt + \int_{-\infty}^{0} [\nu(\{s \in S | \ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma\cdot[\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)]+(1-\gamma)\cdot\min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M\cdot[\gamma\cdot[\pi\cdot\beta_h+(1-\pi)\cdot\beta_\ell]+(1-\gamma)\cdot\beta_\ell]+(1-M)\cdot\alpha_2\\ &= M\cdot[\gamma\cdot\pi\cdot\beta_h+(1-\gamma\cdot\pi)\cdot\beta_\ell]+(1-M)\cdot\alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu\in[0,1]$ and $\tau\in[0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu)=0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell\cdot\mu}{\beta_\ell\cdot\mu+\alpha_2\cdot(1-\mu)},\\ \overline{\tau}(\mu) &:= \frac{\beta_h\cdot\mu}{\beta_h\cdot\mu+\alpha_2\cdot(1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu)=\begin{cases}\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}+\mu & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ \dfrac{\mu}{\tau} & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{1\ell}(\tau,\mu)=\begin{cases}\mu-\left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ 0 & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \textit{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

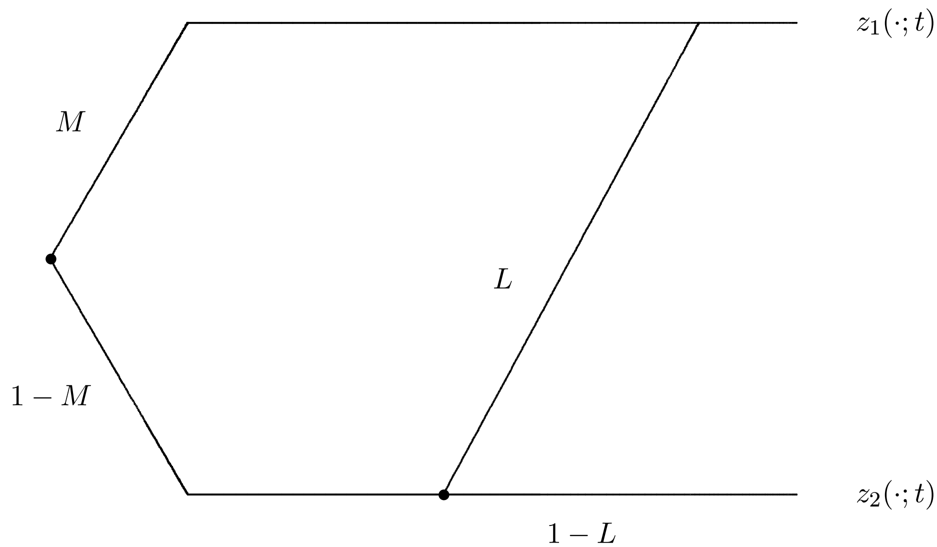

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{array}{rll} z_1(M,L) := & M + \alpha_1 \cdot L \cdot (1-M) & \text{and} \\ z_2(M,L) := & \alpha_2 \cdot (1-L) \cdot (1-M). & \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) :=\ & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ =\ & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

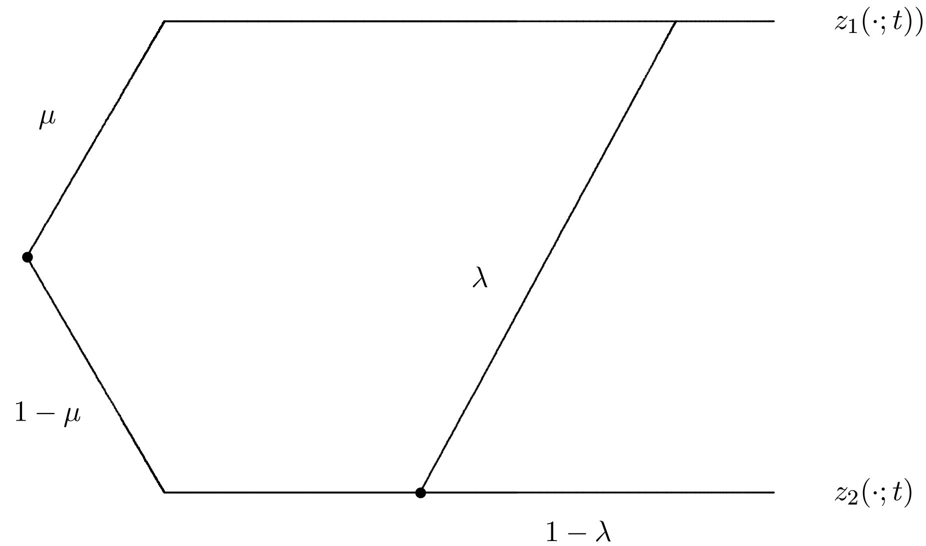

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda,z_h,z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

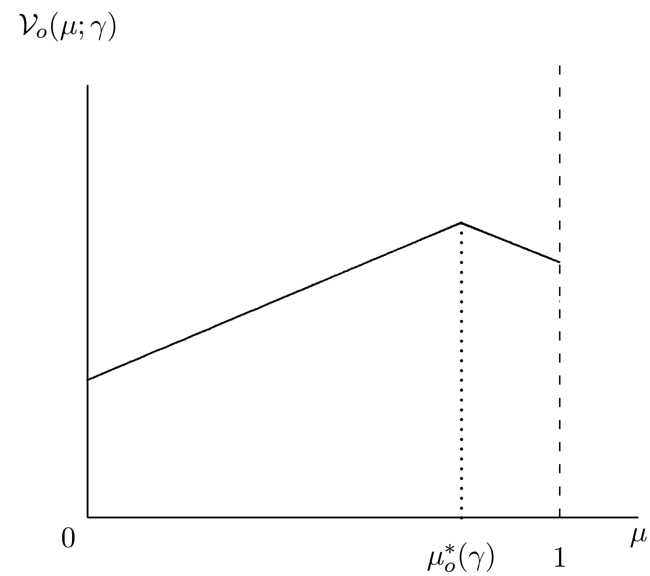

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

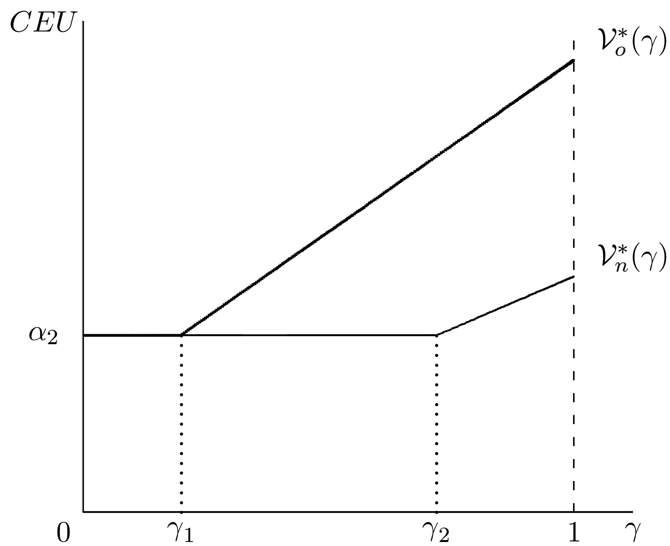

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

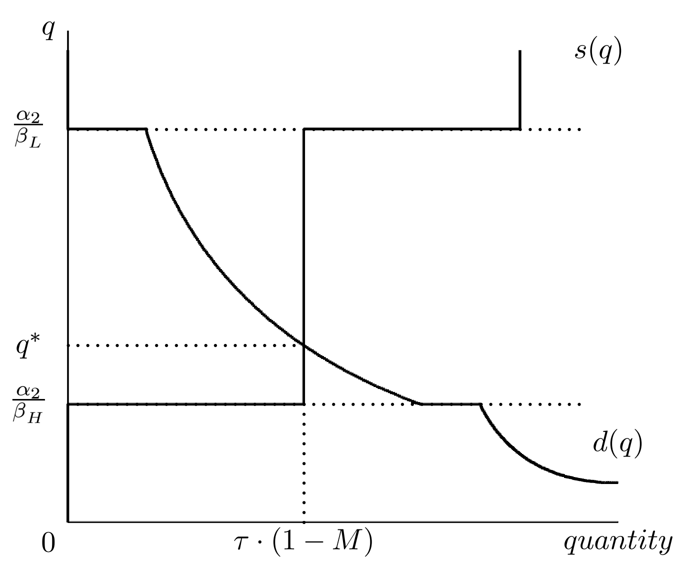

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

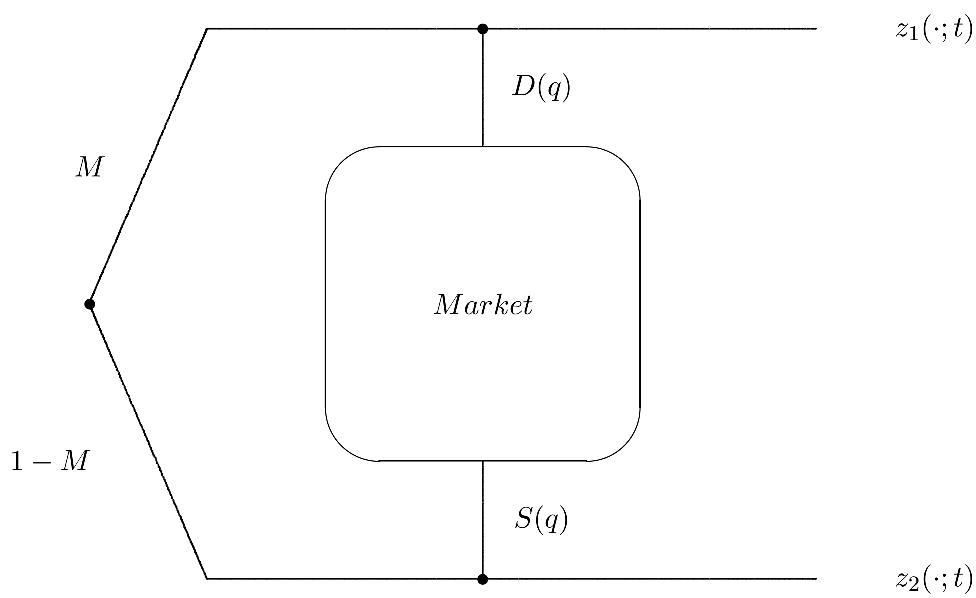

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

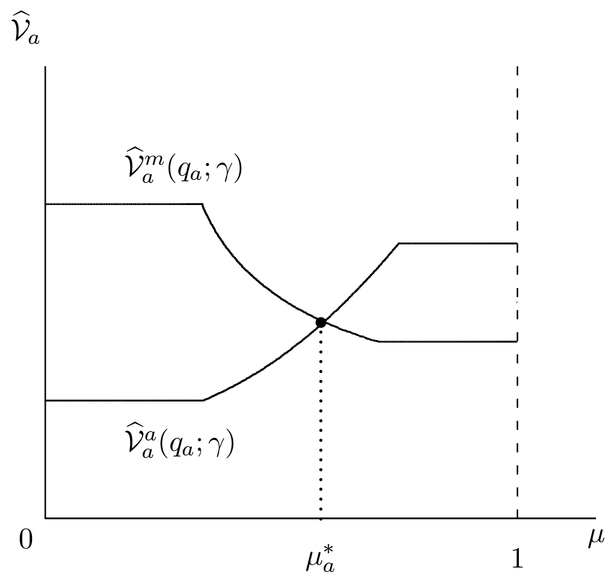

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

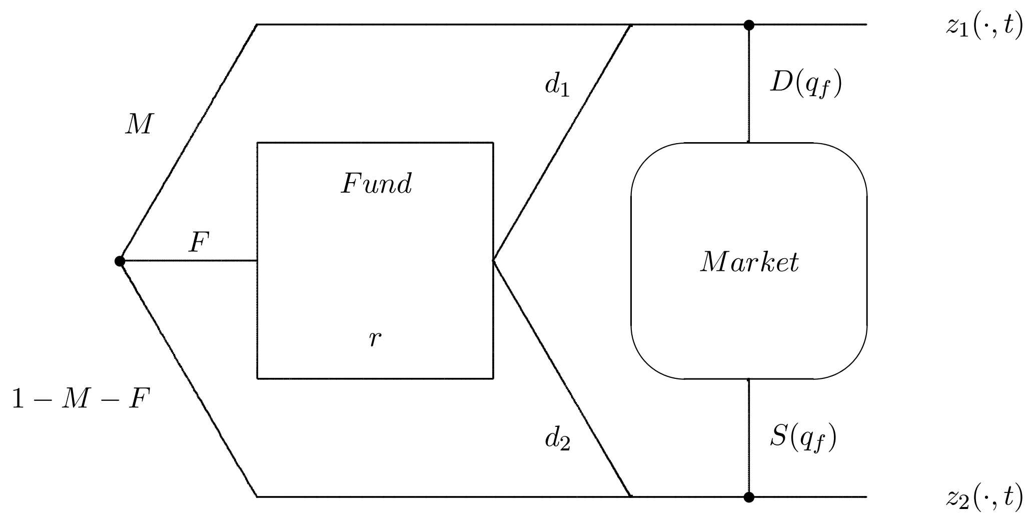

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h)\,+\,(1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

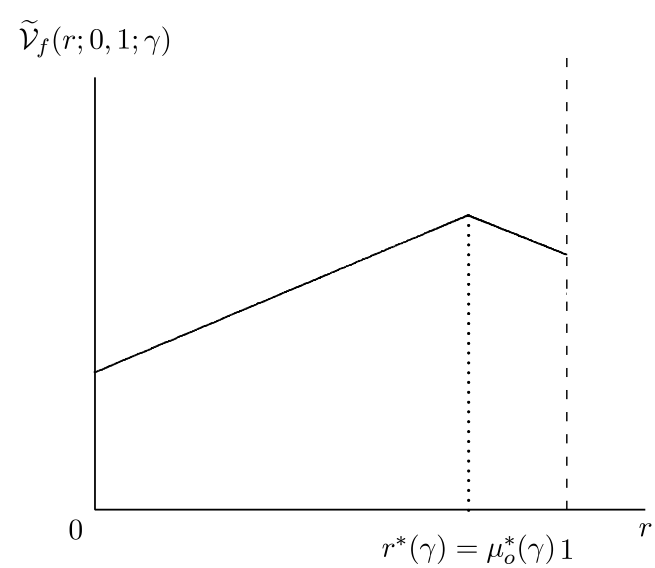

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26]See Appendix A, Remark 10.

[27]See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

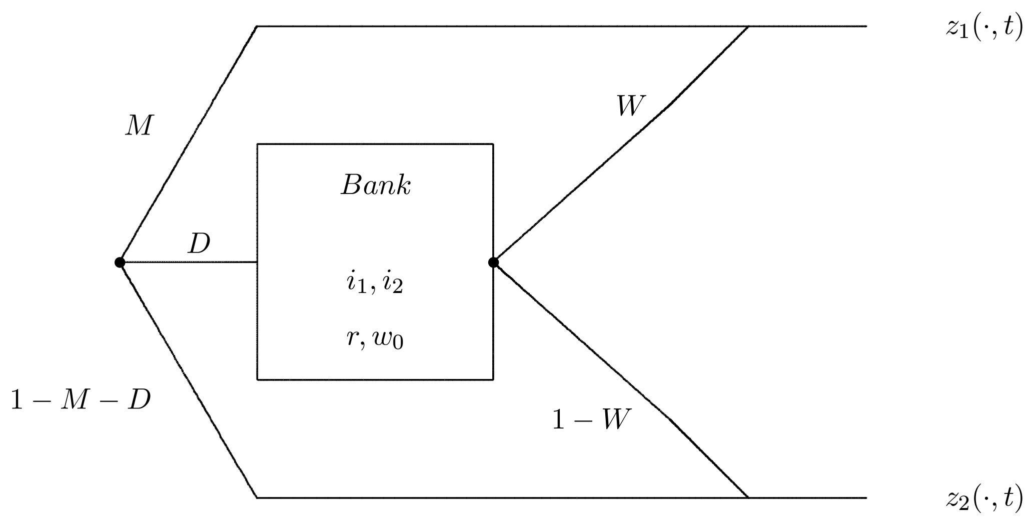

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

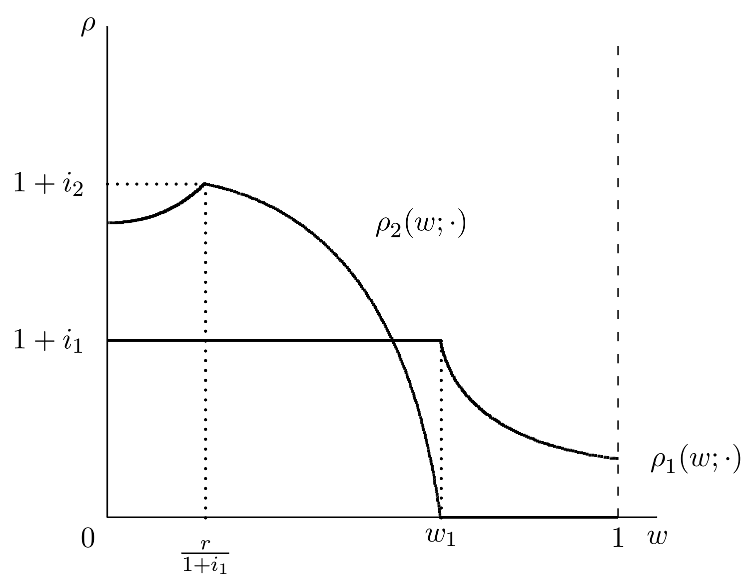

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$
$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

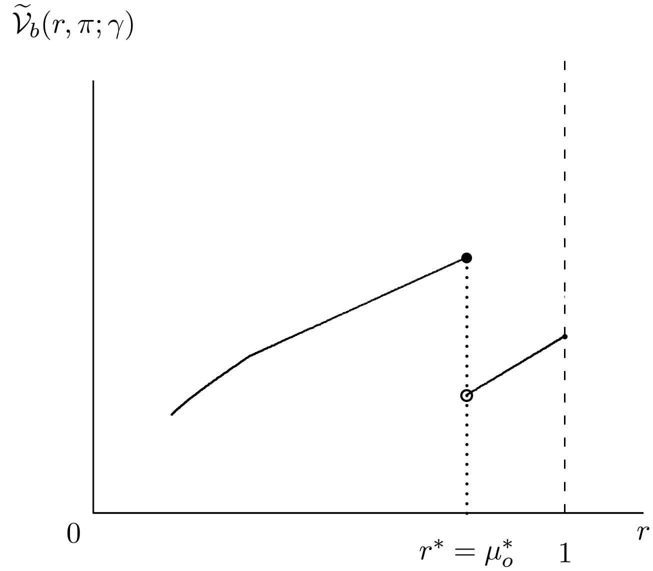

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

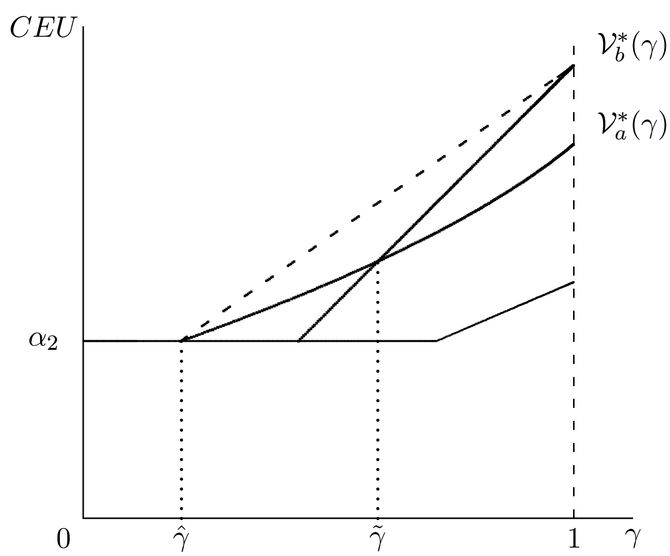

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t,\tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S |\ f(s) \geq t\}$ and $\{s \in S |\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^{\infty} \nu(\{s \in S |\ f(s) \geq t\}) \, dt + \int_{-\infty}^{0} [\nu(\{s \in S |\ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma\cdot[\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)]+(1-\gamma)\cdot\min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M\cdot[\gamma\cdot[\pi\cdot\beta_h+(1-\pi)\cdot\beta_\ell]+(1-\gamma)\cdot\beta_\ell]+(1-M)\cdot\alpha_2\\ &= M\cdot[\gamma\cdot\pi\cdot\beta_h+(1-\gamma\cdot\pi)\cdot\beta_\ell]+(1-M)\cdot\alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu\in[0,1]$ and $\tau\in[0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu)=0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell\cdot\mu}{\beta_\ell\cdot\mu+\alpha_2\cdot(1-\mu)},\\ \overline{\tau}(\mu) &:= \frac{\beta_h\cdot\mu}{\beta_h\cdot\mu+\alpha_2\cdot(1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu)=\begin{cases}\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}+\mu & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ \dfrac{\mu}{\tau} & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{1\ell}(\tau,\mu)=\begin{cases}\mu-\left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ 0 & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0,\overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu),1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0,\overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu),1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau)\cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1)\cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0,\underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell]\cdot\mu & \text{if} \quad \pi \in [\underline{\tau}(\mu),1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ*. This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$*, depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := & \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

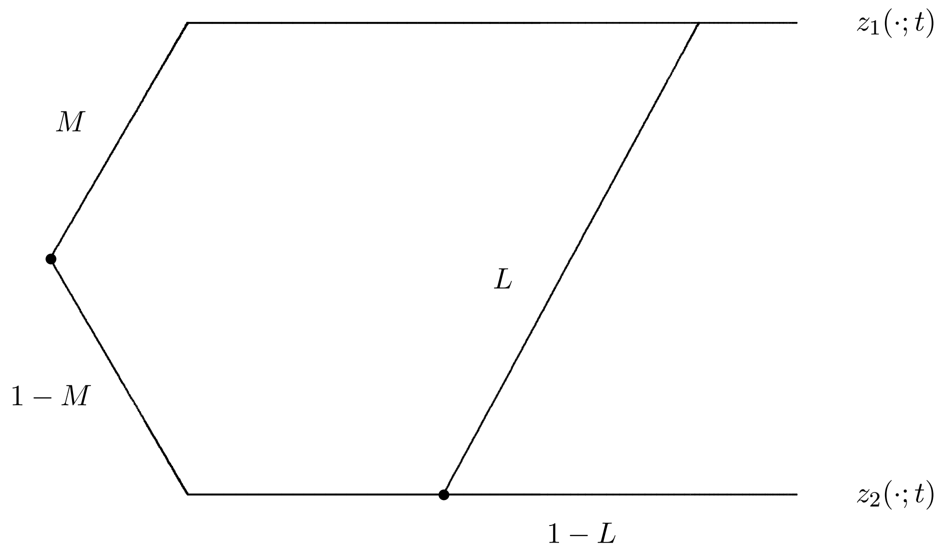

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) &:= \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) &:= \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2 \,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

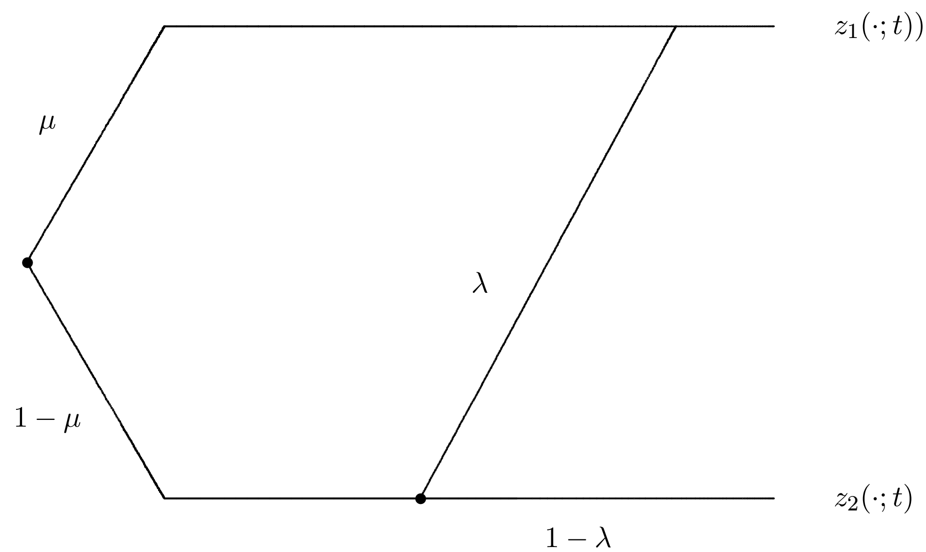

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \text{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \text{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \text{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \text{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \text{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

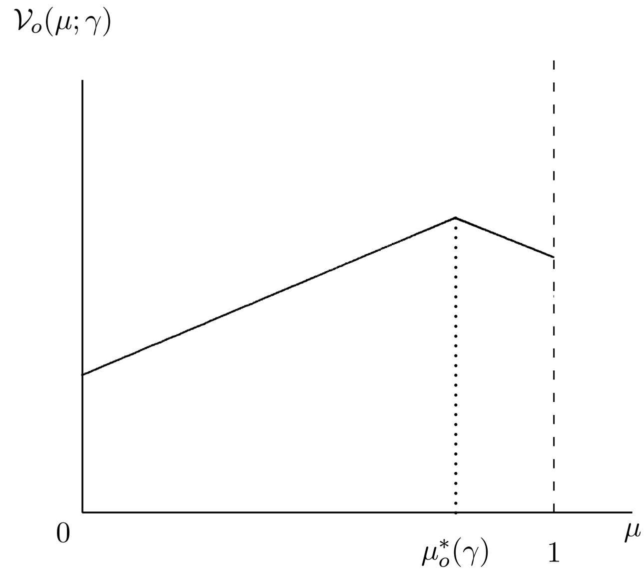

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

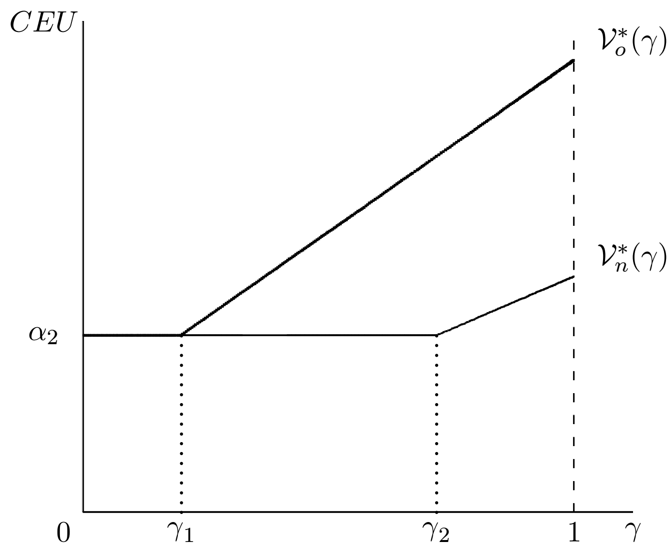

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

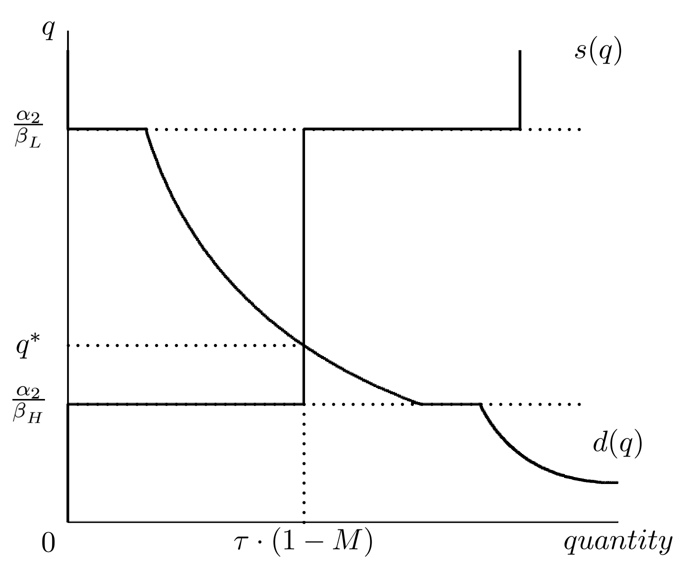

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

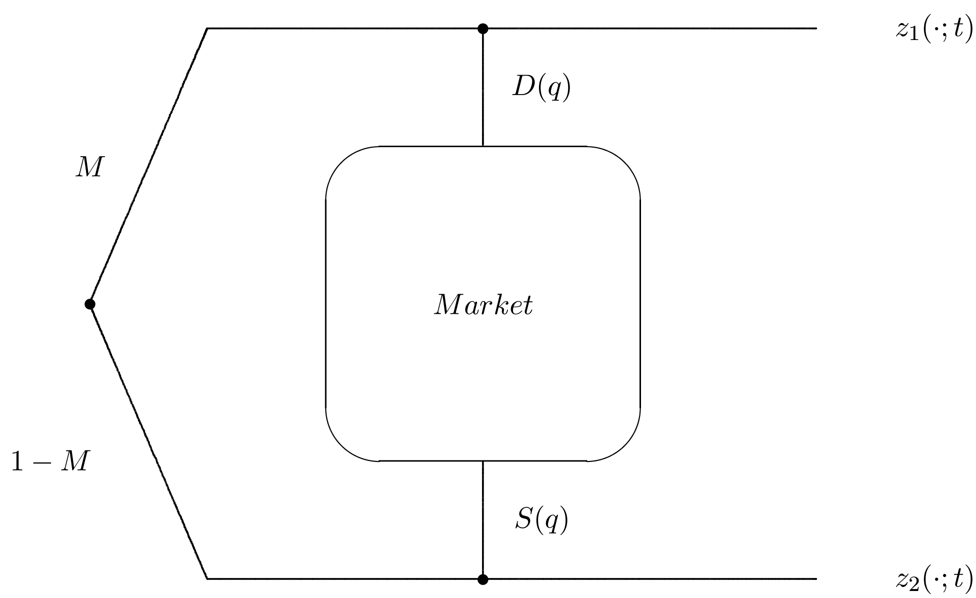

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

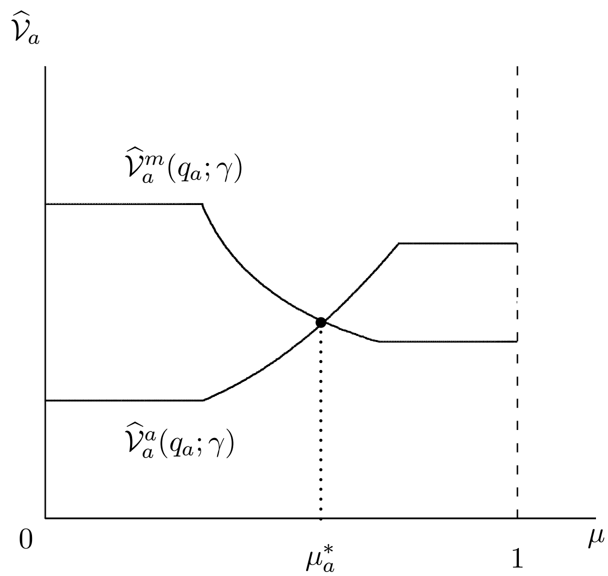

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

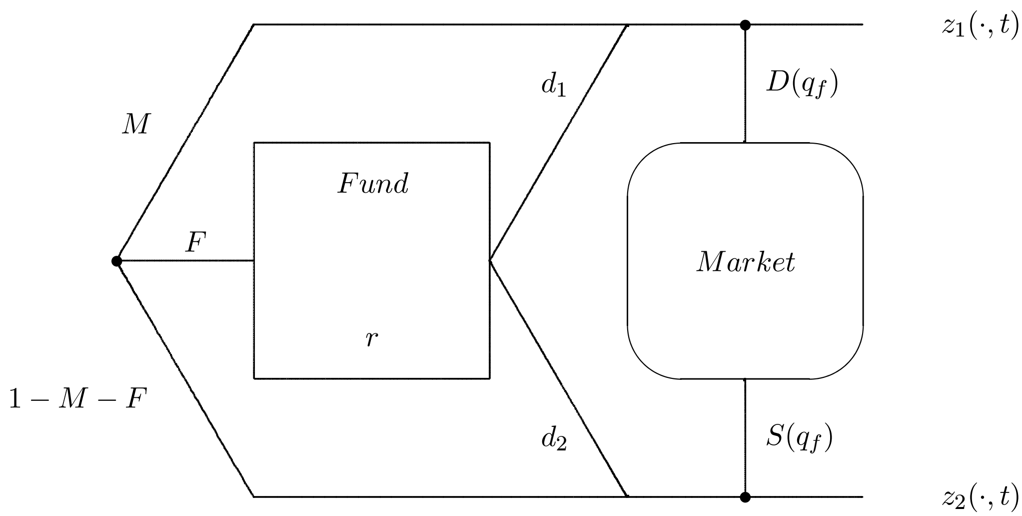

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau)\cdot\mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r)\cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

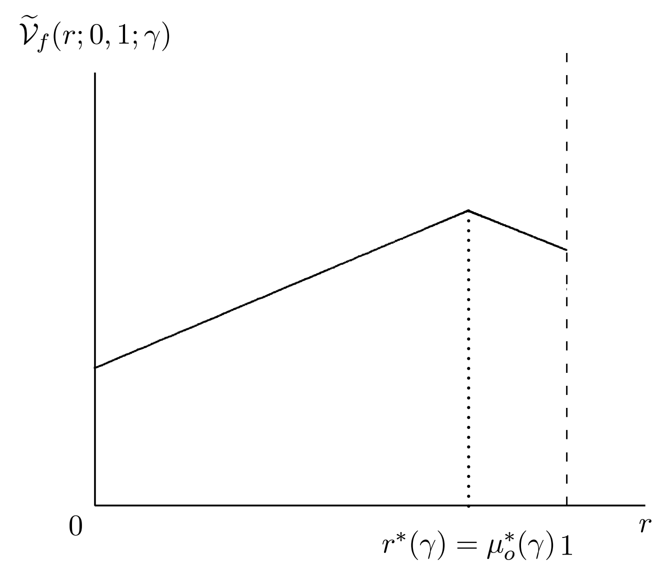

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26]See Appendix A, Remark 10.
[27]See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

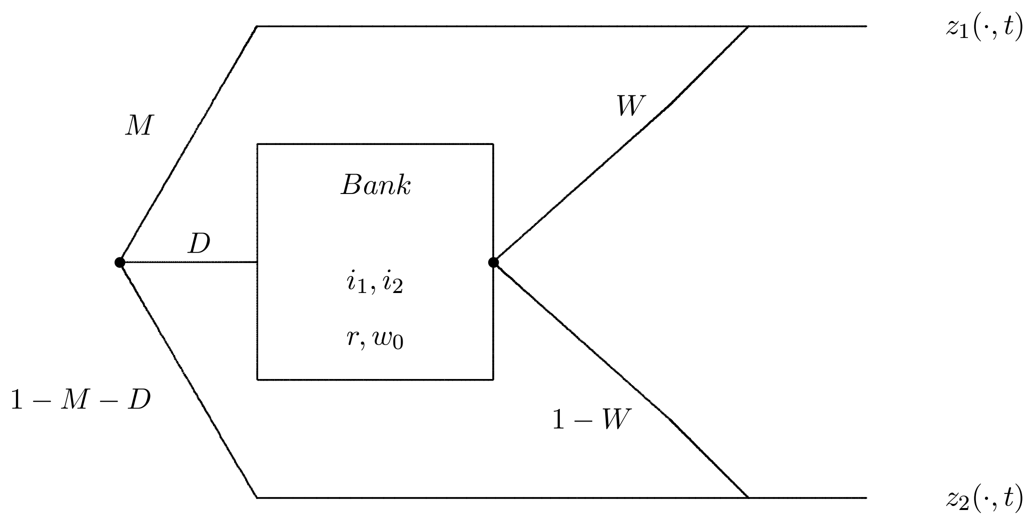

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

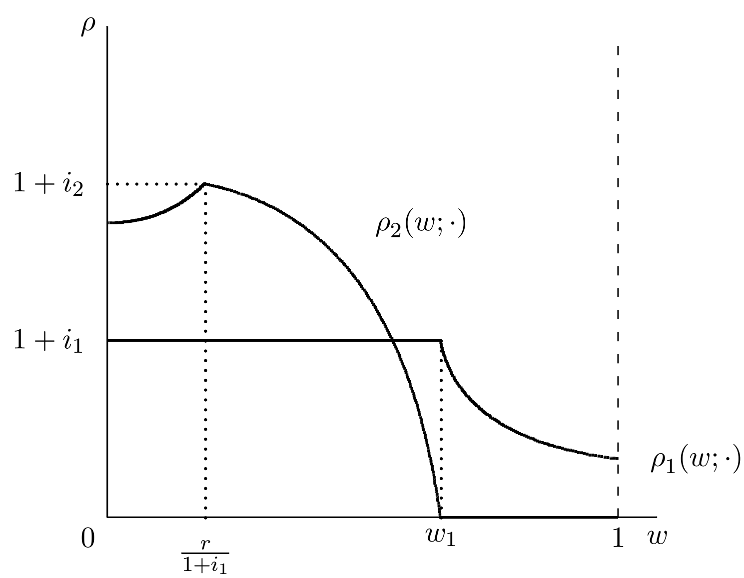

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30]See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}&\mathcal{V}_b(D, M; r, w_0; \gamma)\\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau\\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$
$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

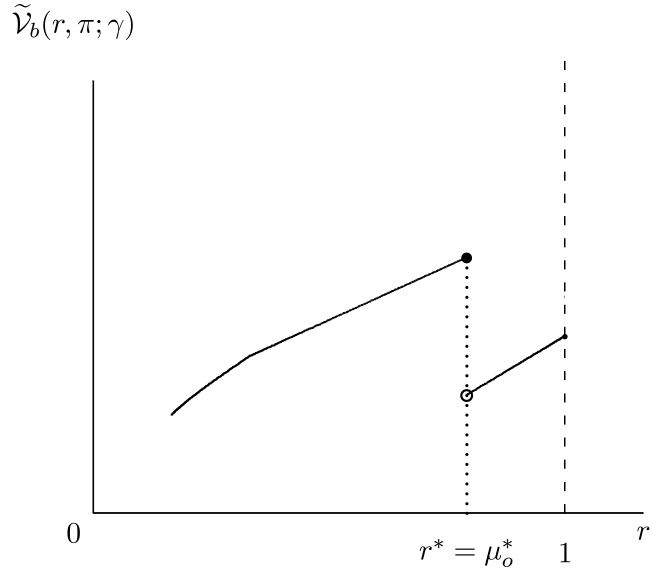

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

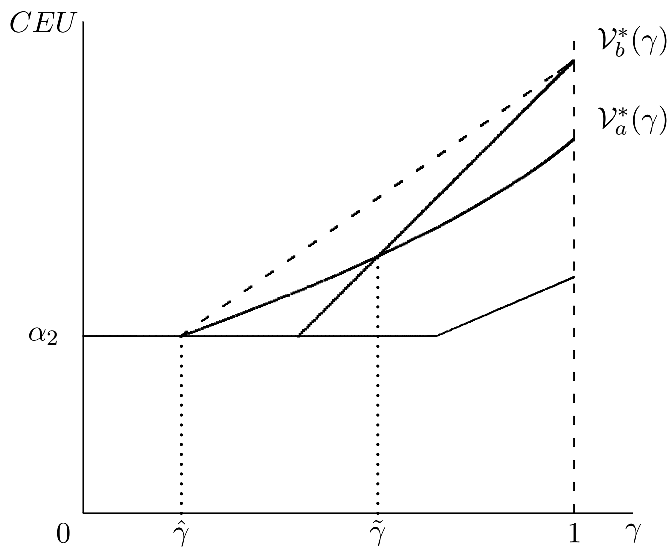

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S |\ f(s) \geq t\}$ and $\{s \in S |\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S |\ f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S |\ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau)\cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1+\gamma\cdot(\frac{\beta_h}{\beta_\ell}-1)\cdot\tau]\cdot[\alpha_2\cdot(1-\mu)+\beta_\ell\cdot\mu] & \text{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2\cdot(1-\mu) + [\gamma\cdot\pi\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell]\cdot\mu & \text{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) :=& \quad \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ =& \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} &\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ &+ \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market exists and the price*

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma, \pi, \alpha_2, \beta_h, \beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) := \;& \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma)\\ =\;& \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell,\\ & \gamma \cdot [\pi \cdot \max\{\beta_h \cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot \max\{\beta_\ell \cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot \alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a; \gamma) = \gamma \cdot [\pi \cdot R^m(q_a; h) + (1-\pi)\cdot R^m(q_a; \ell)] + (1-\gamma)\cdot \beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a; \gamma) = \gamma \cdot [\pi \cdot R^a(q_a; h) + (1-\pi)\cdot R^a(q_a; \ell)] + (1-\gamma)\cdot \alpha_2.$$

The worst case for holding money occurs for $(t, \tau) = (\ell, 0)$*. For investing in the illiquid asset it occurs for* $(t, \tau) = (\ell, \tau)$ *for any* $\tau \in [0, 1]$*. From this it follows that for an asset price* q_a *the ex-ante Choquet expected utility for a consumer who holds a fraction* M *of his initial one unit of wealth as money is*

$$M \cdot \widehat{\mathcal{V}}_a^m(q_a; \gamma) + (1-M)\cdot \widehat{\mathcal{V}}_a^a(q_a; \gamma).$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

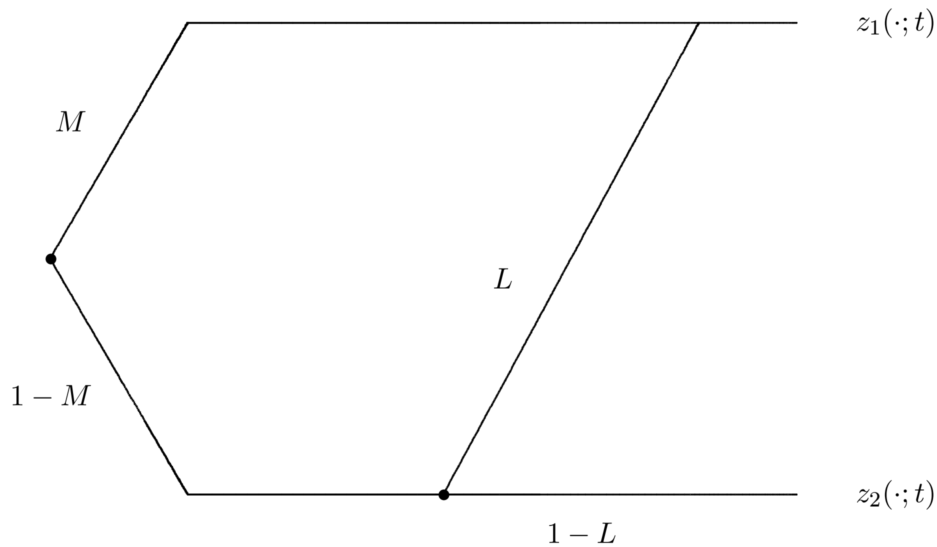

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M; t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

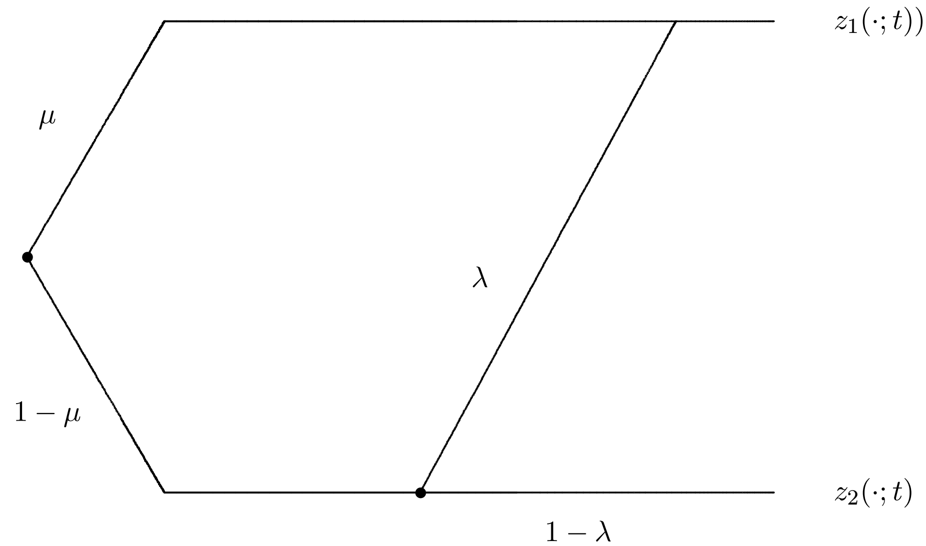

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

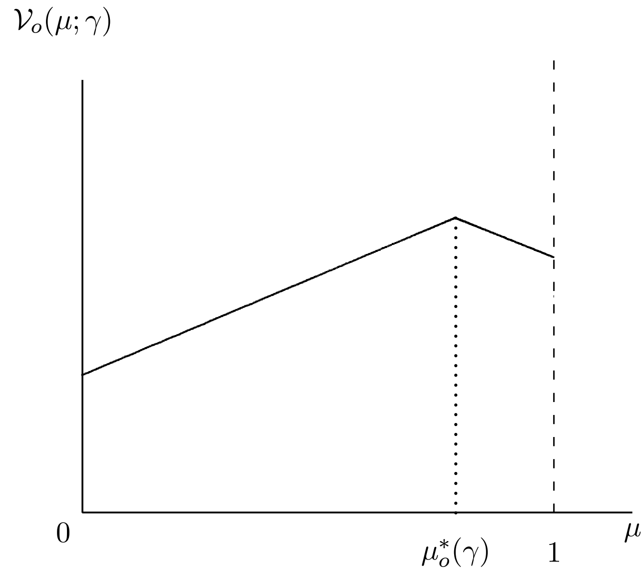

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

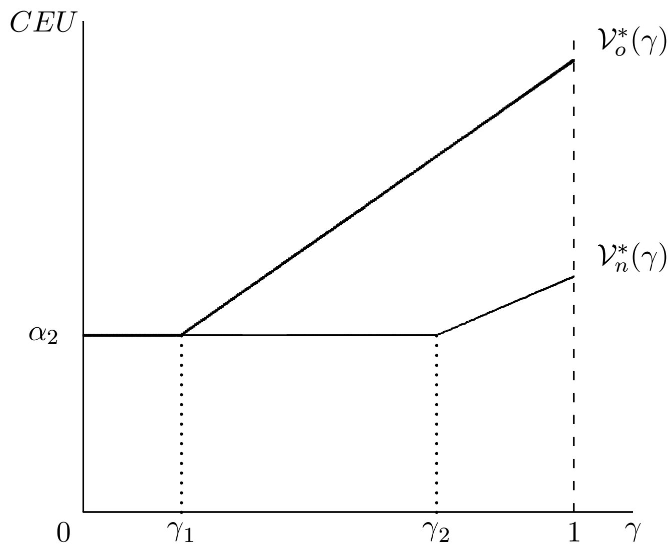

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

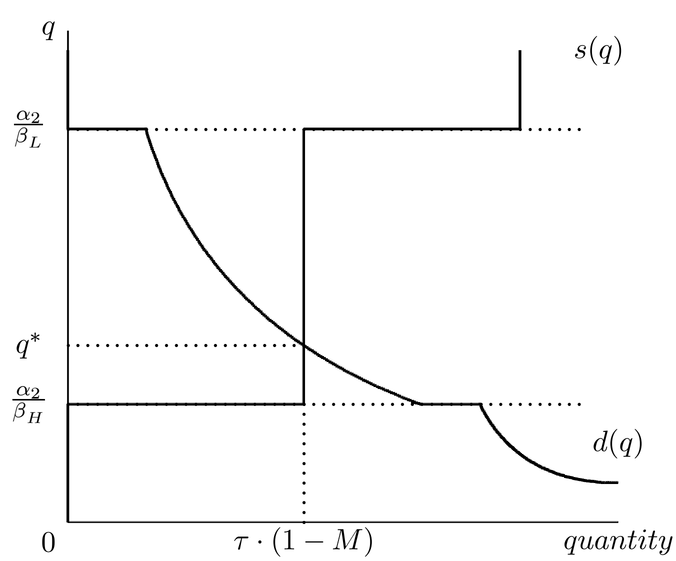

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau, \mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau, \mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

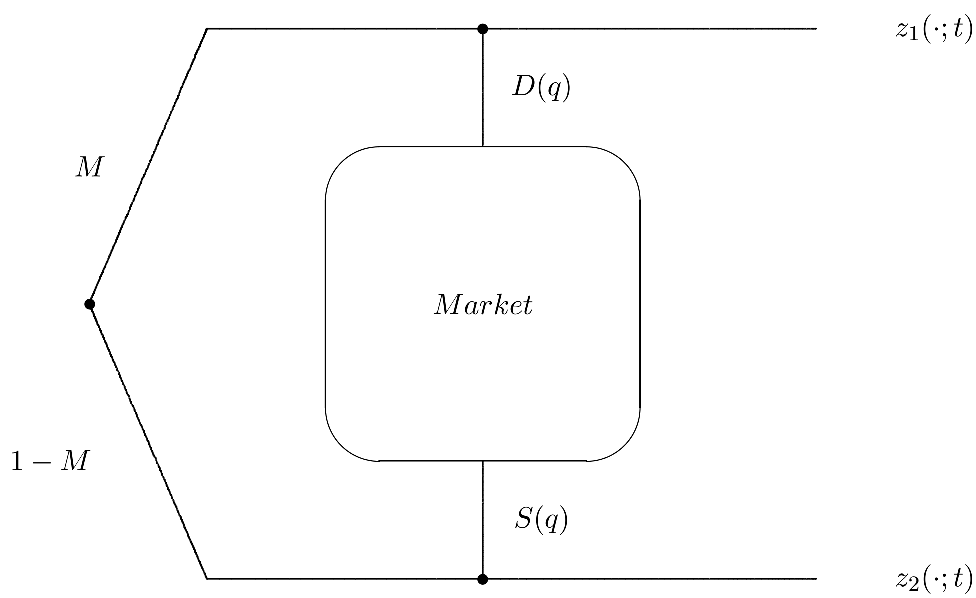

Figure 6: Asset market

Denote by $R^m(q_a; t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a; t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M; \tau, \mu; t)$, from the investment decision M is easily computed:

$$v_a(M; \tau, \mu; t) := M \cdot R^m(q_a^E(\tau, \mu); t) + (1 - M) \cdot R^a(q_a^E(\tau, \mu); t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] .\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma), \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi,$
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi),$
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}].$

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

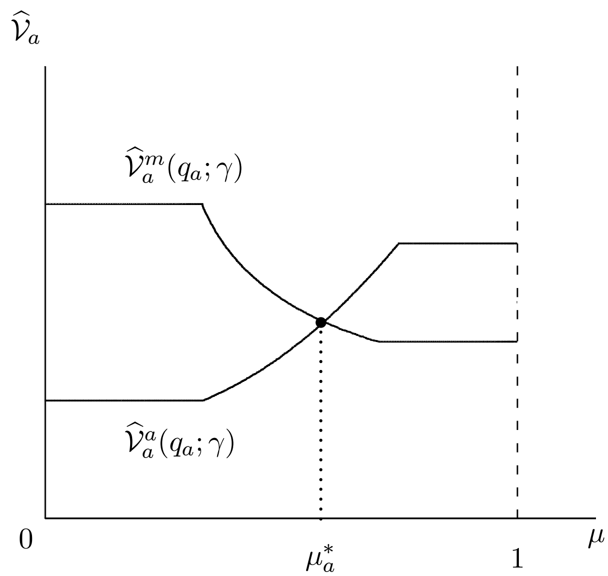

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

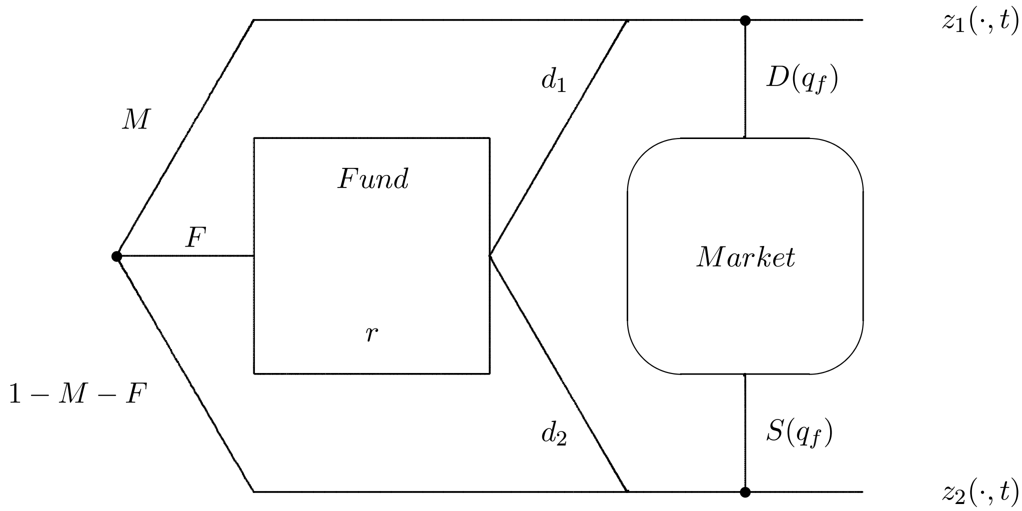

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= r \\ d_2 &:= \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] \\ +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

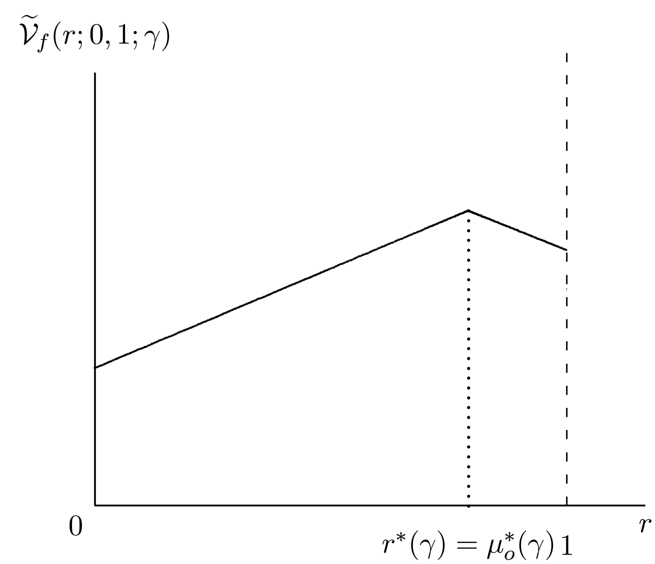

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

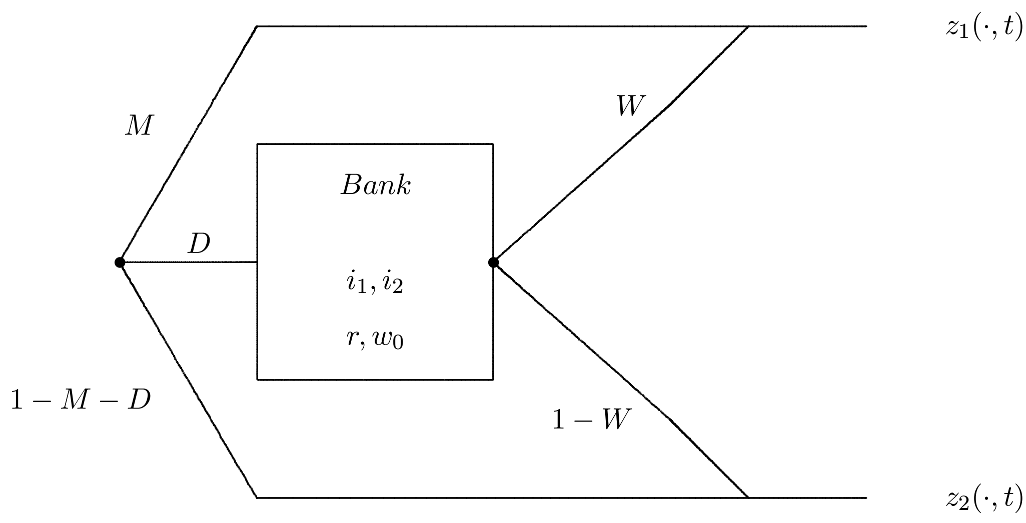

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

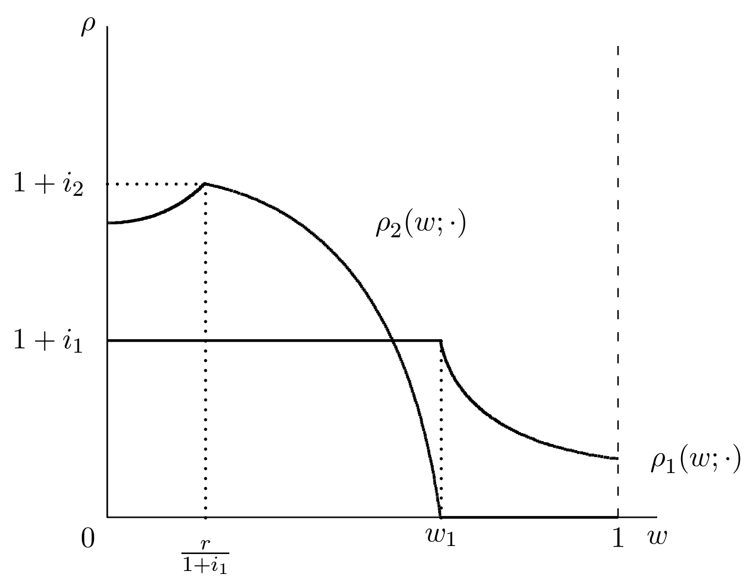

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

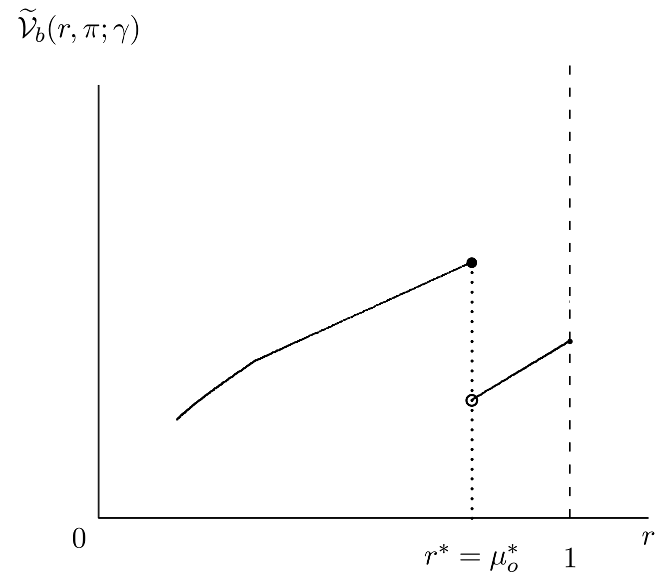

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \ \widetilde{\mathcal{V}}_b(r^*, w_0^*;\gamma) \\ = & \ \max\{\mathcal{V}_n^*(\gamma), \ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1-r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1-r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

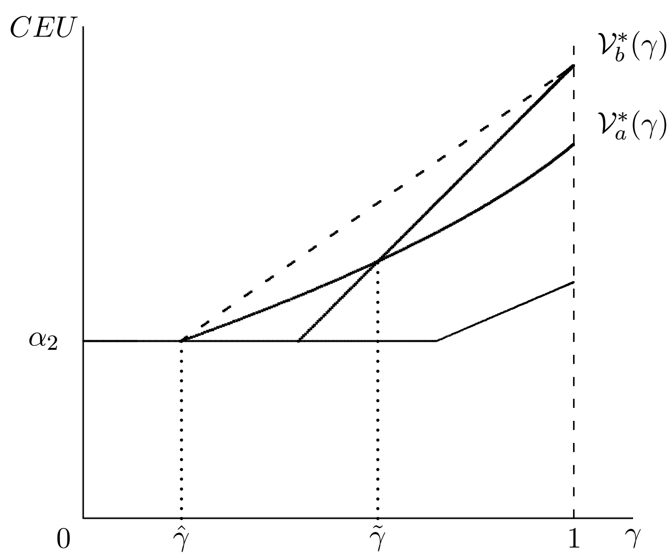

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t,\tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S|\ f(s) \geq t\}$ and $\{s \in S|\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\int^{C} f\, d\nu = \int_0^{\infty} \nu(\{s \in S|\ f(s) \geq t\})\, dt + \int_{-\infty}^{0} [\nu(\{s \in S|\ f(s) \geq t\}) - 1]\, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma\cdot[\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)]+(1-\gamma)\cdot\min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M\cdot[\gamma\cdot[\pi\cdot\beta_h+(1-\pi)\cdot\beta_\ell]+(1-\gamma)\cdot\beta_\ell]+(1-M)\cdot\alpha_2\\ &= M\cdot[\gamma\cdot\pi\cdot\beta_h+(1-\gamma\cdot\pi)\cdot\beta_\ell]+(1-M)\cdot\alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu\in[0,1]$ and $\tau\in[0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu)=0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu):=\frac{\beta_\ell\cdot\mu}{\beta_\ell\cdot\mu+\alpha_2\cdot(1-\mu)},$$
$$\overline{\tau}(\mu):=\frac{\beta_h\cdot\mu}{\beta_h\cdot\mu+\alpha_2\cdot(1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu)=\begin{cases}\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}+\mu & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ \dfrac{\mu}{\tau} & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{1\ell}(\tau,\mu)=\begin{cases}\mu-\left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ 0 & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0,\overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu),1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0,\overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu),1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0,\underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu),1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ. *This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$, *depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \quad \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & \quad + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market exists and the price*

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi$,
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi)$,
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) :=\; & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma)\\ =\; & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell,\\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$*. For investing in the illiquid asset it occurs for* $(t,\tau) = (\ell,\tau)$ *for any* $\tau \in [0,1]$*. From this it follows that for an asset price* q_a *the ex-ante Choquet expected utility for a consumer who holds a fraction* M *of his initial one unit of wealth as money is*

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M) \in [0,1] \times [0,1] \\ s.t.\ F+M \leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \, + \, (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+ (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

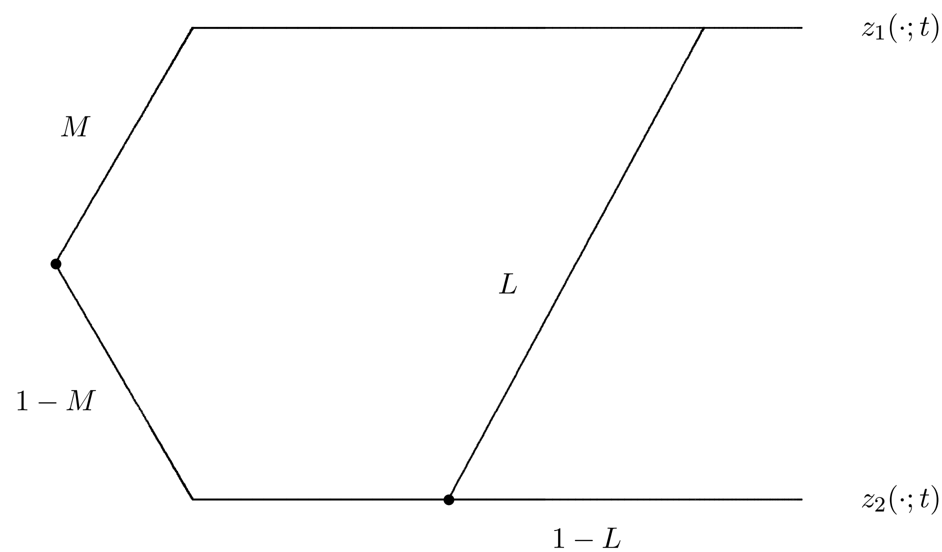

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) :=& \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) :=& \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13]Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

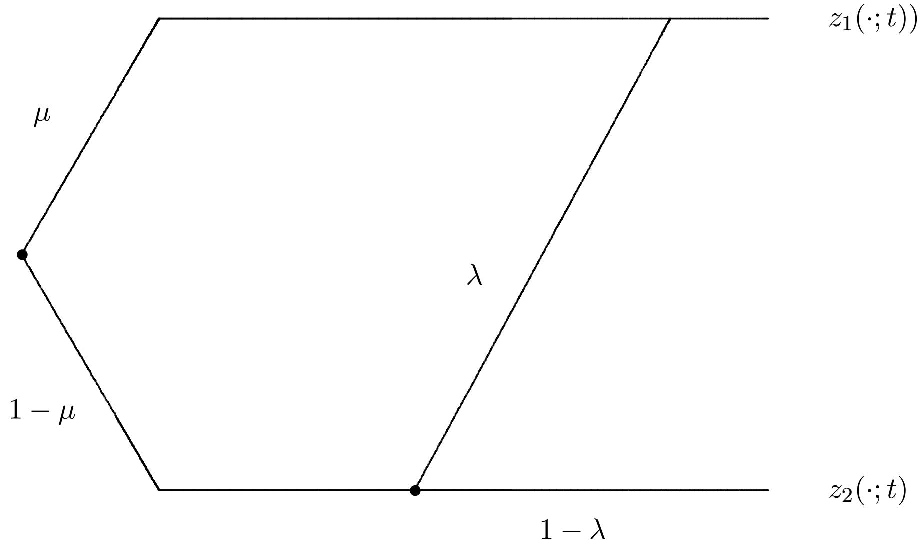

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{lllll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

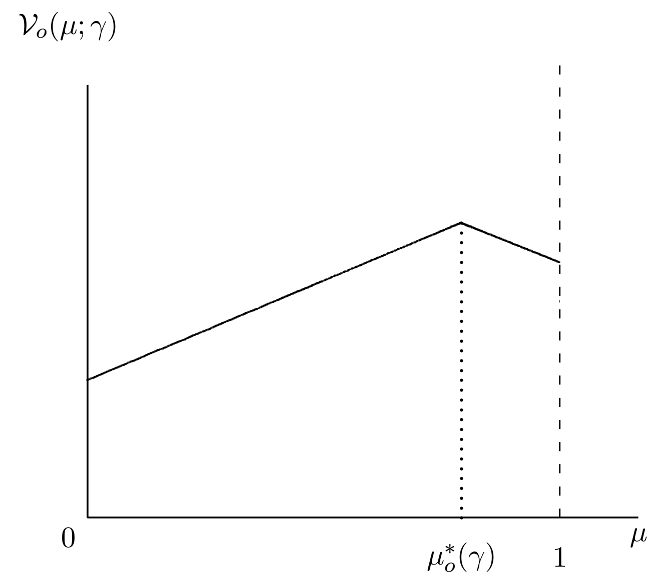

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

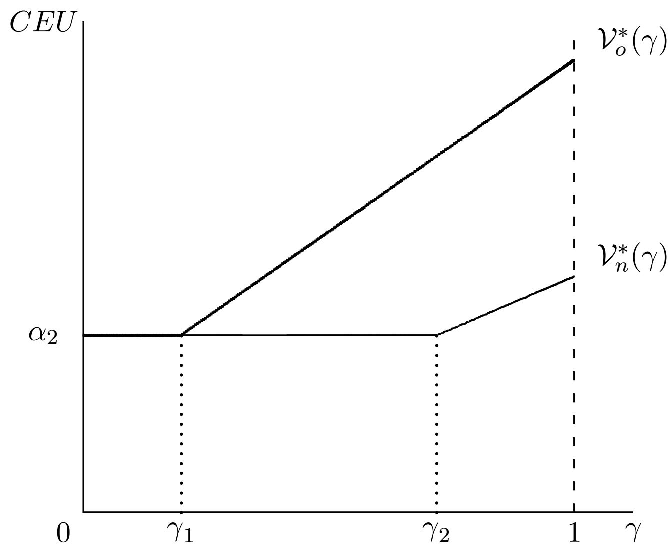

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

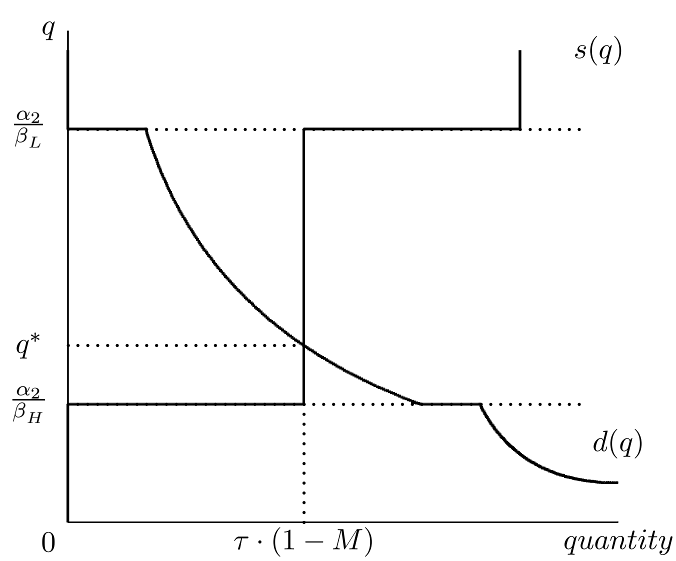

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

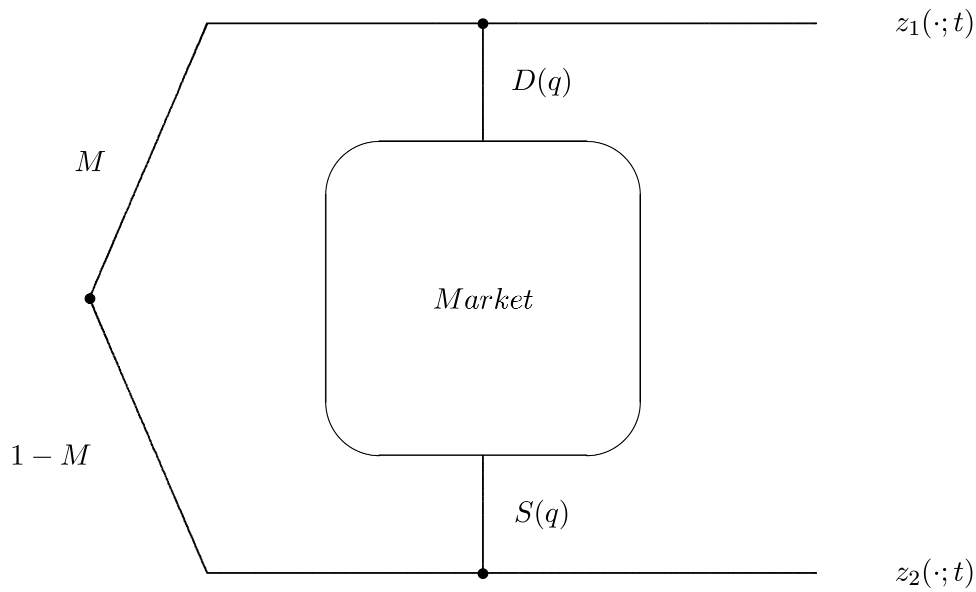

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] \,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi,$
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi),$
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}].$

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

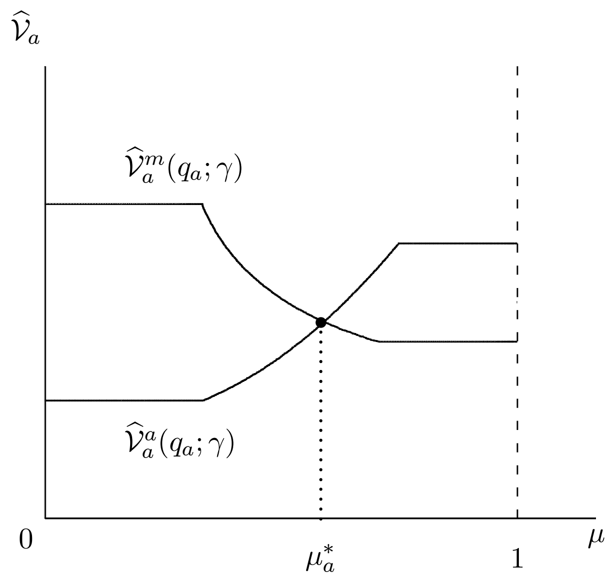

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

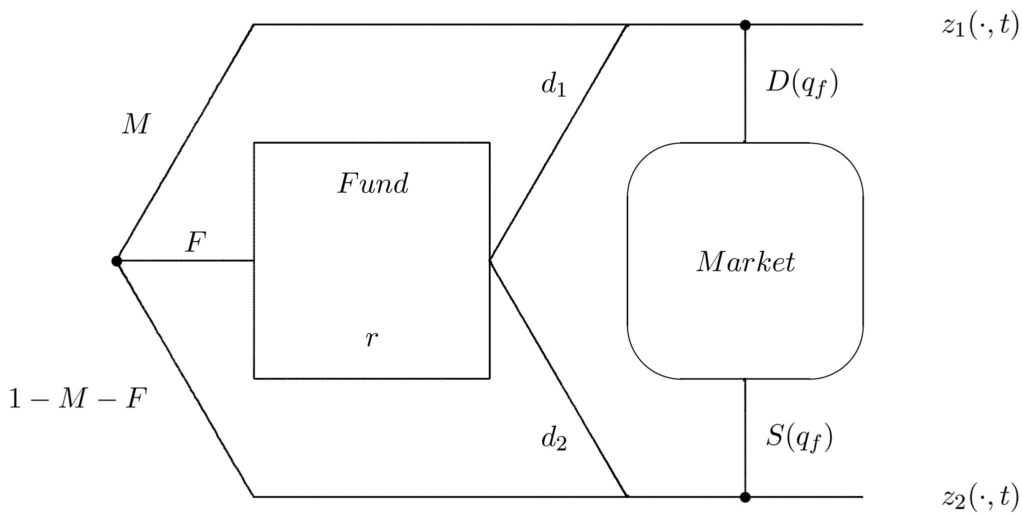

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau)\cdot\mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r)\cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] \\ +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

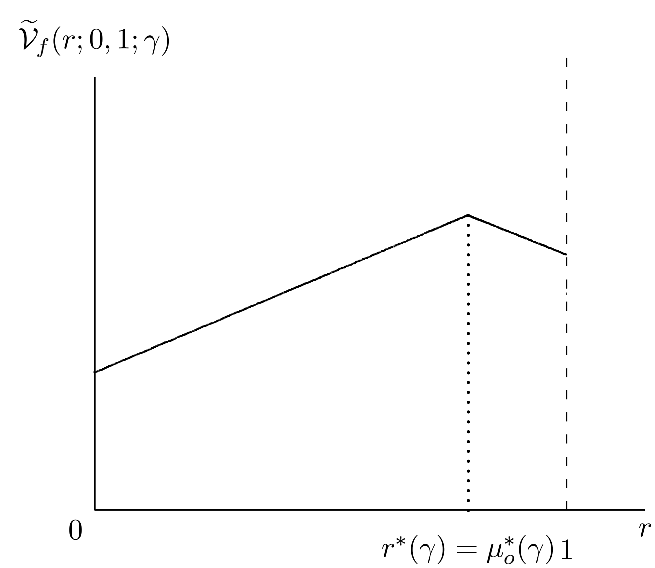

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.
For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26]See Appendix A, Remark 10.
[27]See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

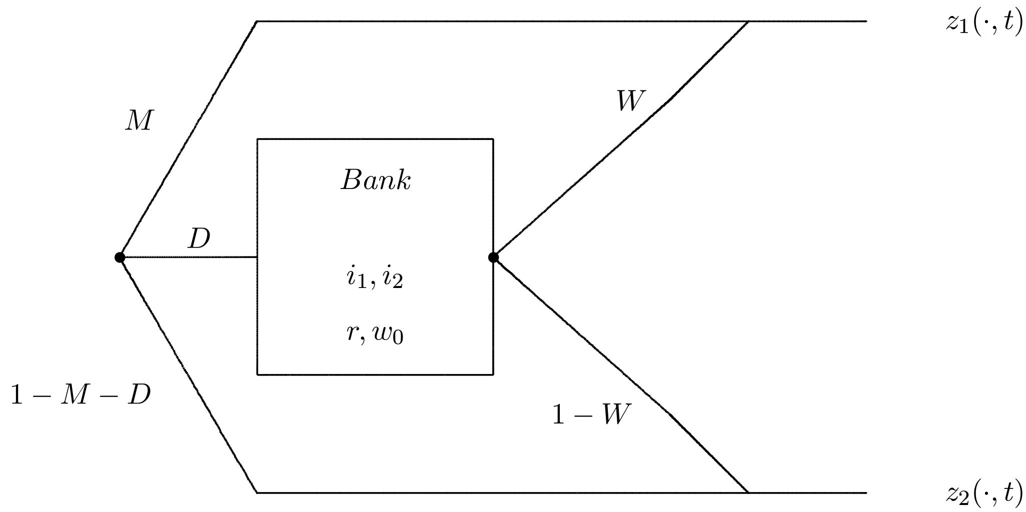

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

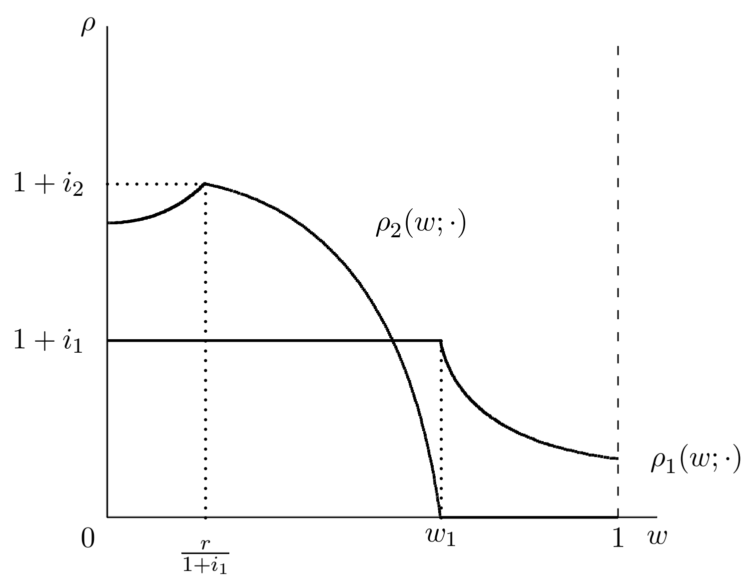

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) :=& \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ =& \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

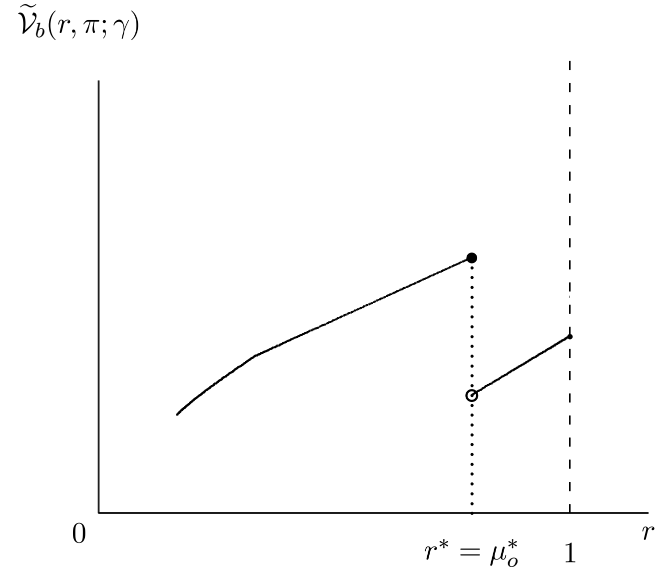

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

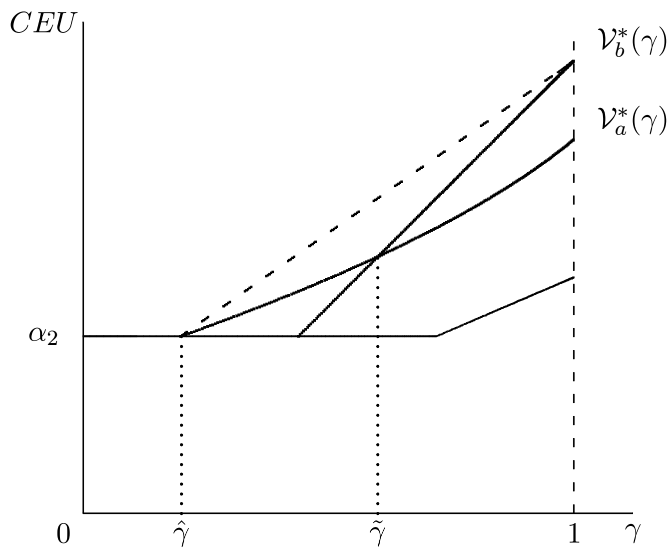

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t,\tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S|\ f(s) \geq t\}$ and $\{s \in S|\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\int^{C} f\, d\nu = \int_0^\infty \nu(\{s \in S|\ f(s) \geq t\})\, dt + \int_{-\infty}^0 [\nu(\{s \in S|\ f(s) \geq t\}) - 1]\, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma\cdot[\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)]+(1-\gamma)\cdot\min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M\cdot[\gamma\cdot[\pi\cdot\beta_h+(1-\pi)\cdot\beta_\ell]+(1-\gamma)\cdot\beta_\ell]+(1-M)\cdot\alpha_2\\ &= M\cdot[\gamma\cdot\pi\cdot\beta_h+(1-\gamma\cdot\pi)\cdot\beta_\ell]+(1-M)\cdot\alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu\in[0,1]$ and $\tau\in[0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu)=0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell\cdot\mu}{\beta_\ell\cdot\mu+\alpha_2\cdot(1-\mu)},\\ \overline{\tau}(\mu) &:= \frac{\beta_h\cdot\mu}{\beta_h\cdot\mu+\alpha_2\cdot(1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu)=\begin{cases}\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}+\mu & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ \dfrac{\mu}{\tau} & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{1\ell}(\tau,\mu)=\begin{cases}\mu-\left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ 0 & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma);\gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := & \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & + \sqrt{ \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h } \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market exists and the price*

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) :=\ & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\ =\ & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$*. For investing in the illiquid asset it occurs for* $(t,\tau) = (\ell,\tau)$ *for any* $\tau\in[0,1]$*. From this it follows that for an asset price* q_a *the ex-ante Choquet expected utility for a consumer who holds a fraction* M *of his initial one unit of wealth as money is*

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+ (1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot\alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ,φ)*. Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$.
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1, i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \leq \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

Jürgen Eichberger [2] and Willy Spanjers [3]

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0,1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

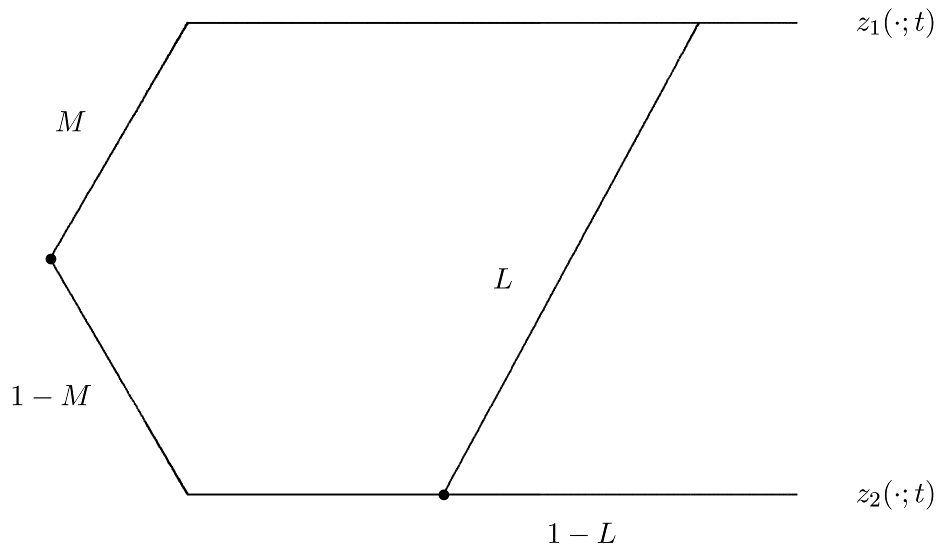

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\; d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2 \,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

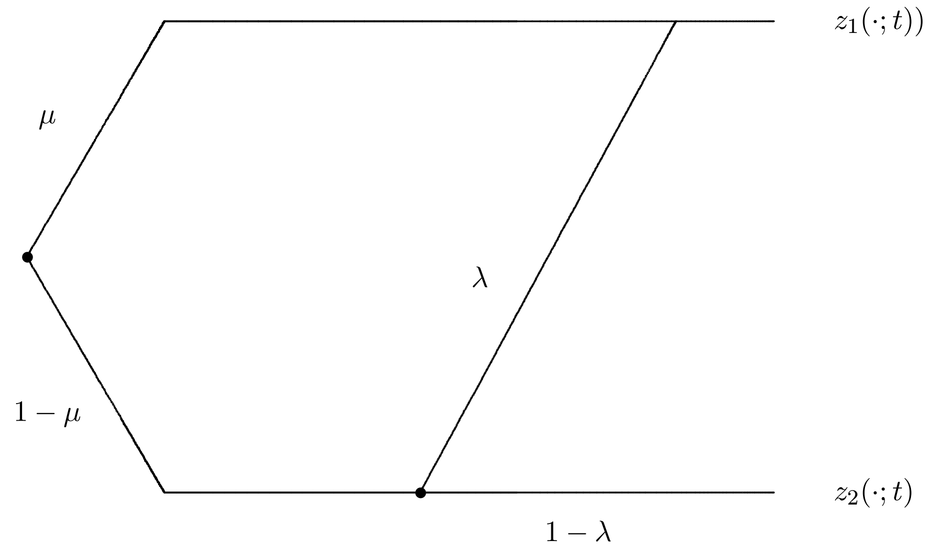

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \multicolumn{3}{l}{\lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0.} & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

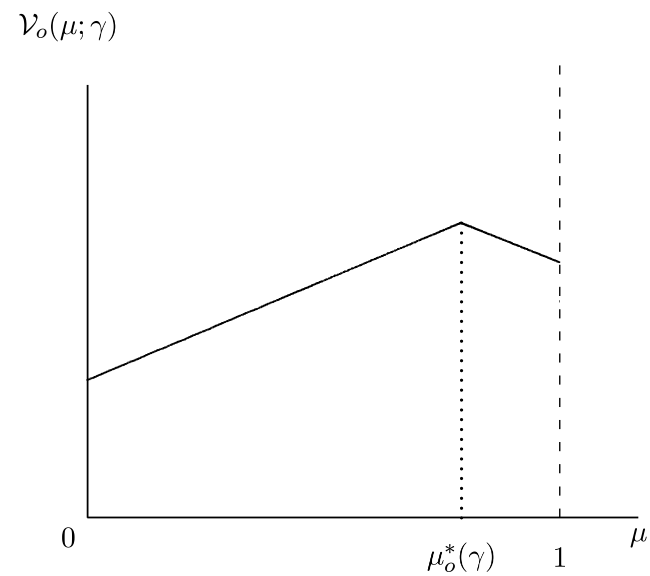

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

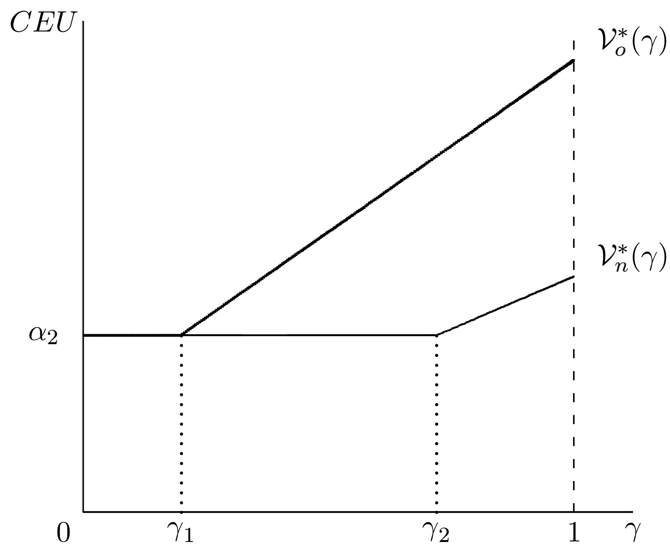

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

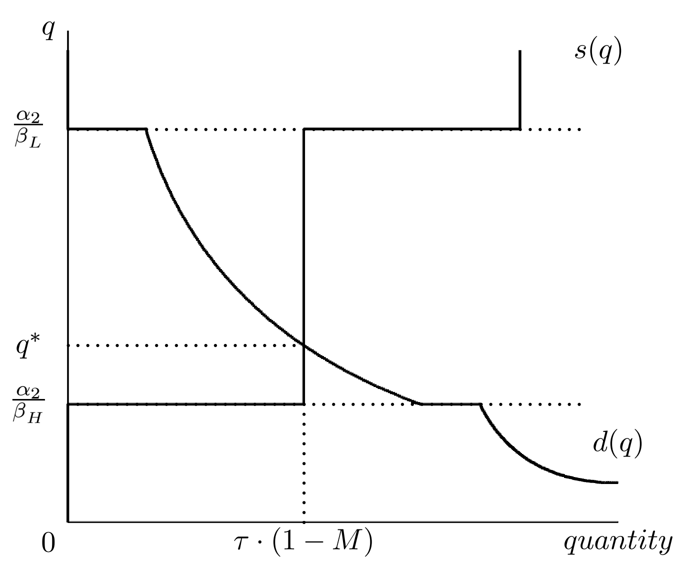

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau, \mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau, \mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

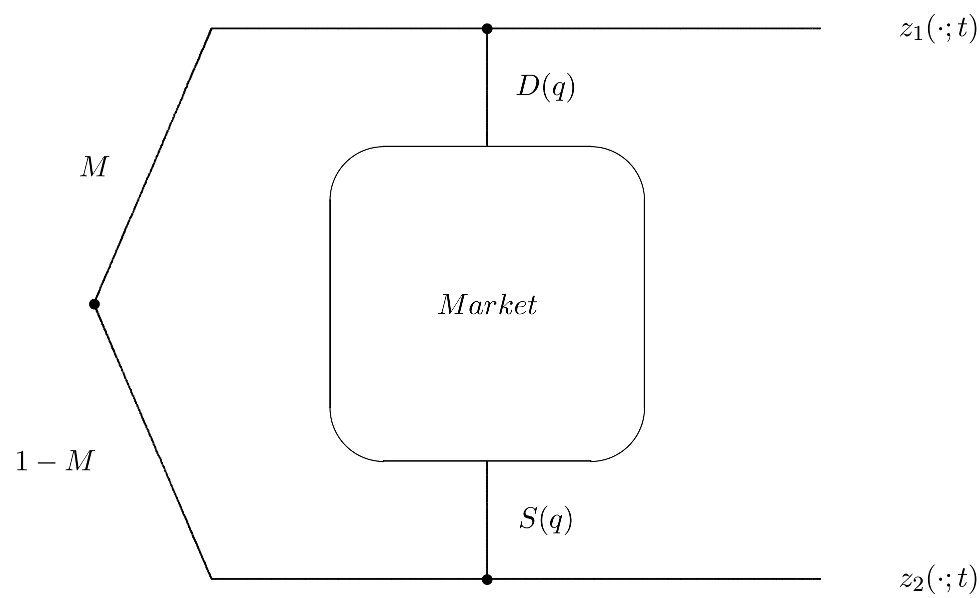

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M) \cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

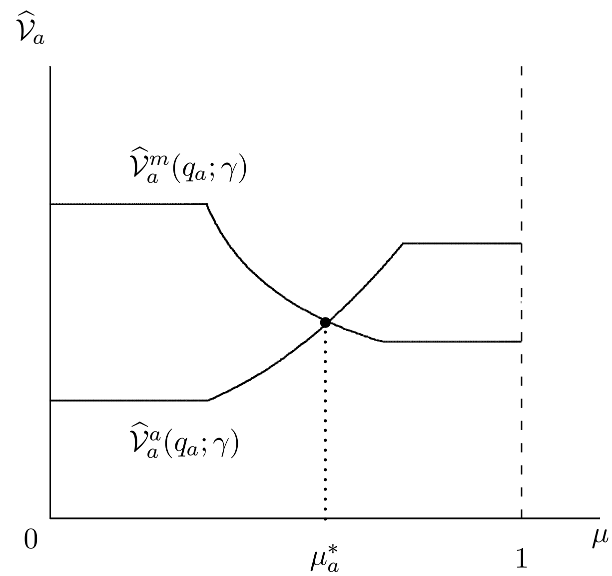

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

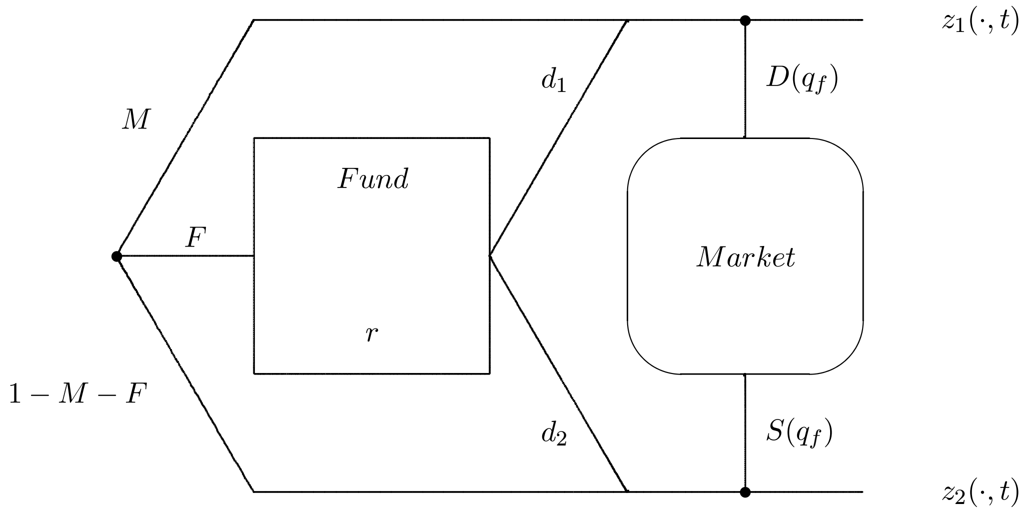

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 :=& \quad r \\ d_2 :=& \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) \,+\, (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] \\ +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

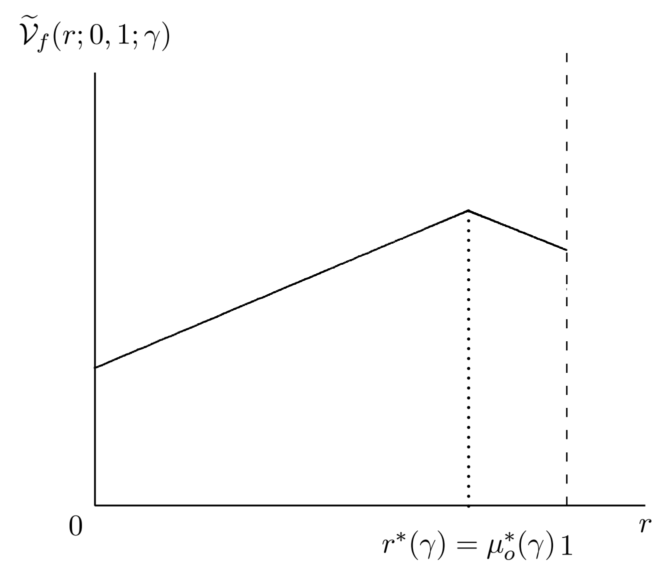

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.
For all $\gamma \in [0, 1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.
The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

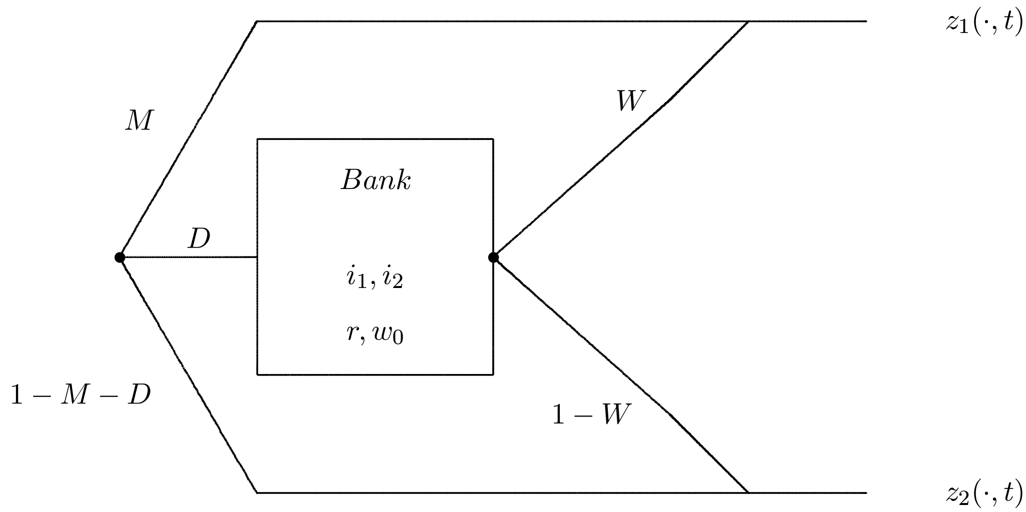

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

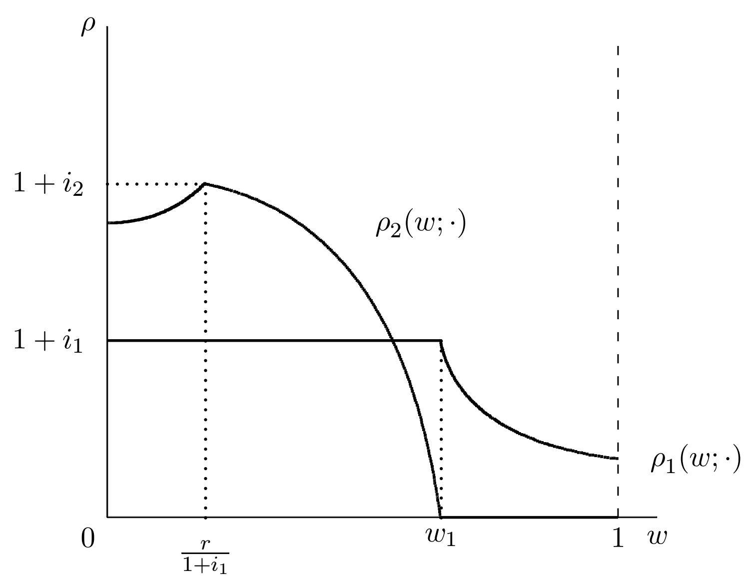

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}&\mathcal{V}_b(D,M;r,w_0;\gamma)\\ &:= \gamma\cdot\int[\tau\cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)]\cdot p(\tau)\,d\tau\\ &\qquad + (1-\gamma)\cdot\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t).\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D,M;r,w_0;\gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D,M;r,w_0;\gamma)$$
$$\text{s.t.} \quad D+M\le 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$.

It remains to determine the deposit contract parameters (r,w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau=\pi$ and $\tau=1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r<1$. As a consequence, banks offer deposit contracts on the basis of $w_0=\pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r,\pi;\gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0=0$ and $w_0=1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

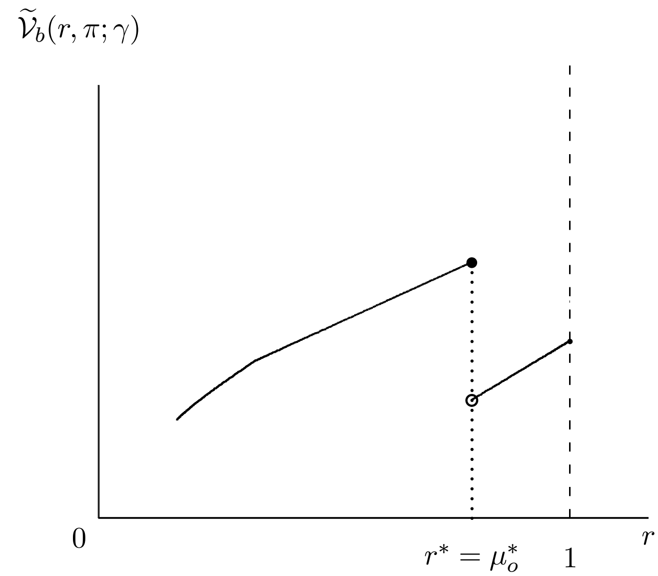

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0, 1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \ \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \ \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

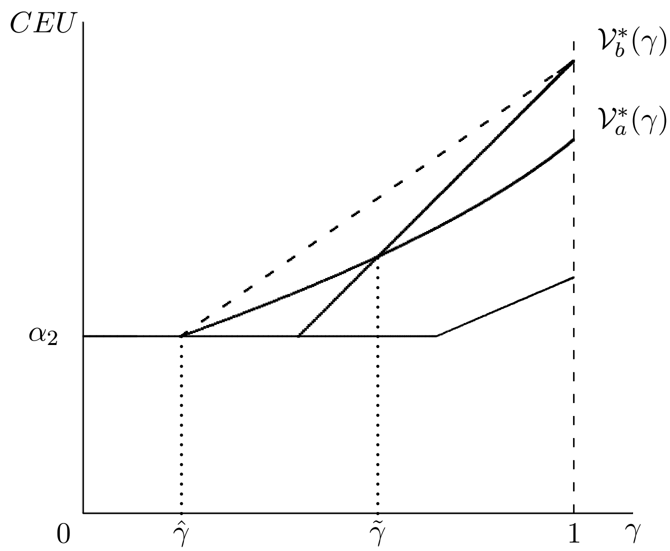

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \ f(s) \geq t\}$ and $\{s \in S | \ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\int^{C} f \, d\nu = \int_0^{\infty} \nu(\{s \in S | \ f(s) \geq t\}) \, dt + \int_{-\infty}^{0} [\nu(\{s \in S | \ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma\cdot[\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)]+(1-\gamma)\cdot\min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M\cdot[\gamma\cdot[\pi\cdot\beta_h+(1-\pi)\cdot\beta_\ell]+(1-\gamma)\cdot\beta_\ell]+(1-M)\cdot\alpha_2\\ &= M\cdot[\gamma\cdot\pi\cdot\beta_h+(1-\gamma\cdot\pi)\cdot\beta_\ell]+(1-M)\cdot\alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu\in[0,1]$ and $\tau\in[0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu)=0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell\cdot\mu}{\beta_\ell\cdot\mu+\alpha_2\cdot(1-\mu)},\\ \overline{\tau}(\mu) &:= \frac{\beta_h\cdot\mu}{\beta_h\cdot\mu+\alpha_2\cdot(1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu)=\begin{cases}\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}+\mu & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ \dfrac{\mu}{\tau} & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{1\ell}(\tau,\mu)=\begin{cases}\mu-\left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ 0 & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0,\overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu),1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0,\overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu),1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0,\underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu),1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ. *This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$, *depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) :=\ & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma)\\ =\ & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell,\\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau \in [0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$
$$+(1-\gamma)\cdot [r\cdot \beta_\ell + \alpha_2 \cdot (1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ, φ). *Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$.
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1, i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \textit{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \textit{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0*, one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period 1, i.e.,* $w=\tau$*, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ *the constraint*

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution “competing banks” under conditions that are most favourable for consumers.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$*, where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

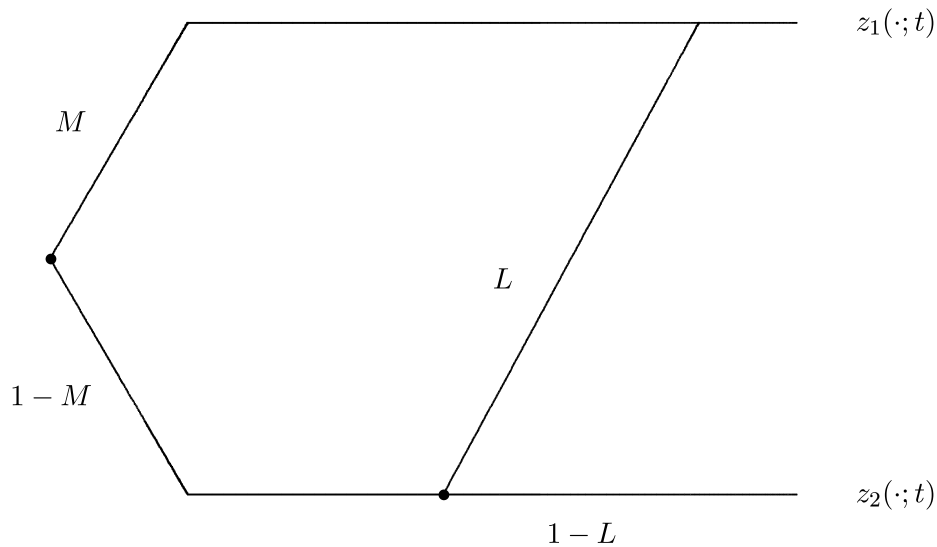

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) &:= \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) &:= \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1-M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & \qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & \qquad + (1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

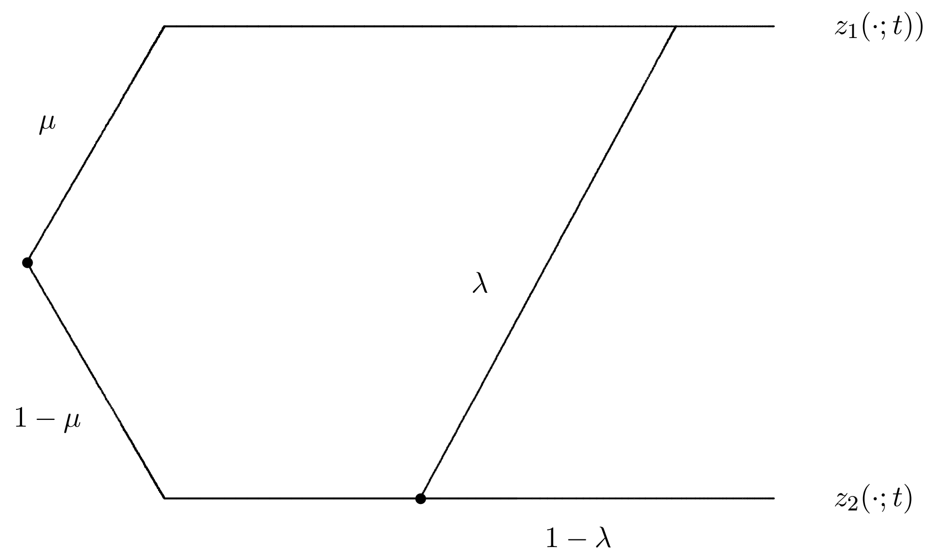

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$U(z_h, z_\ell; \tau) := \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell)$$
$$:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}].$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0, 1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0, 1]$,

- the pay-offs $(z_{1h}(\tau, \mu), z_{2h}(\tau, \mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau, \mu), z_{2\ell}(\tau, \mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau, \mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$
\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}
$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := & \quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

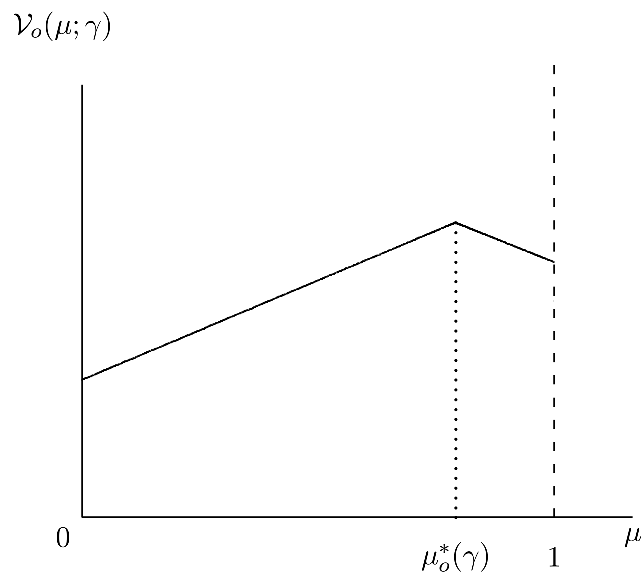

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

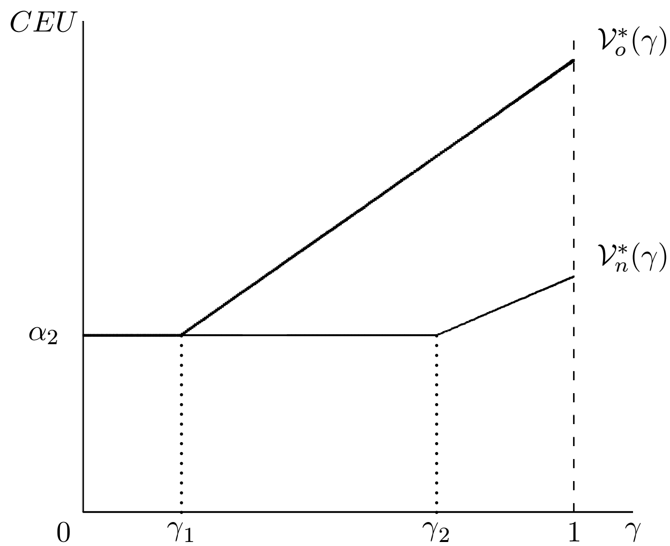

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

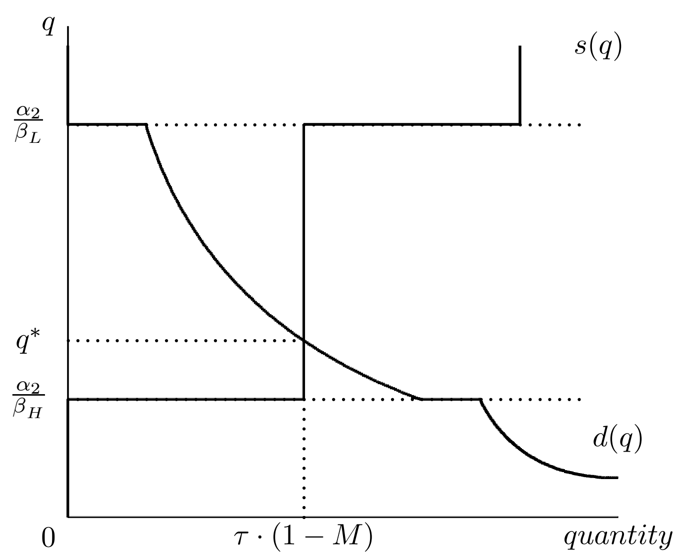

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

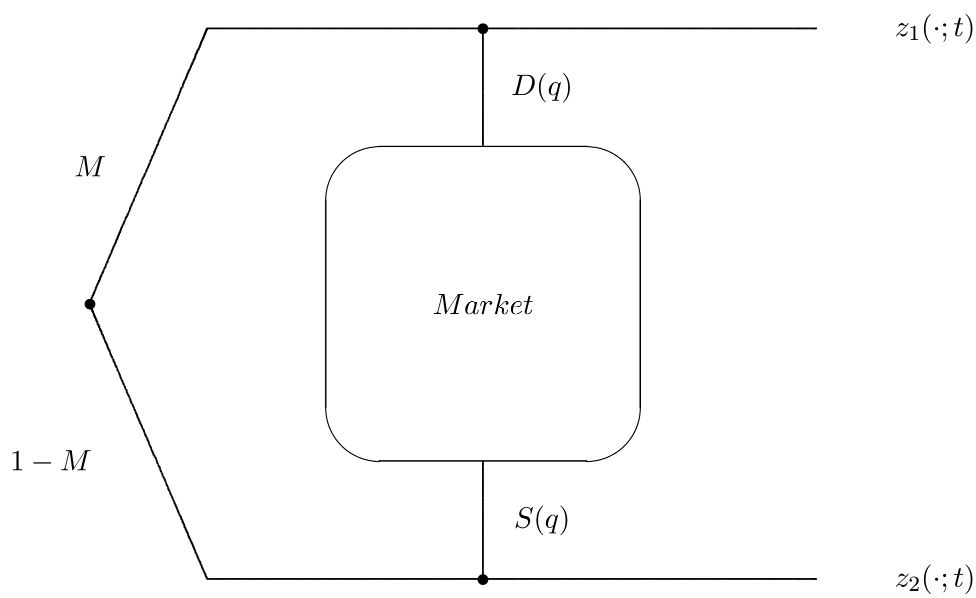

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t\cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M\cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]}[v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

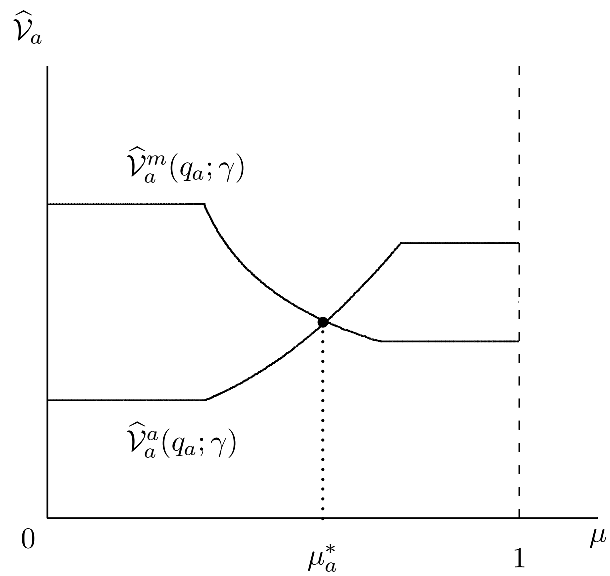

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

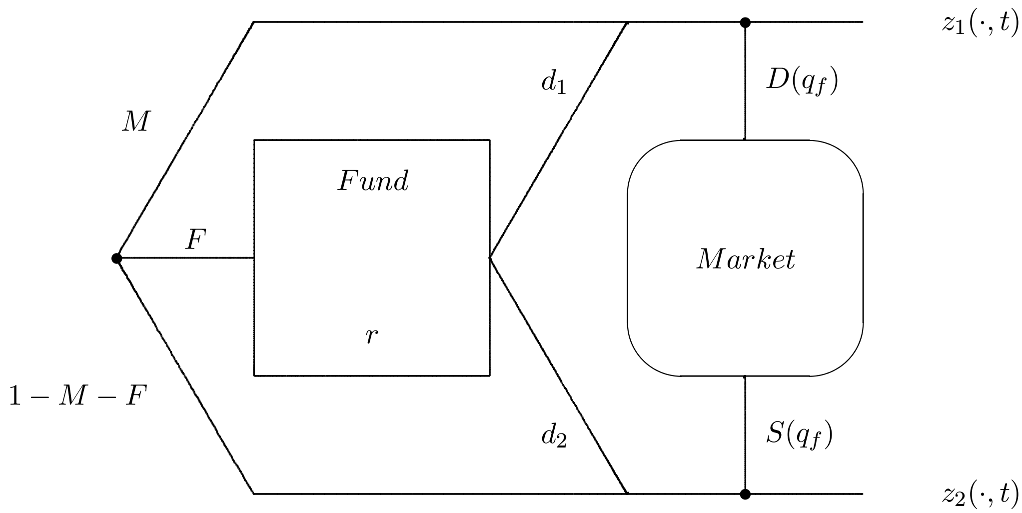

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1-r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma \cdot [\pi \cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)]$$
$$+(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$
$$\text{s.t. } F+M \le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

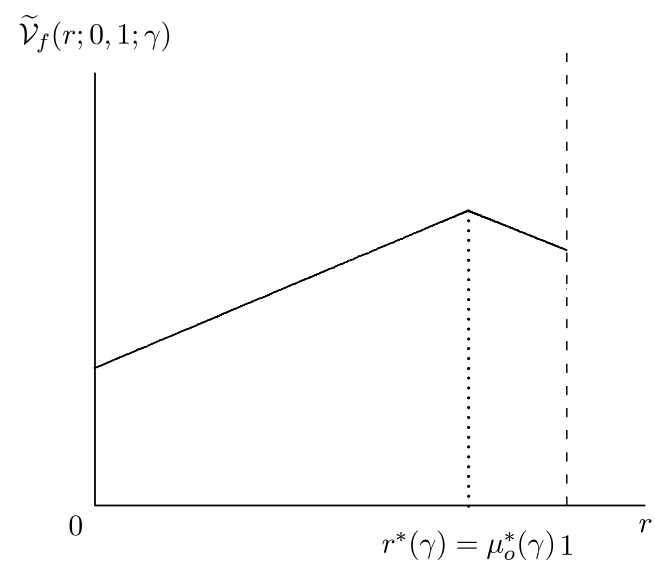

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*
For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

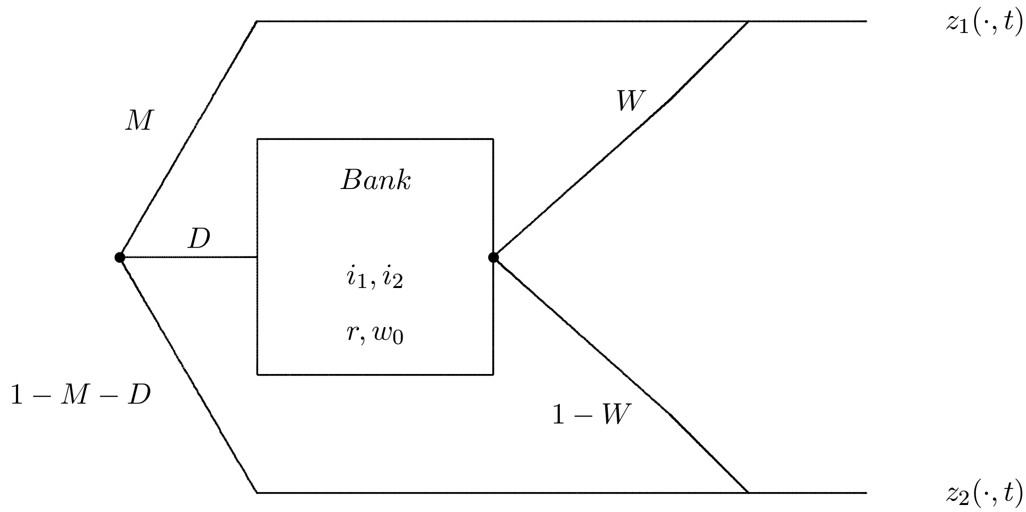

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

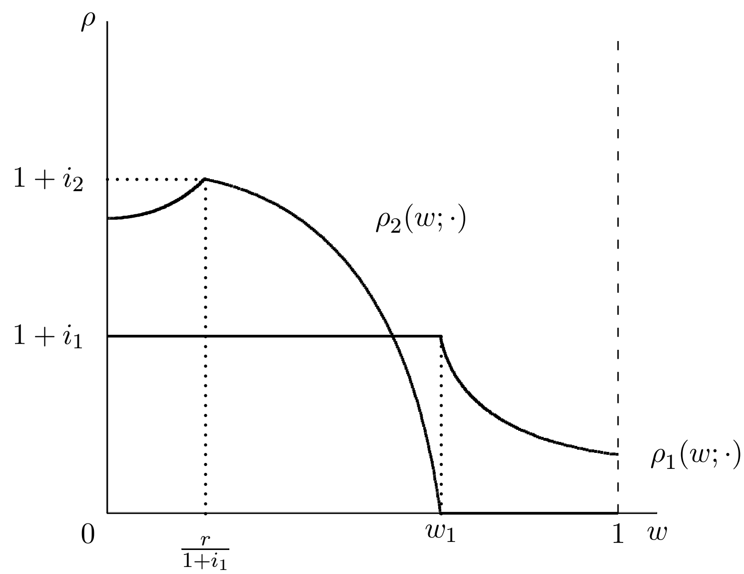

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$
$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

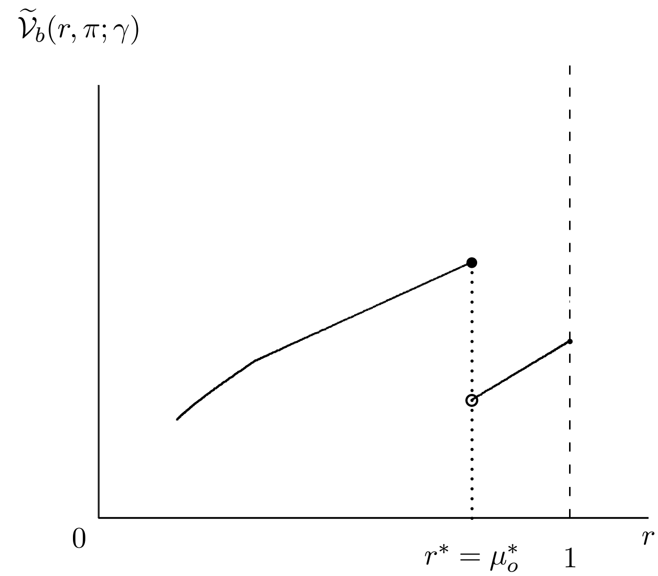

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \ \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \ \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. BANKS, MUTUAL FUNDS, OR ASSET MARKETS?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

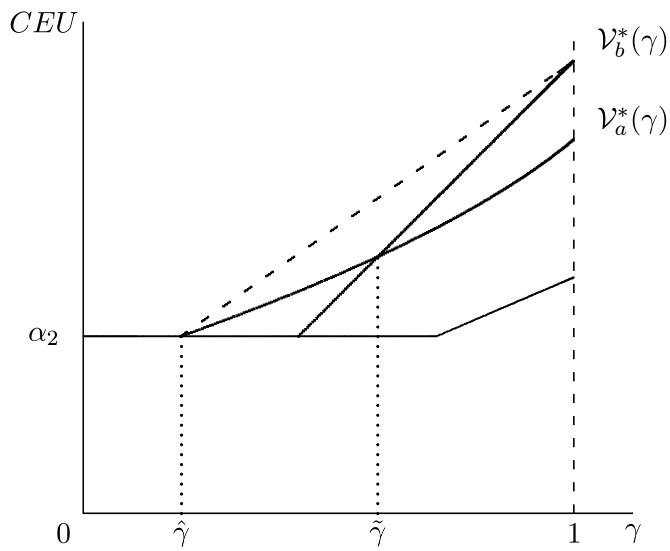

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. CONCLUDING REMARKS

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S|\ f(s) \geq t\}$ and $\{s \in S|\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S|\ f(s) \geq t\})\, dt + \int_{-\infty}^0 [\nu(\{s \in S|\ f(s) \geq t\}) - 1]\, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}
\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)] + (1-\gamma)\cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\
&= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_\ell] + (1-\gamma)\cdot \beta_\ell] + (1-M)\cdot \alpha_2 \\
&= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.
\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned}\mathcal{V}_o^*(\gamma) :=& \quad \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ =& \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \textit{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \textit{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}\end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned}&\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ &+ \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big].\end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) :=\ & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma)\\ =\ & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell,\\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau \in [0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M) \in [0,1] \times [0,1] \\ s.t.\ F+M \leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \ + \ (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+ (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) \;+\; (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ, φ)*. Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$*.*
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^*\cdot r^* + \psi^* = \mu_a^*(\gamma)$*.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1, i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1+(1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w = \tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau < 1$ the constraint

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h) = 1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell) = 0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) := & \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ = & \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ & + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}_+^2 \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

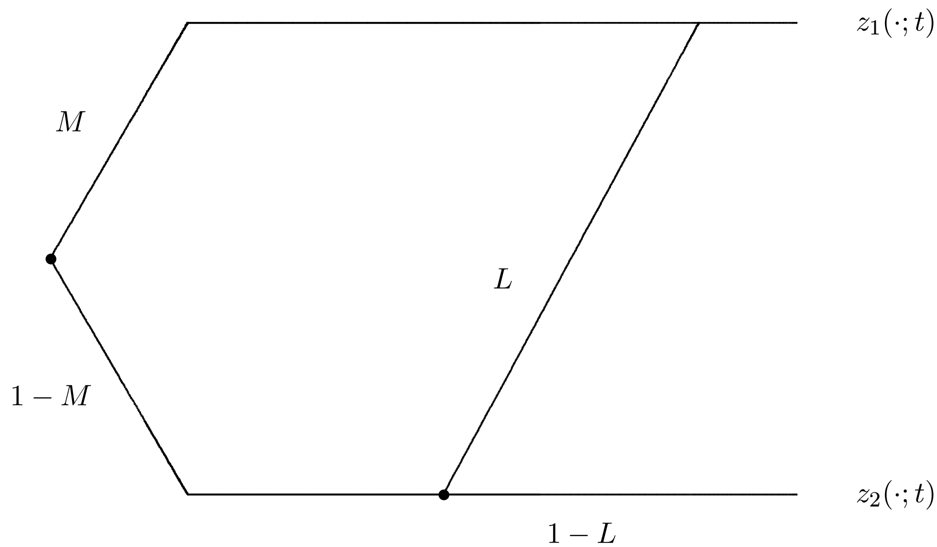

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) &:= \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) &:= \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & +(1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\} \cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\}) \cdot v_n(M;\ell)] \\ & +(1-\gamma) \cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M) \cdot \alpha_2 \geq \beta_\ell \cdot M + (1-M) \cdot \alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

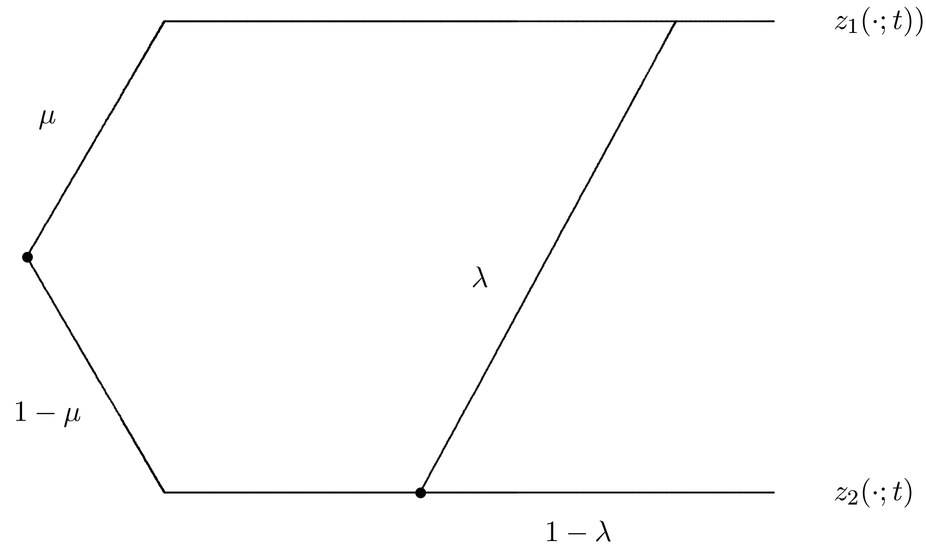

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := & \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := & \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda,z_h,z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := & \quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

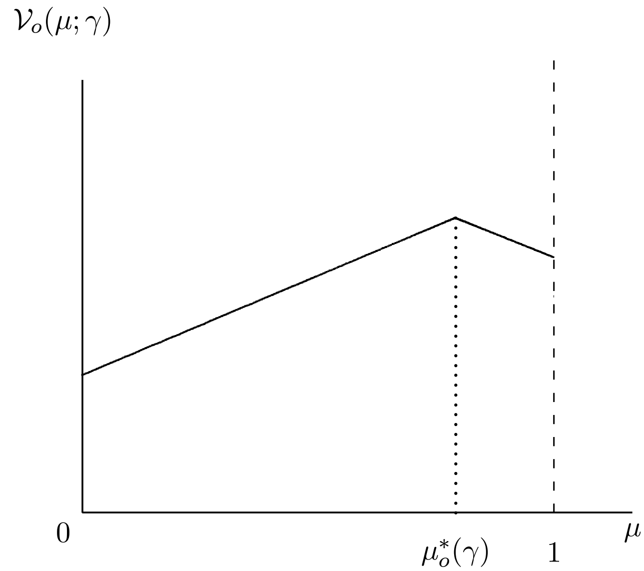

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

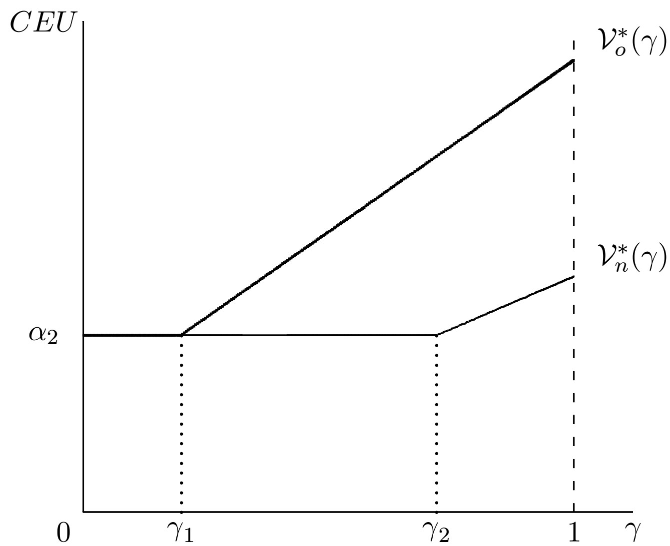

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

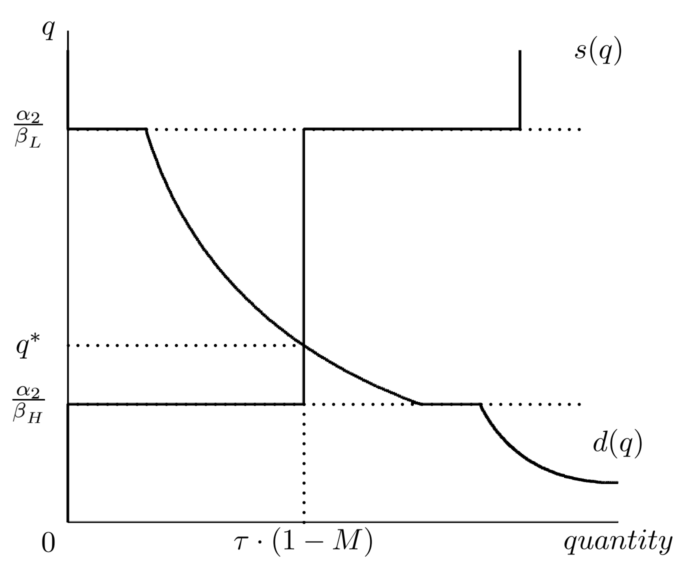

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

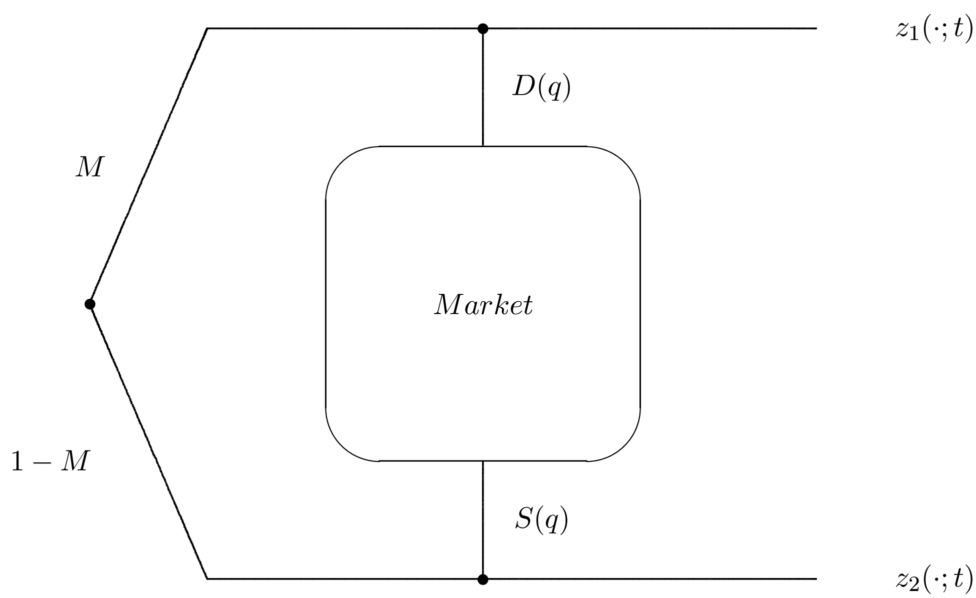

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M; \tau, \mu; t)] \,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi,$
- *if* $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- *if* $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

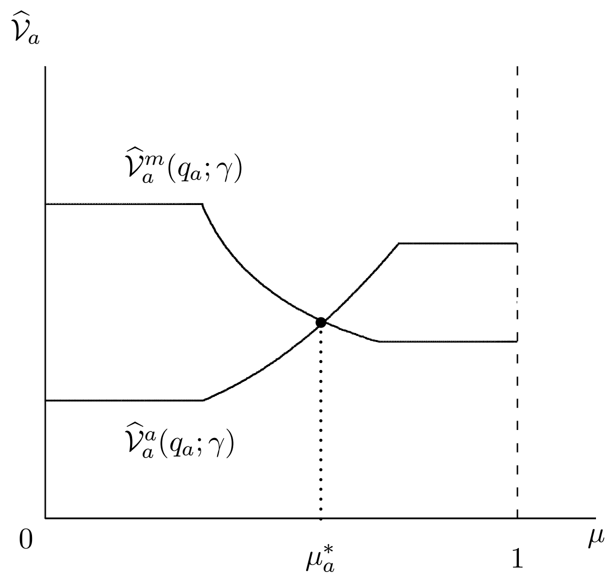

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

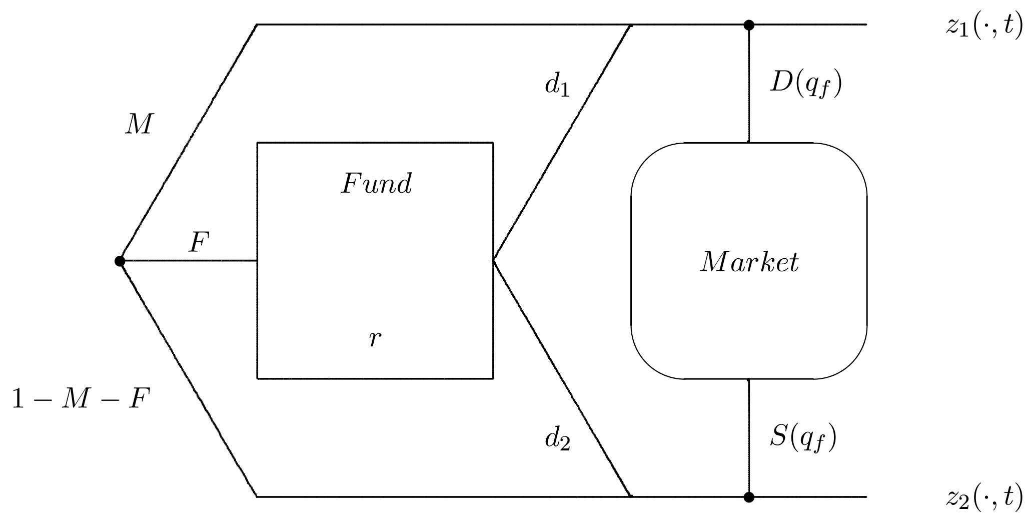

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

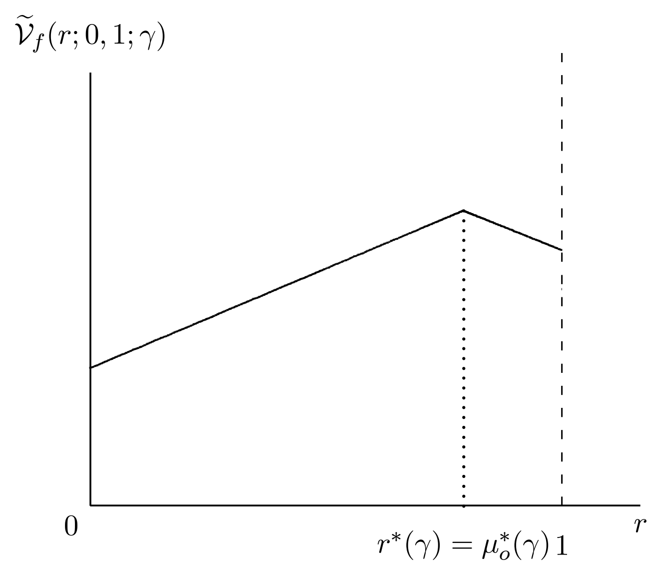

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*
For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

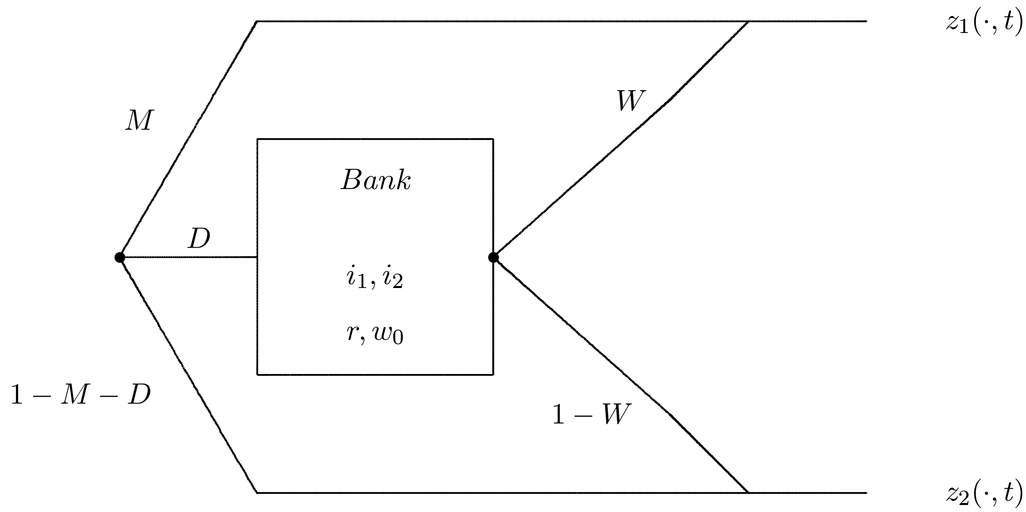

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

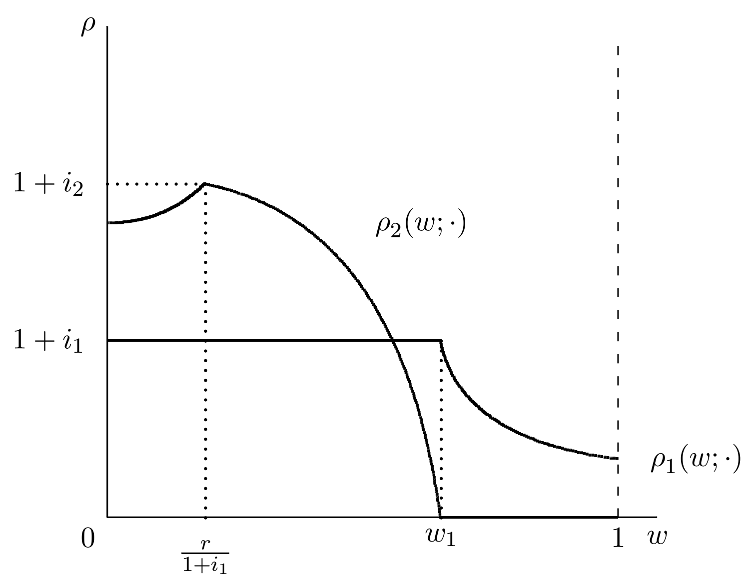

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1 - \tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

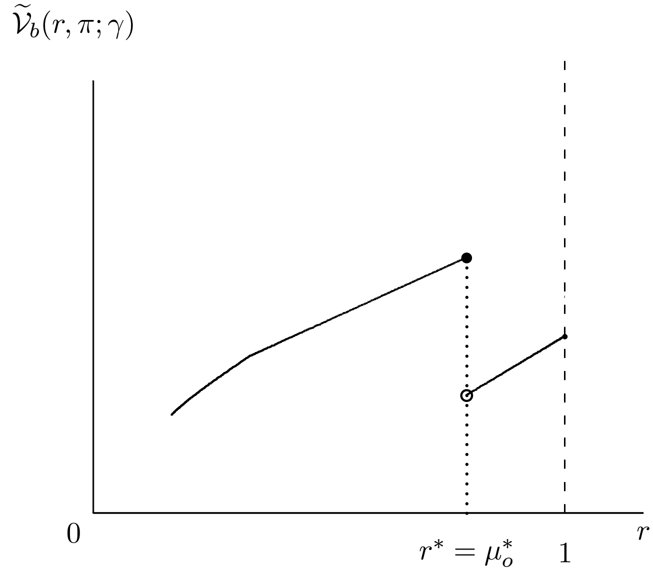

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. BANKS, MUTUAL FUNDS, OR ASSET MARKETS?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

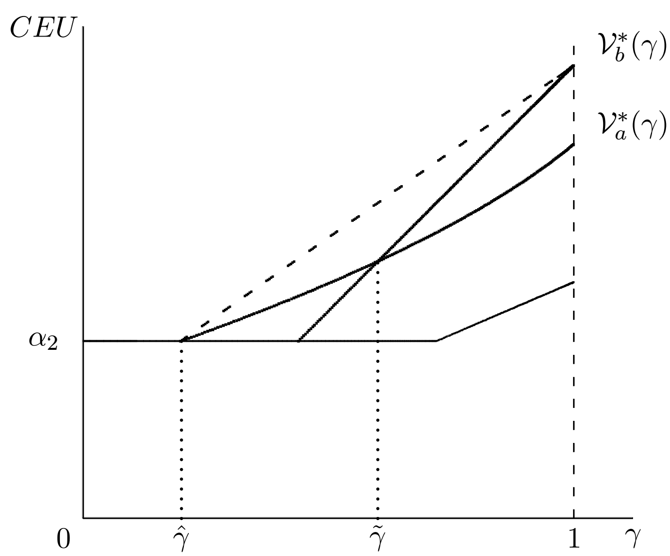

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \; f(s) \geq t\}$ and $\{s \in S | \; f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S | \; f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S | \; f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)] + (1-\gamma)\cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_\ell] + (1-\gamma)\cdot \beta_\ell] + (1-M)\cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0,\underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu),1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0,\underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu),1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\frac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \textit{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma);\gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} &\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\quad \Bigg[-\left(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell)\right) \\ &\quad + \sqrt{\left(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell)\right)^2 + 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h} \Bigg]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}
\mathcal{V}_a^*(\gamma) :=\; & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\
=\; & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\
& \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.
\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau \in [0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t,\tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t,τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M) \in [0,1] \times [0,1] \\ s.t.\ F + M \leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+ (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) \;+\; (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ,φ). Thus, its decision problem for given (ψ,φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \textit{if} \quad w \leq \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \textit{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w=\tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ the constraint

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0)\le\beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0)>\beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) :=& \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ =& \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ & + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0, 1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \;& \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \;& \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S. *The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

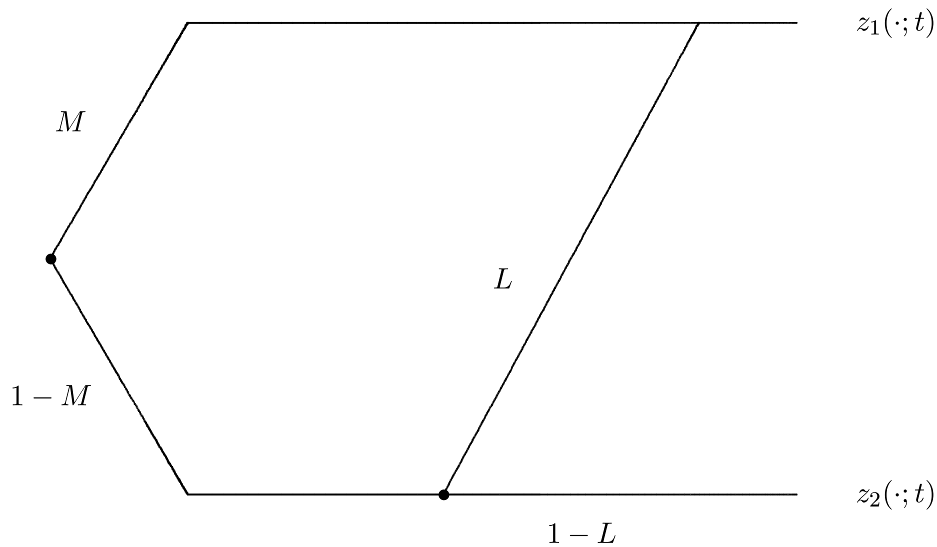

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=& \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M,L) :=& \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types
The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

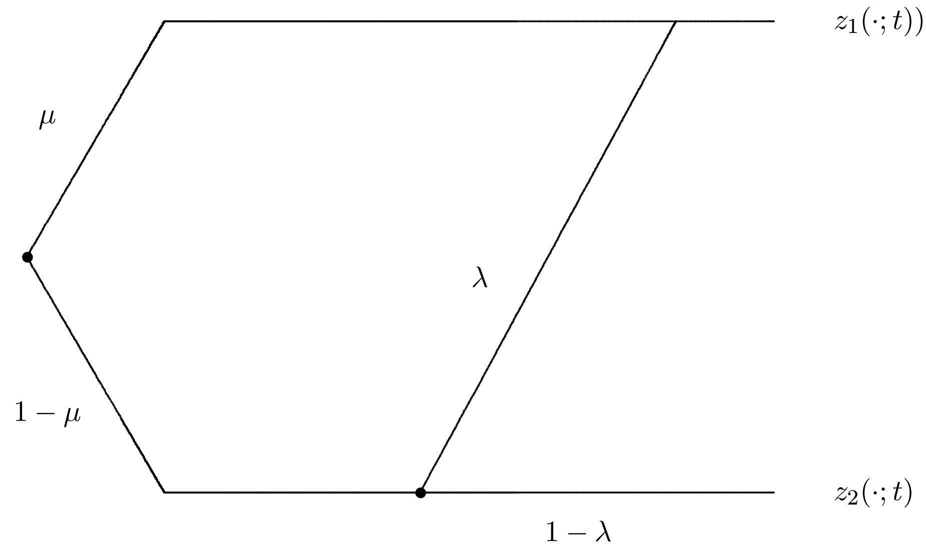

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{lllll} \max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\ \text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\ & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\ & \tau \cdot z_{1h} + (1-\tau)\cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu)\cdot \lambda & \mathrm{F}_1 \\ & \tau \cdot z_{2h} + (1-\tau)\cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu)\cdot(1-\lambda) & \mathrm{F}_2 \\ & \lambda \geq 0,\ z_{1h}\geq 0,\ z_{1\ell}\geq 0,\ z_{2h}\geq 0,\ z_{2\ell}\geq 0. & & & \mathrm{NN} \end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := & \quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t, \tau) \in \{h, \ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

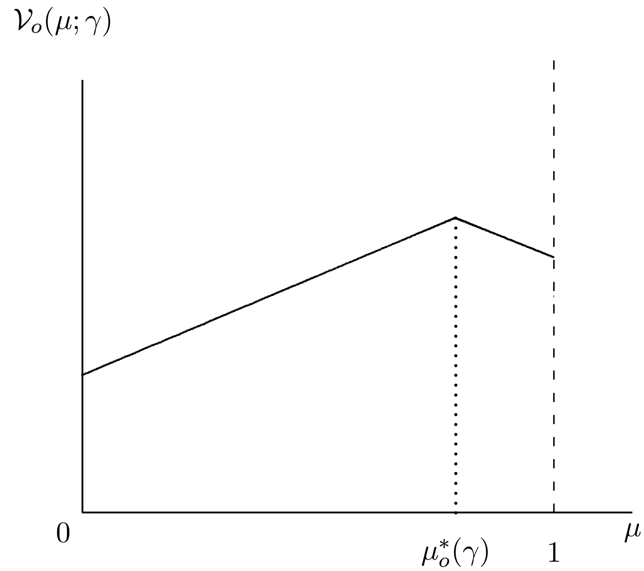

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

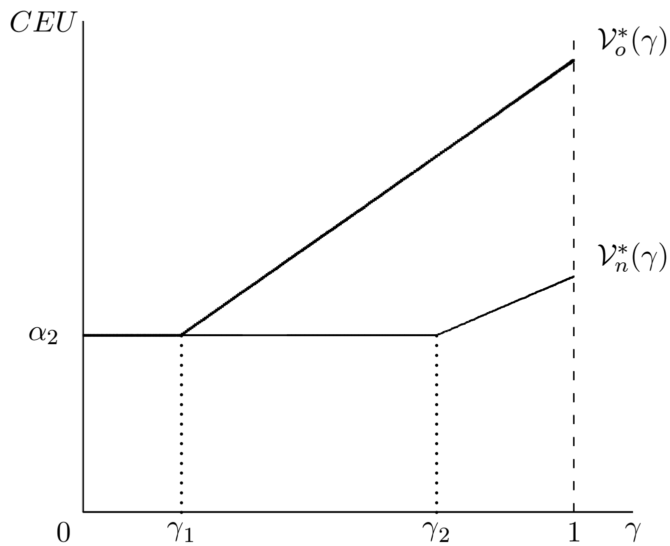

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

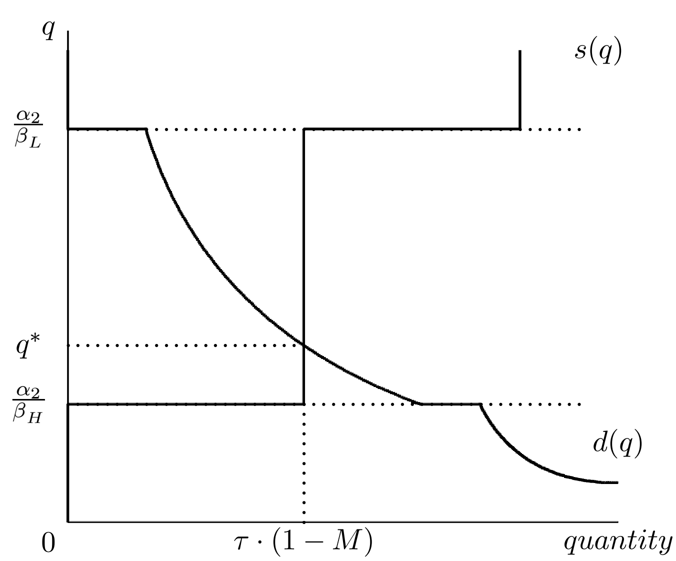

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

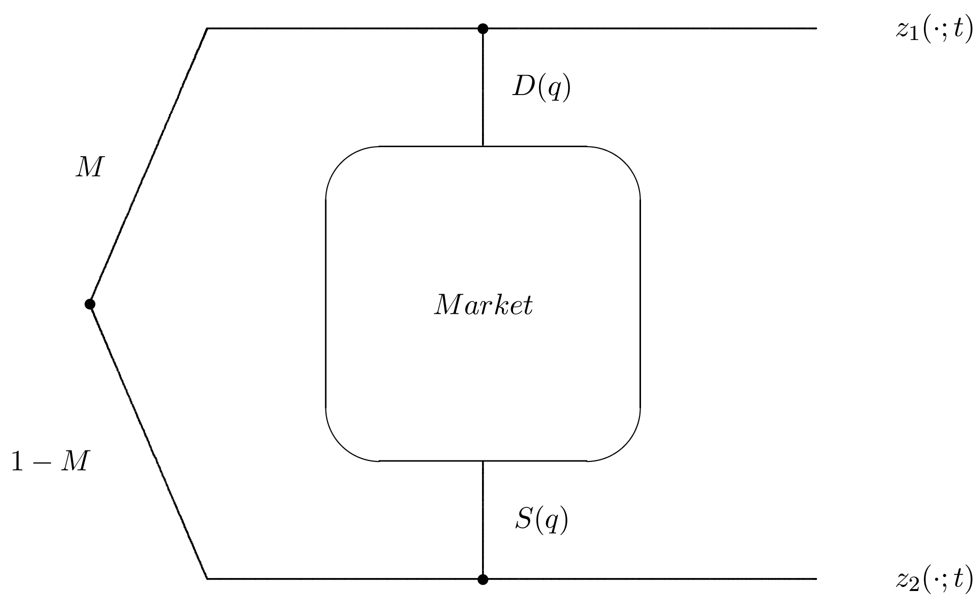

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

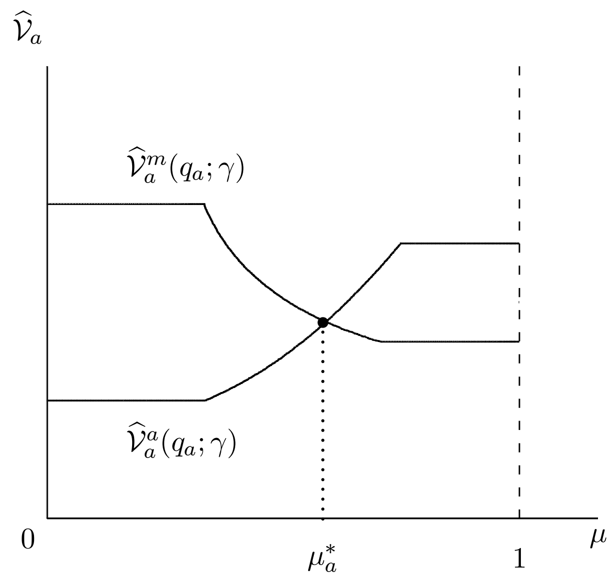

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

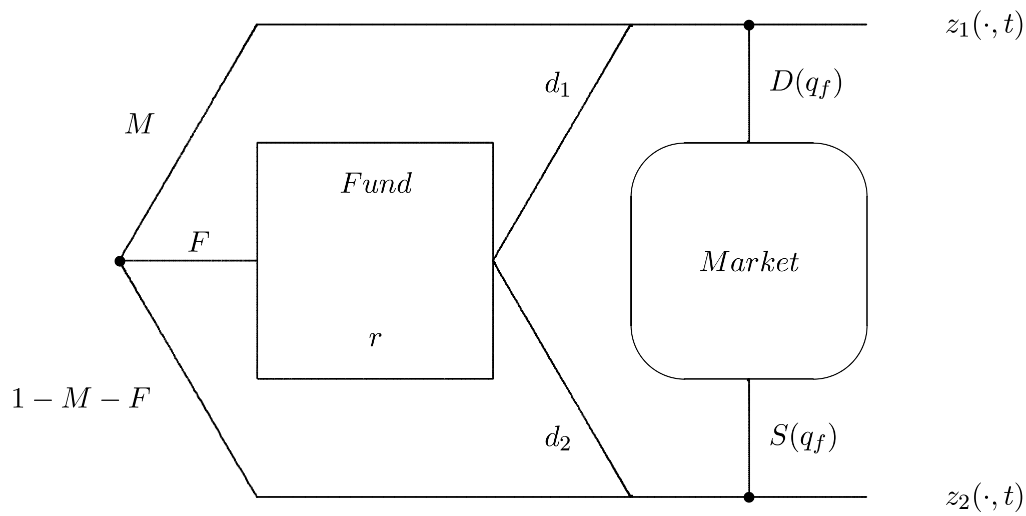

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

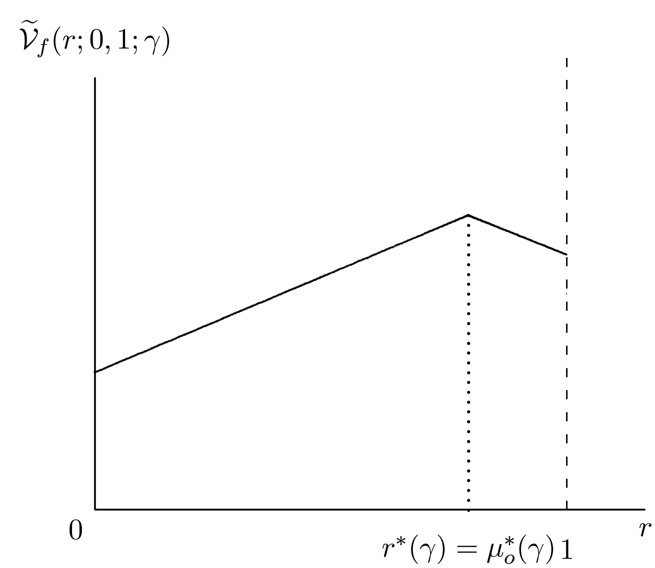

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

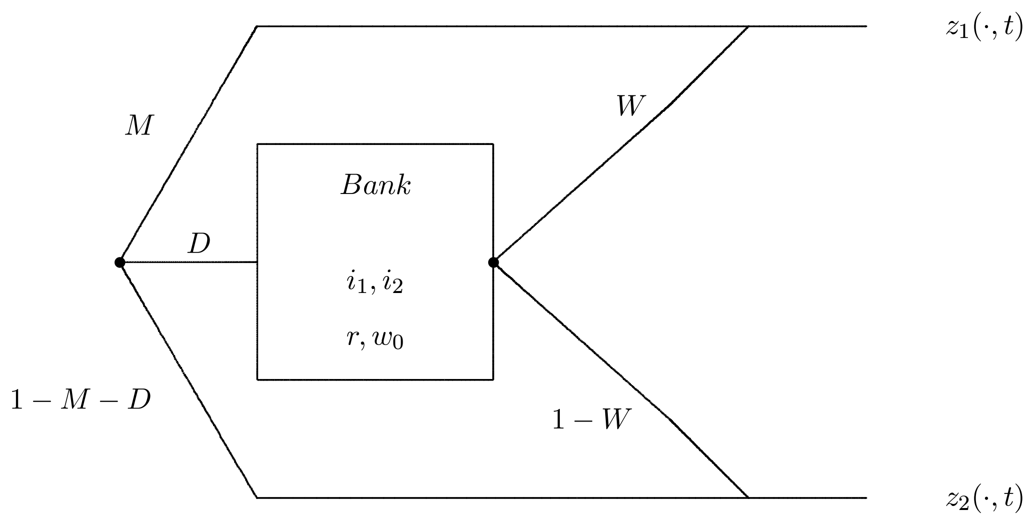

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

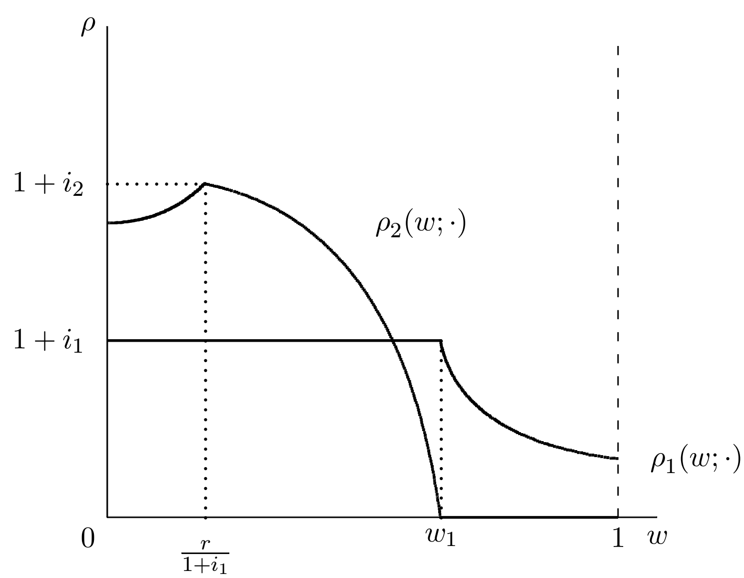

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}&\mathcal{V}_b(D, M; r, w_0; \gamma)\\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

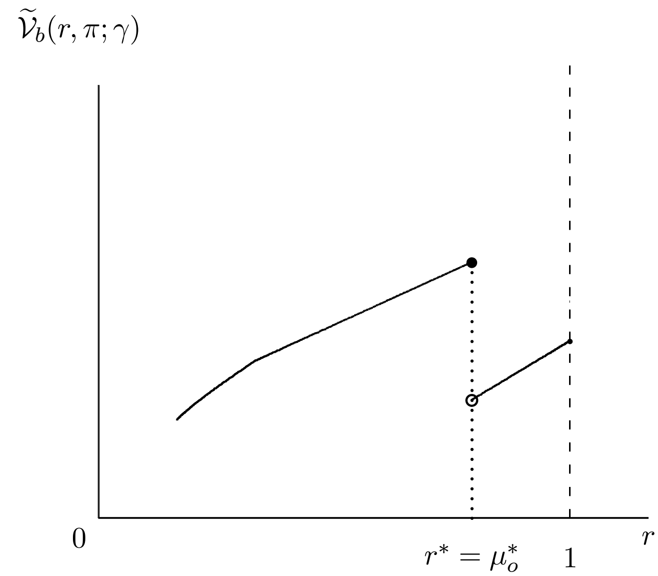

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ =& \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

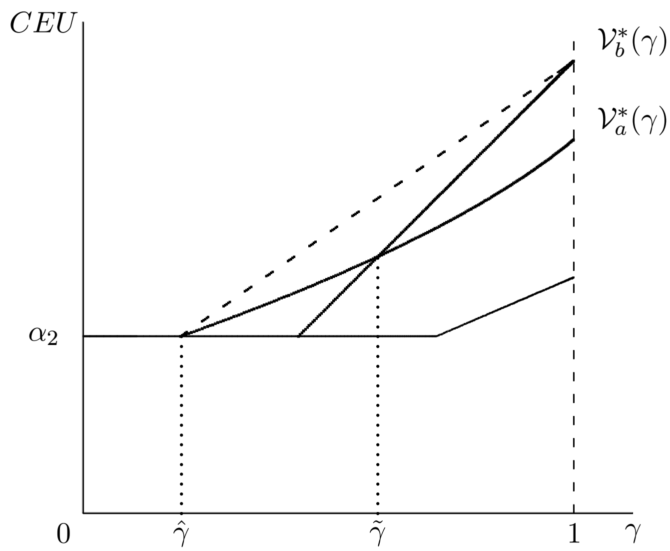

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. CONCLUDING REMARKS

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \; f(s) \geq t\}$ and $\{s \in S | \; f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\int^{C} f \, d\nu = \int_{0}^{\infty} \nu(\{s \in S | \; f(s) \geq t\}) \, dt + \int_{-\infty}^{0} [\nu(\{s \in S | \; f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$
\begin{aligned}
\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\
&= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2 \\
&= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.
\end{aligned}
$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau, \mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$
\begin{aligned}
\underline{\tau}(\mu) &:= \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)}, \\
\overline{\tau}(\mu) &:= \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.
\end{aligned}
$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$
z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \textit{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \textit{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}
$$

$$
z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \textit{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \textit{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}
$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0,\overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu),1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0,\overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu),1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau)\cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1+\gamma\cdot(\frac{\beta_h}{\beta_\ell}-1)\cdot\tau]\cdot[\alpha_2\cdot(1-\mu)+\beta_\ell\cdot\mu] & \textit{if} \quad \pi \in [0,\underline{\tau}(\mu)) \\ \alpha_2\cdot(1-\mu) + [\gamma\cdot\pi\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell]\cdot\mu & \textit{if} \quad \pi\in[\underline{\tau}(\mu),1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ. *This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$, *depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \quad \left[-\left(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \right) \right. \\ & \quad \left. + \sqrt{\left(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \right)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \right]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma, \pi, \alpha_2, \beta_h, \beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}
\mathcal{V}_a^*(\gamma) := \quad & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\
= \quad & \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\
& \gamma \cdot [\pi \cdot \max\{\beta_h \cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot \max\{\beta_\ell \cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot \alpha_2\}.
\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau \in [0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M) \in [0,1] \times [0,1] \\ s.t.\ F+M \leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ,φ)*. Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$*.*
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$*.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1, i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w; i_1, i_2, r) := \begin{cases} \dfrac{\alpha_2 \cdot (1-r) + r - (1+i_1) \cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1 + i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)} \cdot [(1-r) - \dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1 + (1-\alpha_1) \cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1 + (1-\alpha_1) \cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0*, one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w; r, w_0) := \quad \widehat{\rho}_1(w; i_1(r, w_0), i_2(r, w_0), r),$$

$$\rho_2(w; r, w_0) := \quad \widehat{\rho}_2(w; i_1(r, w_0), i_2(r, w_0), r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period 1, i.e.,* $w = \tau$*, the following self selection constraints must hold:* [36]

$$\rho_2(\tau; r, w_0) \leq \beta_h \cdot \rho_1(\tau; r, w_0) \qquad S_h$$

$$\beta_\ell \cdot \rho_1(\tau; r, w_0) \leq \rho_2(\tau; r, w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau < 1$ *the constraint*

$$\beta_\ell \cdot \rho_1(\tau; r, w_0) > \rho_2(\tau; r, w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau; r, w_0; h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) \leq \beta_h \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) > \beta_h \cdot \rho_1(\tau; r, w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau; h) = 1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau; \ell) = 0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\ d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0, 1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) := & \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ = & \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ & \qquad + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0, 1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := {} & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = {} & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu\in[\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \le \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \ge \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

[1]We would like to thank seminar participants at the Norwegian School of Management (Sandvika), the University of Oslo, the University of Birmingham, the Bundesbank (Frankfurt a.M.), the Free University in Berlin, the University of Frankfurt a.M., Keele University, University of Hohenheim and Chemnitz University of Technology for helpful comments and suggestions.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. NON-ADDITIVE BELIEFS AND THE CHOQUET INTEGRAL

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S. *The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

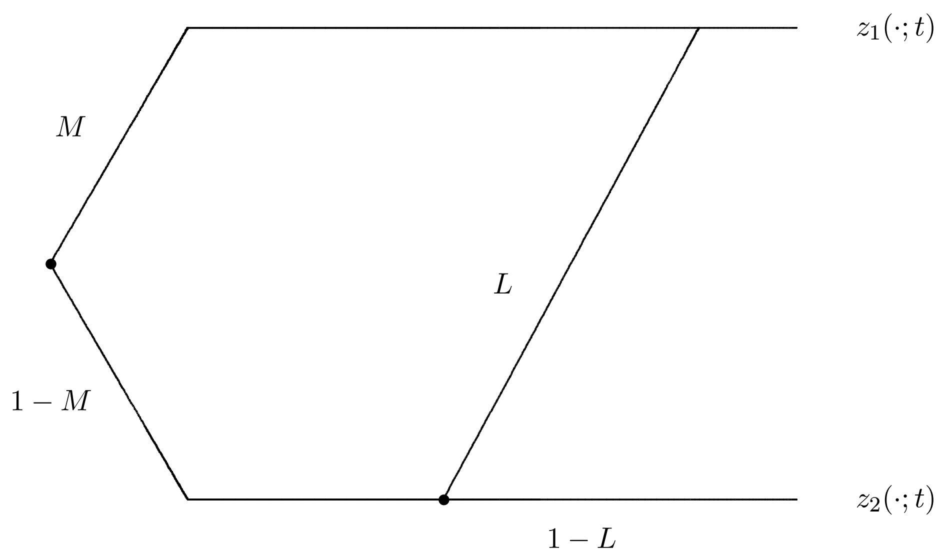

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) :=& \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) :=& \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1-M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1-\tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) :=\ & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & +(1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ =\ & \gamma \cdot [\mathbb{E}_p\{\tau\} \cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\}) \cdot v_n(M;\ell)] \\ & +(1-\gamma) \cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M) \cdot \alpha_2 \geq \beta_\ell \cdot M + (1-M) \cdot \alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

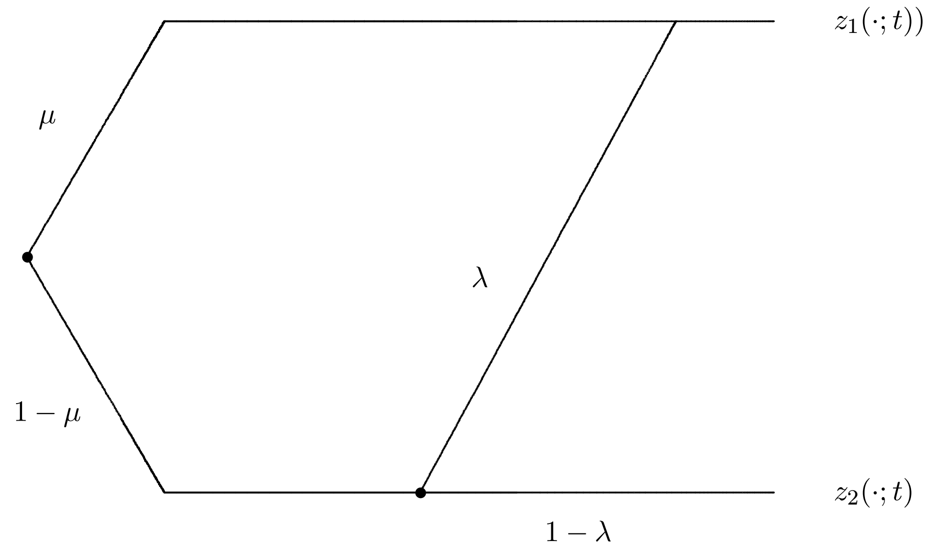

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := & \quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

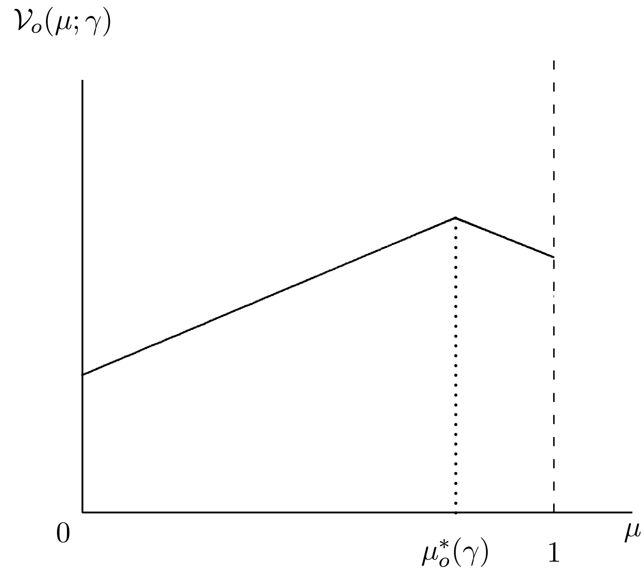

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

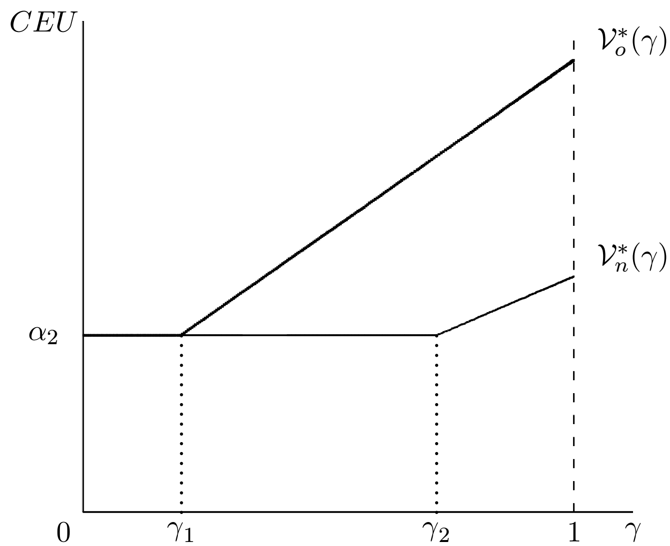

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

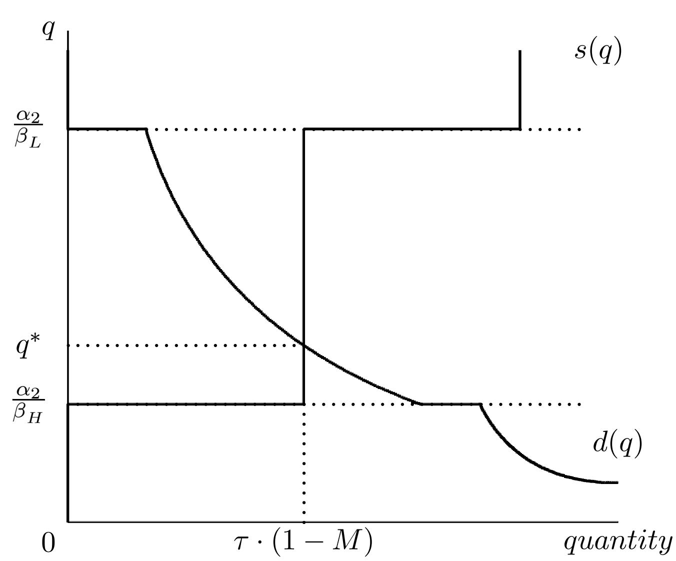

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

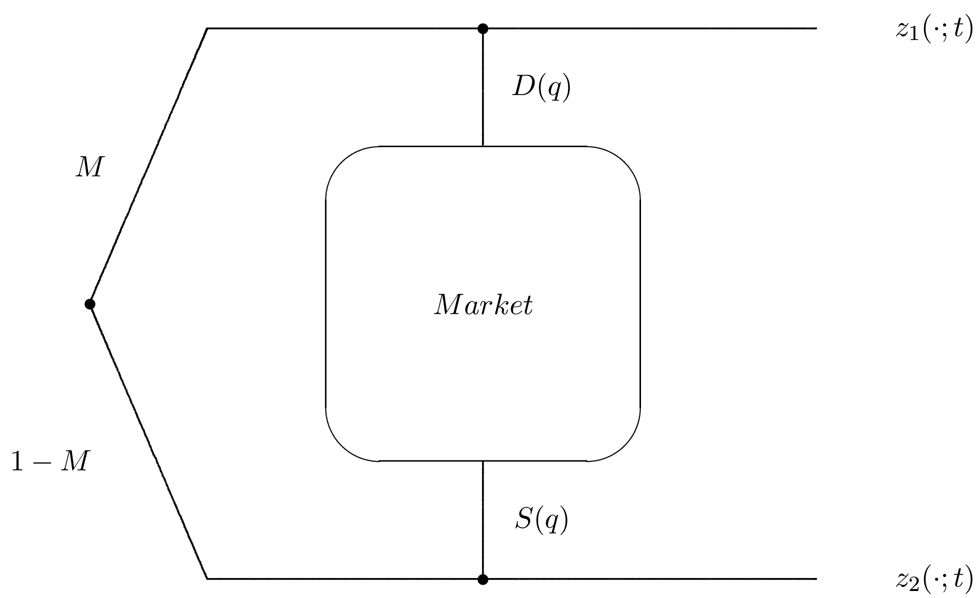

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M) \cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] .\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma), \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

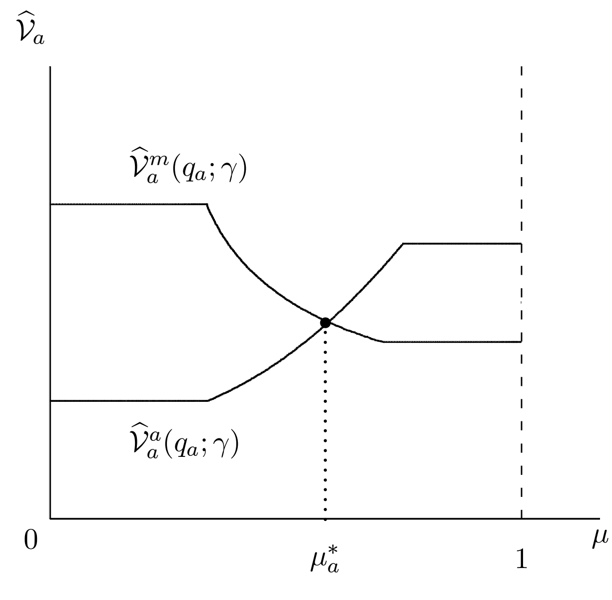

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

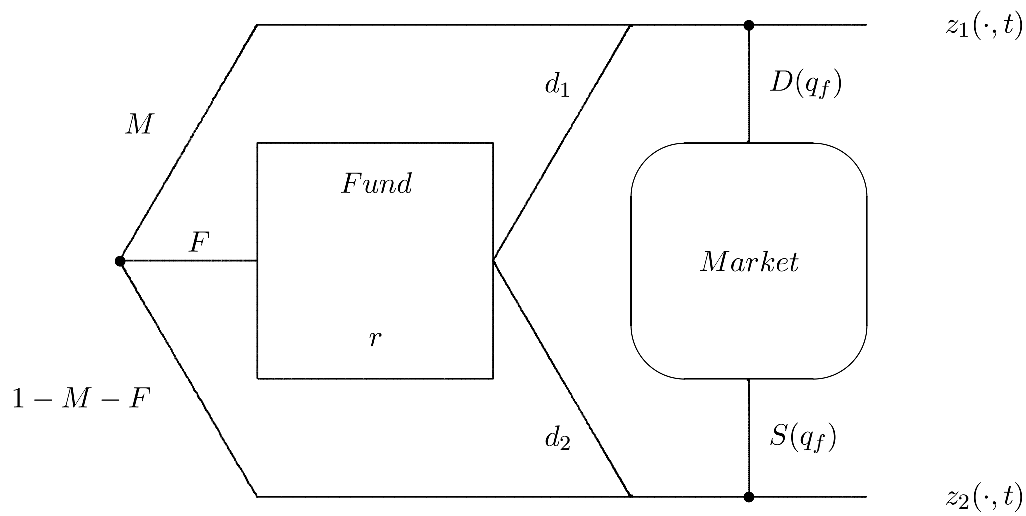

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau)\cdot\mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r)\cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$
$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

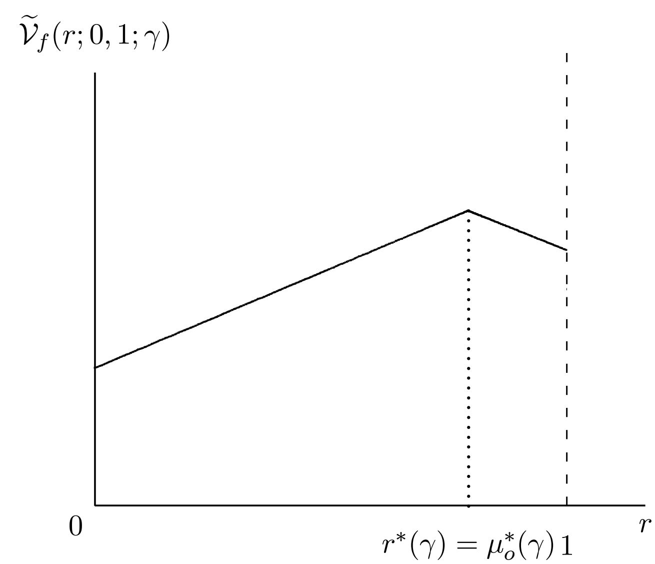

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.
For all $\gamma \in [0, 1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.
The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

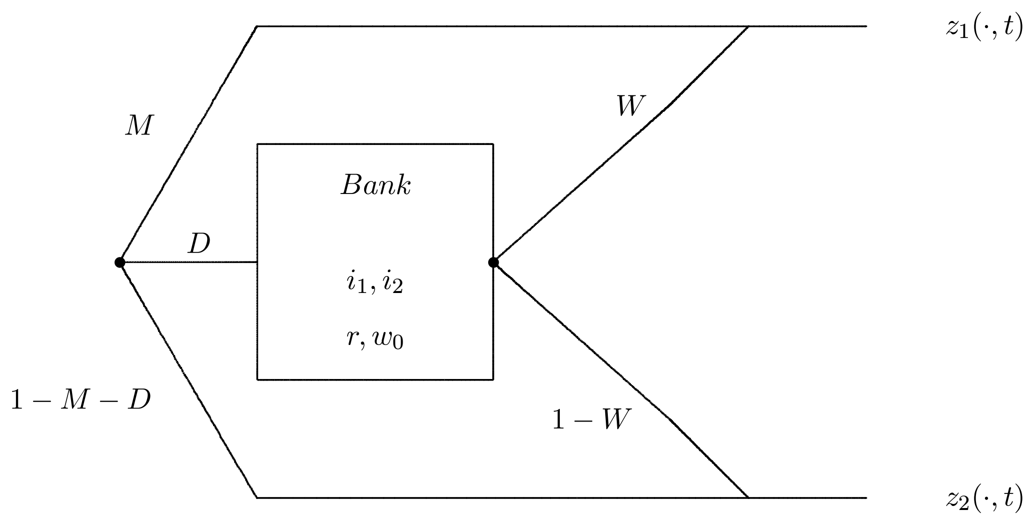

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

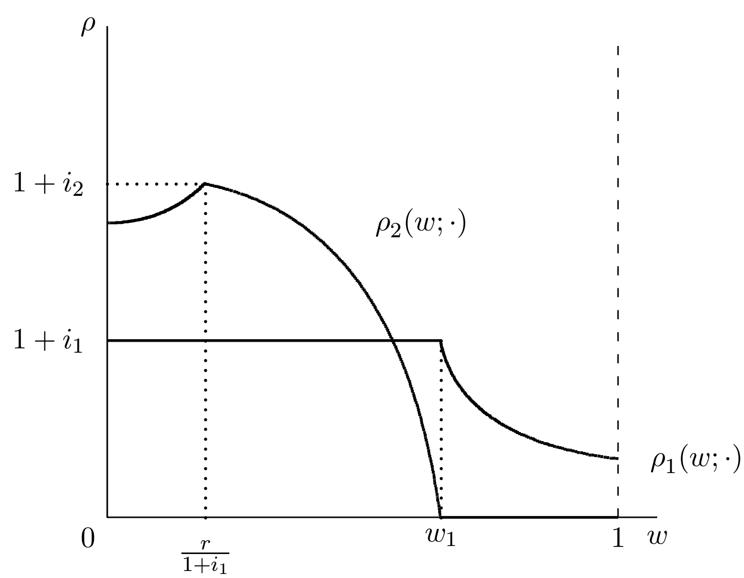

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

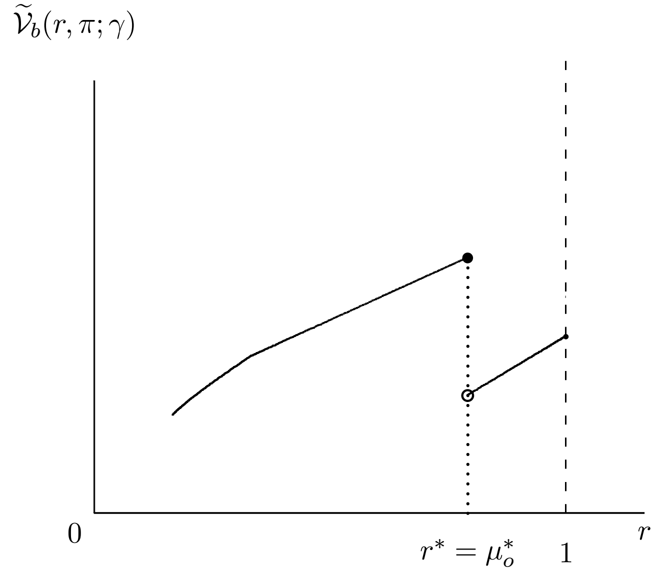

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

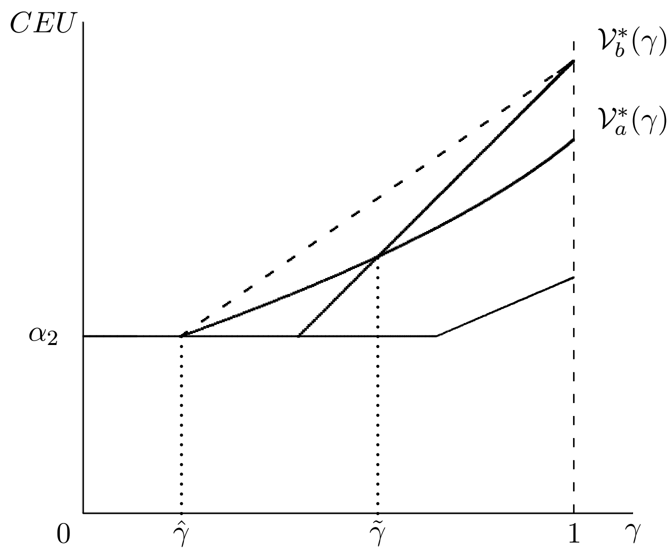

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t,\tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \ f(s) \geq t\}$ and $\{s \in S | \ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\int^{C} f \, d\nu = \int_{0}^{\infty} \nu(\{s \in S | \ f(s) \geq t\}) \, dt + \int_{-\infty}^{0} [\nu(\{s \in S | \ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)}, \\ \overline{\tau}(\mu) &:= \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \textit{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \quad \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & \quad + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}
\mathcal{V}_a^*(\gamma) :=\; & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\
=\; & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\
& \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.
\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau \in [0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. When this is not the case, investment policy M may affect the state (t, τ) for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of $\widehat{\mathcal{V}}_a^m$ and $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and invest the remaining fraction $(1 - F - M)$ in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, and

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of $h-$types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \; + \; (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ consumers, the worst case occurs when $q_f^*(\tau; \psi, \varphi, r)$ attains its upper bound at $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. In this case a consumer's utility equals

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M and in F. Consumers of type h can always obtain a utility level at least as high as the consumers of type ℓ by copying the latter's behaviour.

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot [r\cdot \beta_\ell + \alpha_2 \cdot (1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ, φ). Thus, its decision problem for given (ψ, φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \frac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \frac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \frac{r}{1+i_1} \\ \frac{\alpha_2}{(1-w)}\cdot[(1-r)-\frac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0*, one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period* 1*, i.e.,* $w=\tau$*, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 *to lie between the type-contingent valuations of the actual return in Period* 1*. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for* $\tau < 1$ *the constraint*

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau;r,w_0;\ell) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) < \beta_\ell \cdot \rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) \geq \beta_\ell \cdot \rho_1(\tau;r,w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D,M;r,w_0;\gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau) \cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D,M;r,w_0;\gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r,w_0;\gamma) := & \max_{\substack{(D,M)\in[0,1]\times[0,1] \\ s.t.\ D+M\leq 1}} \mathcal{V}_b(D,M;r,w_0;\gamma) \\ = & \max\{[\gamma \cdot \{\pi \cdot v_b(\pi;r,w_0;h) + (1-\pi) \cdot v_b(\pi;r,w_0;\ell)\} \\ & + (1-\gamma) \cdot \min_{\tau\in[0,1]} v_b(\tau;r,w_0;t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r,w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r,w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$, taking into account the withdrawal behaviour $W(\tau;r,w_0;t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau=\pi$ and $\tau=1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \le \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \ge \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\,, \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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ABSTRACT. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. INTRODUCTION

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

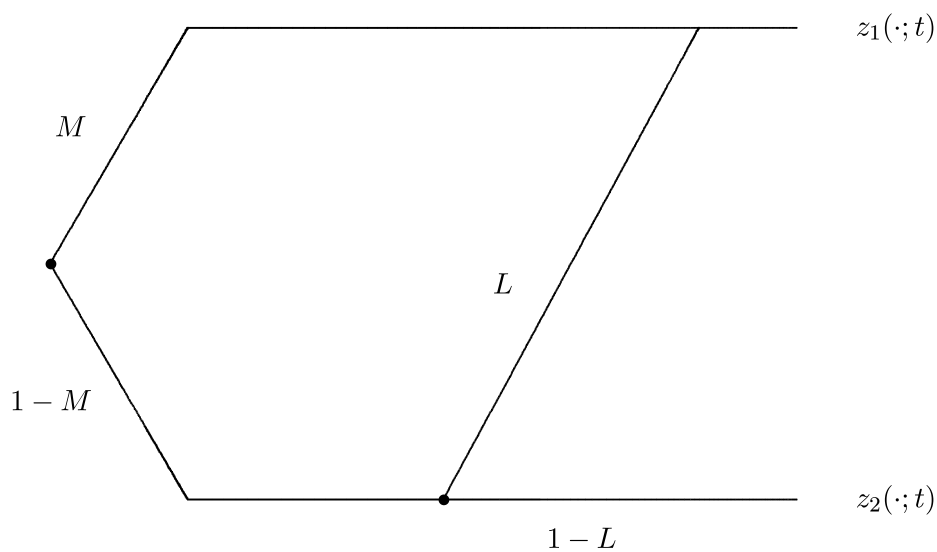

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M; t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

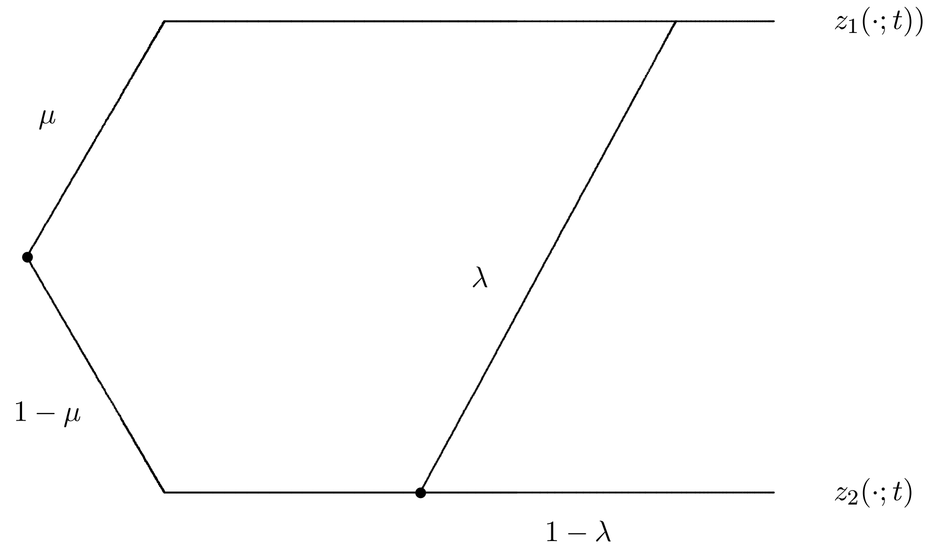

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{lllll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

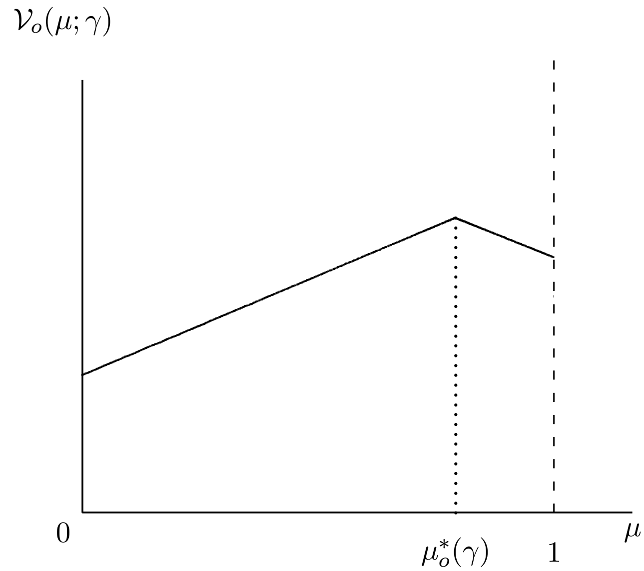

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

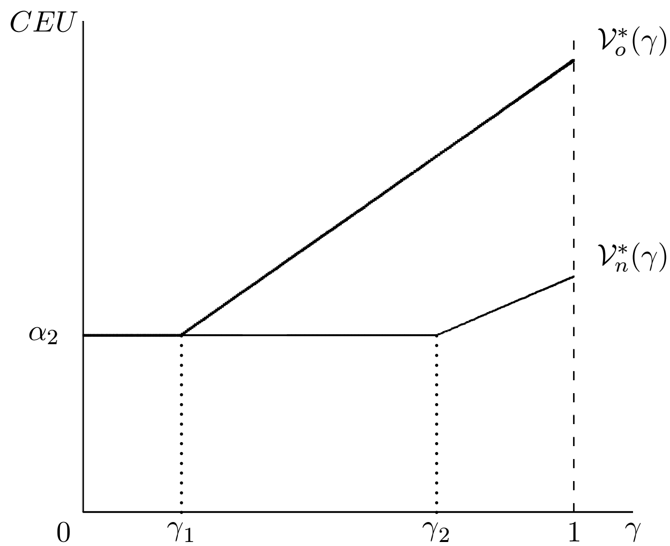

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

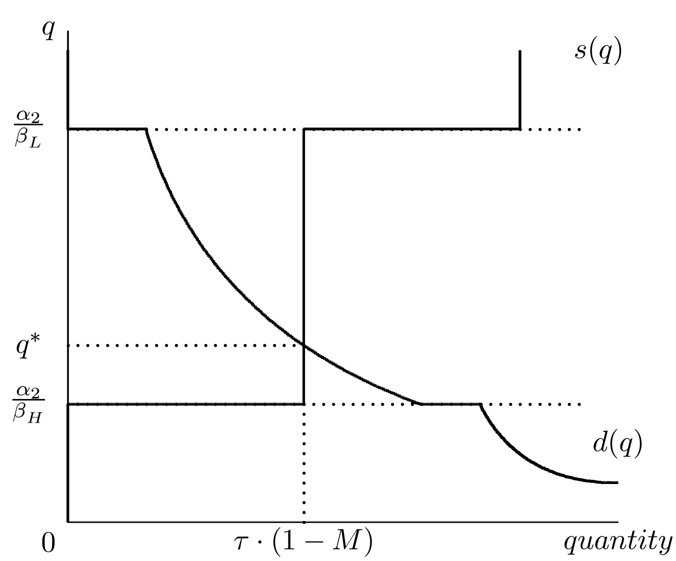

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

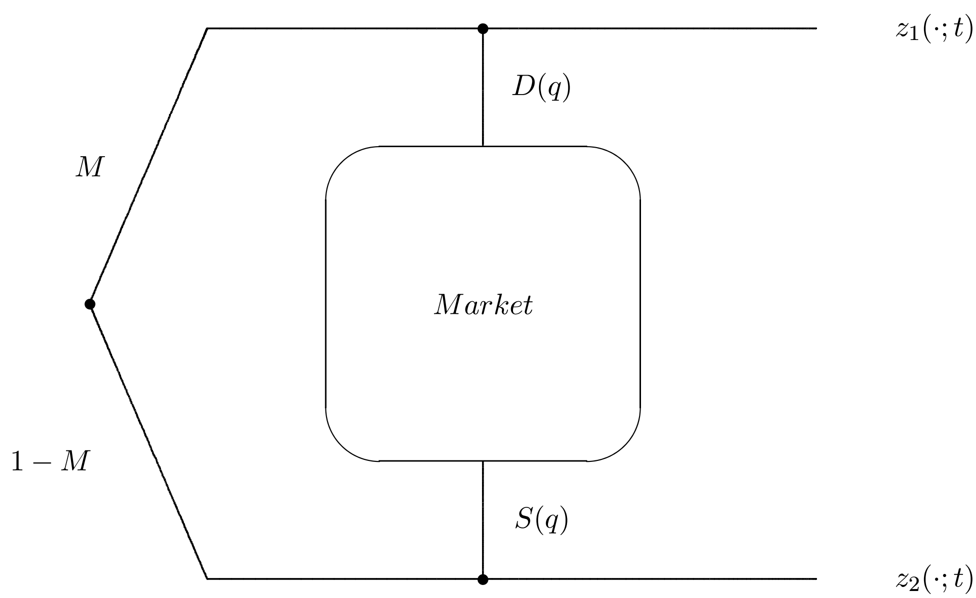

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\mathcal{V}_a(M, \mu; \gamma) := \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] .$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0,1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma), \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

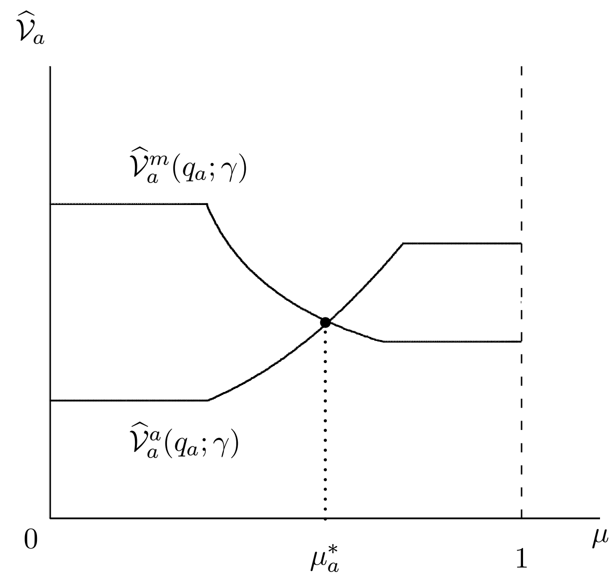

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

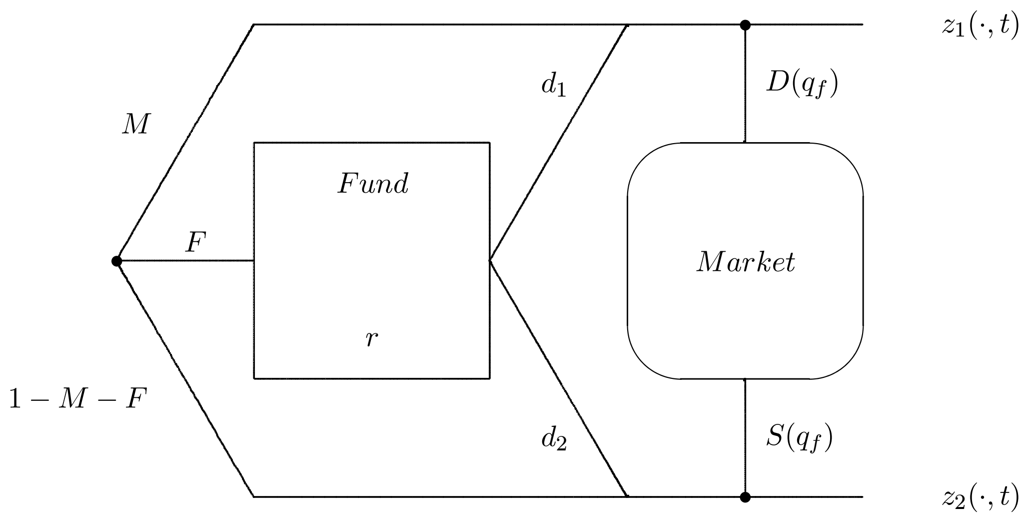

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 :=& \quad r \\ d_2 :=& \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau)\cdot\mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r)\cdot(1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau;\psi,\varphi,r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ,φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F,M;\tau,\psi,\varphi,r;t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r)\cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

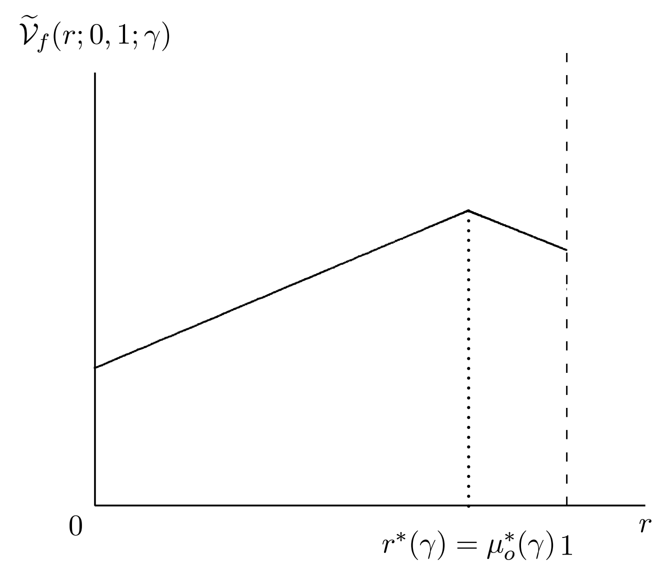

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.

[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

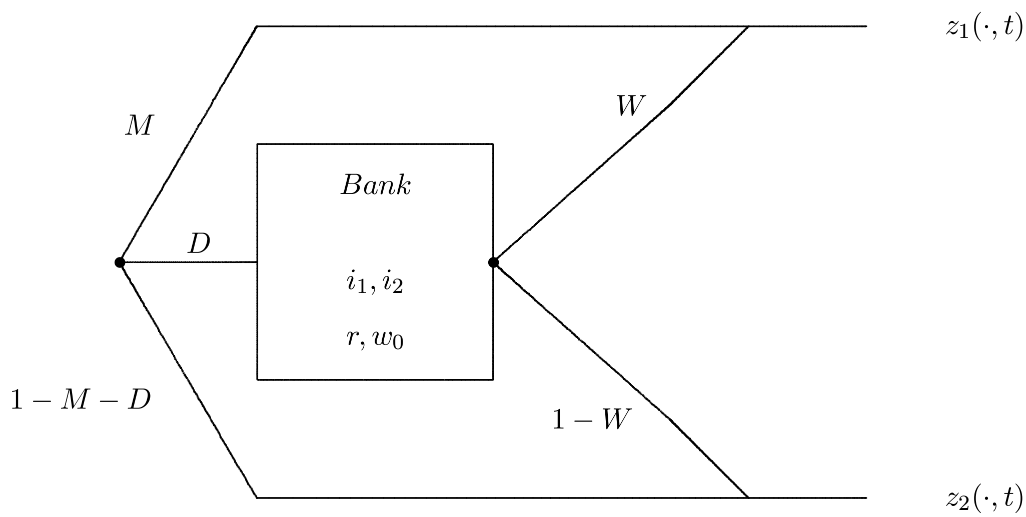

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

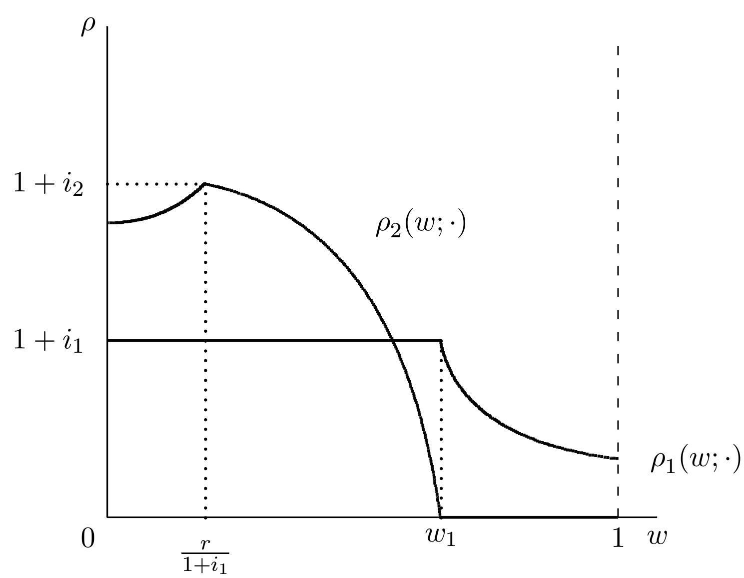

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D,M;r,w_0;\gamma)\\
&:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\
&\qquad\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D,M;r,w_0;\gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D,M;r,w_0;\gamma)$$

$$\text{s.t.} \quad D+M \le 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$.

It remains to determine the deposit contract parameters (r,w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r,\pi;\gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

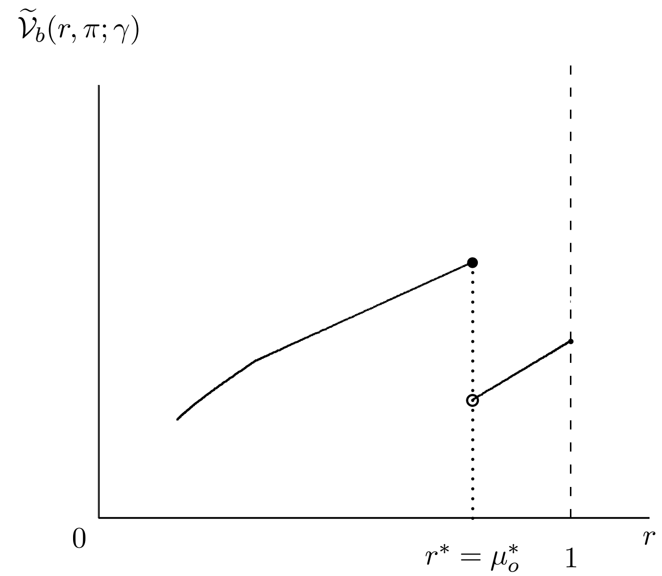

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ =& \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$*. There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

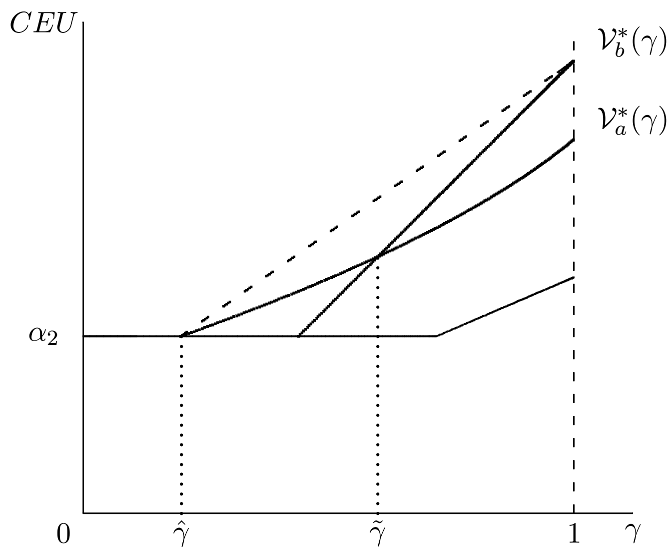

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S},\ A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \ \text{ for all } \ E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S|\ f(s) \geq t\}$ and $\{s \in S|\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\overset{C}{\int} f\, d\nu = \int_0^\infty \nu(\{s \in S|\ f(s) \geq t\})\, dt + \int_{-\infty}^0 [\nu(\{s \in S|\ f(s) \geq t\}) - 1]\, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma\cdot[\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)]+(1-\gamma)\cdot\min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M\cdot[\gamma\cdot[\pi\cdot\beta_h+(1-\pi)\cdot\beta_\ell]+(1-\gamma)\cdot\beta_\ell]+(1-M)\cdot\alpha_2\\ &= M\cdot[\gamma\cdot\pi\cdot\beta_h+(1-\gamma\cdot\pi)\cdot\beta_\ell]+(1-M)\cdot\alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu\in[0,1]$ and $\tau\in[0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu)=0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell\cdot\mu}{\beta_\ell\cdot\mu+\alpha_2\cdot(1-\mu)},\\ \overline{\tau}(\mu) &:= \frac{\beta_h\cdot\mu}{\beta_h\cdot\mu+\alpha_2\cdot(1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu)=\begin{cases}\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}+\mu & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ \dfrac{\mu}{\tau} & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{1\ell}(\tau,\mu)=\begin{cases}\mu-\left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ 0 & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi) \cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned}\mathcal{V}_o^*(\gamma) :=& \quad \mathcal{V}_o(\mu^*(\gamma);\gamma) \\ =& \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}\end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot$$

$$\Bigg[- \left(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell)\right)$$

$$+ \sqrt{\left(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell)\right)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \, \Bigg].$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) := \quad & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\ = \quad & \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma),\alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau\in[0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. When this is not the case, investment policy M may affect the state (t, τ) for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of $\widehat{\mathcal{V}}_a^m$ and $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and invest the remaining fraction $(1 - F - M)$ in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, and

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of $h-$types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M) \in [0,1] \times [0,1] \\ s.t.\ F+M \leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+ (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ consumers, the worst case occurs when $q_f^*(\tau; \psi, \varphi, r)$ attains its upper bound at $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. In this case a consumer's utility equals

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M and in F. Consumers of type h can always obtain a utility level at least as high as the consumers of type ℓ by copying the latter's behaviour.

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$
$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ,φ)*. Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$.
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^*\cdot r^* + \psi^* = \mu_a^*(\gamma)$.

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1,i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \textit{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \textit{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w=\tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ the constraint

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) &:= \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ &\qquad + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1-\pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1-r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1-r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_h + \tau \cdot \alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_\ell + \tau \cdot \alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi+\psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\,, \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$z_{2h}(\tau) := \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right]$$

$$= \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h.$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ is a best reply for $r = \mu_\ell$ and $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell$$

$$= \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell.$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

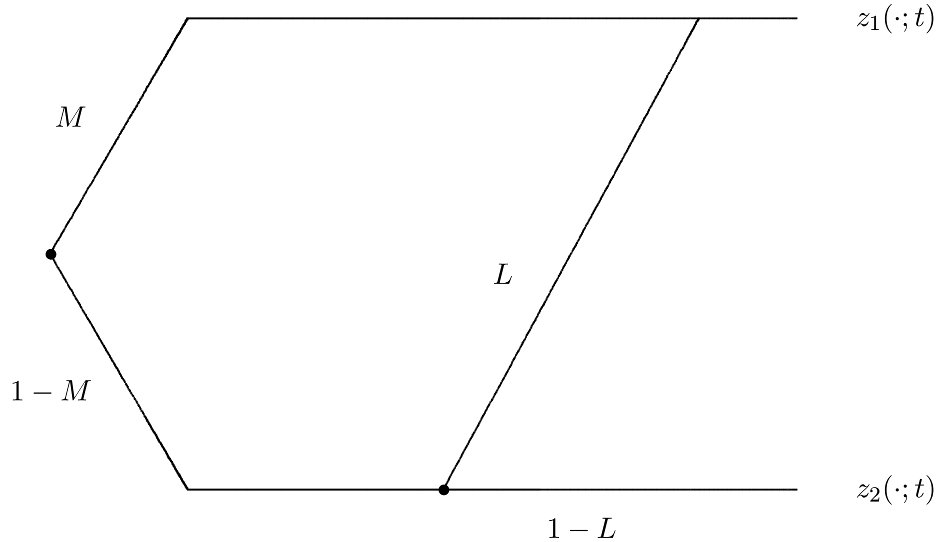

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=& \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) :=& \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

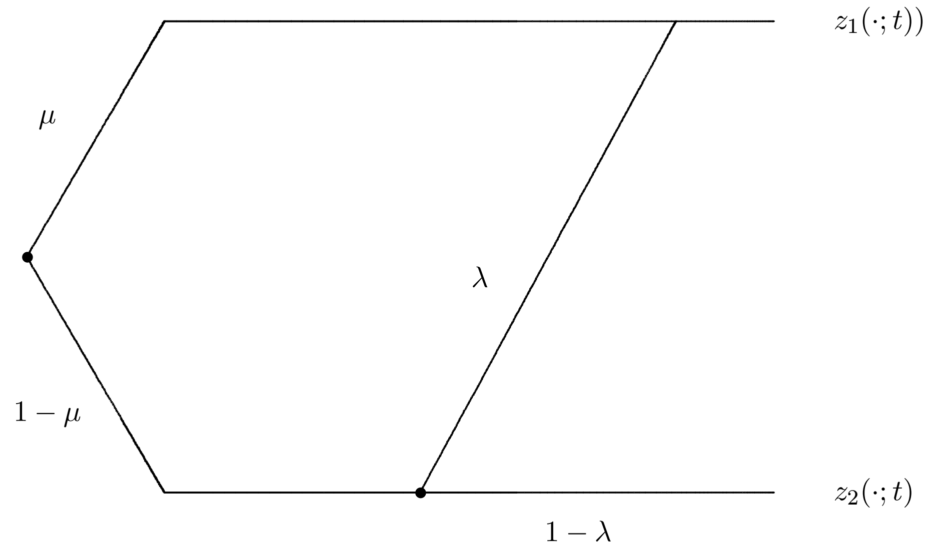

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := & \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := & \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := &\quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = &\quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

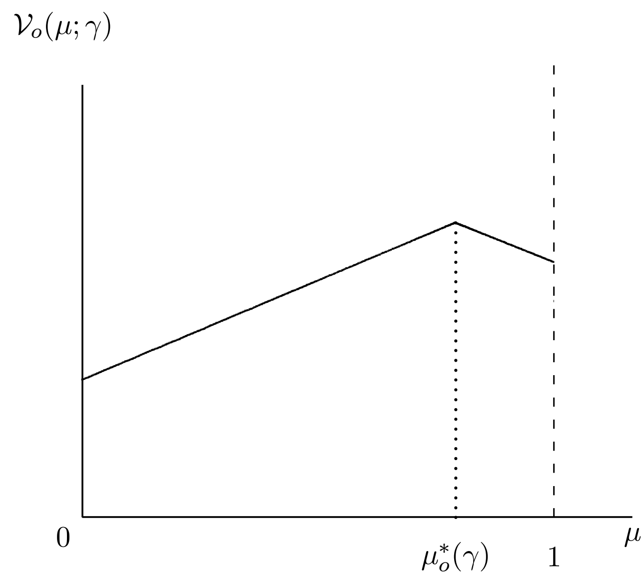

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

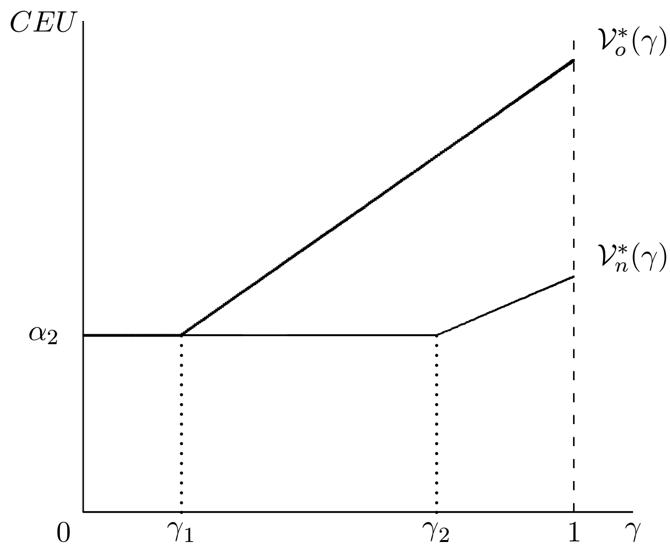

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

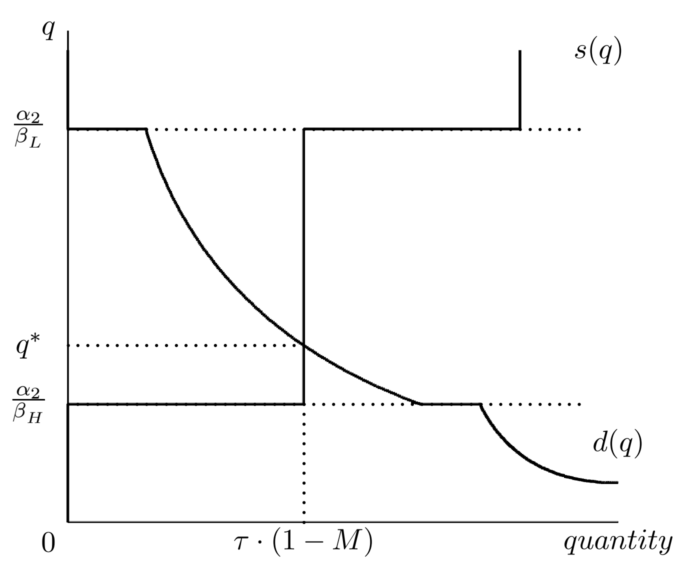

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

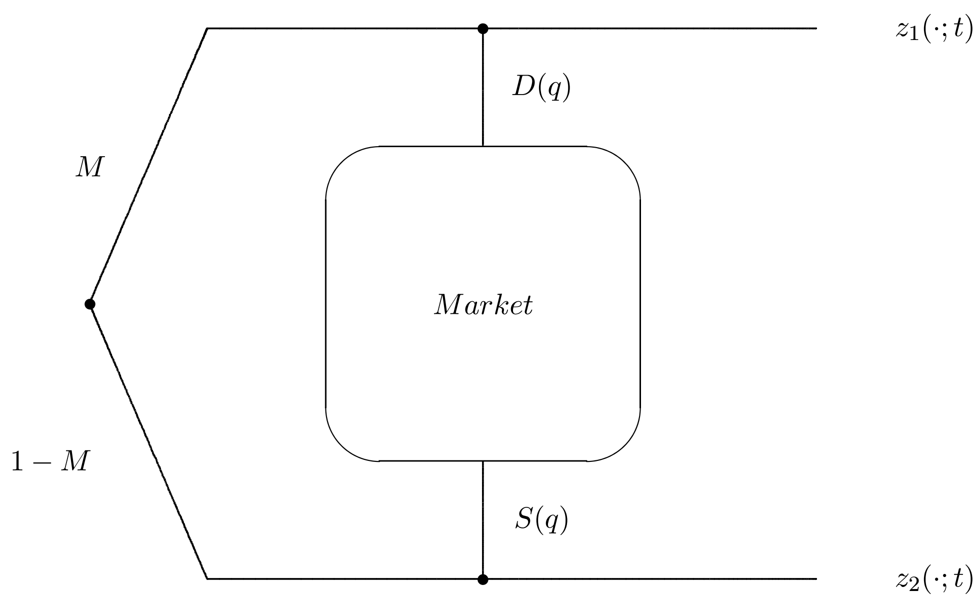

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & +(1-\gamma)\cdot\min_{(t,\tau)\in\{h,\ell\}\times[0,1]}[v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

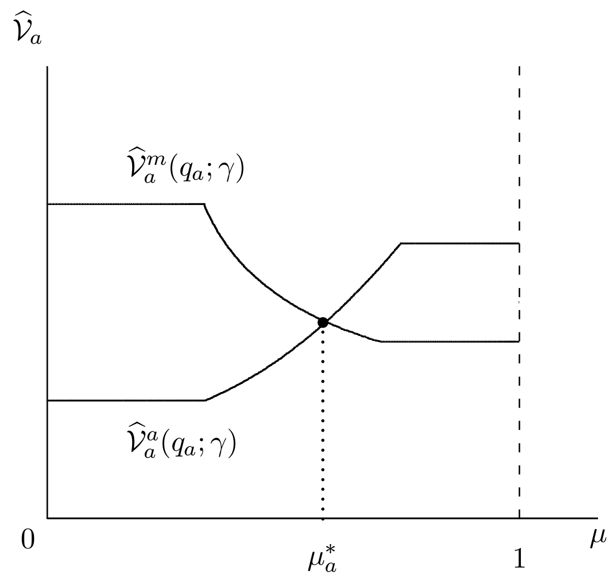

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

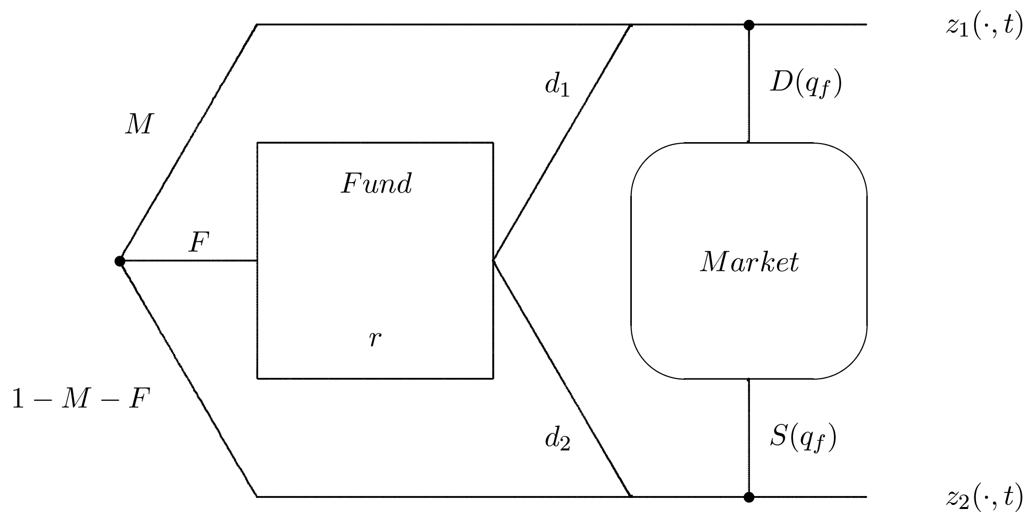

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 :=& \quad r \\ d_2 :=& \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau)\cdot\mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r)\cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

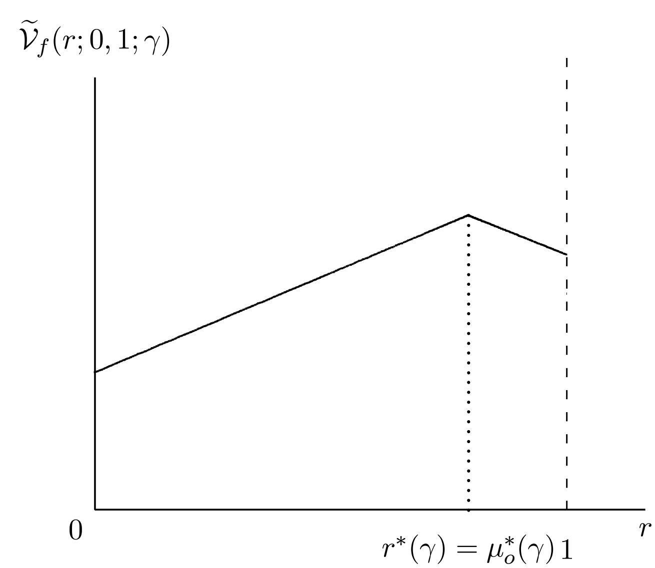

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0, 1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

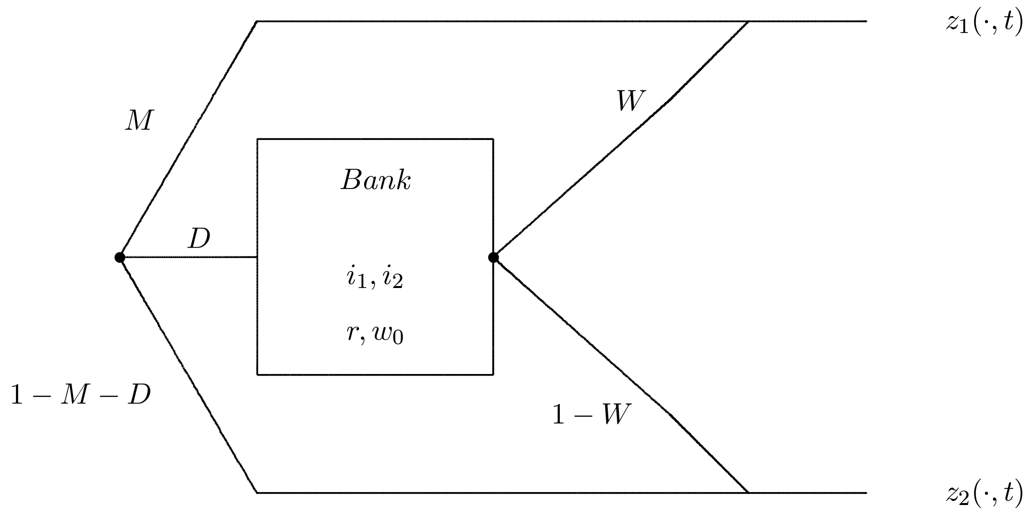

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

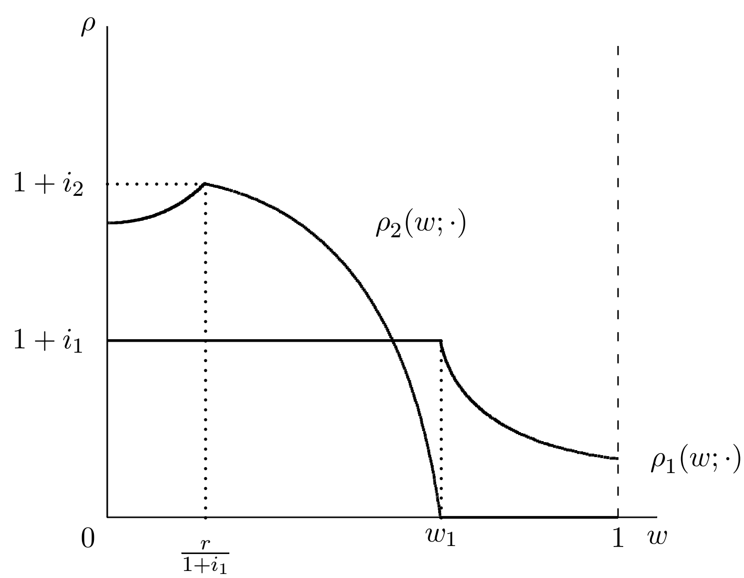

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$
$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

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[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

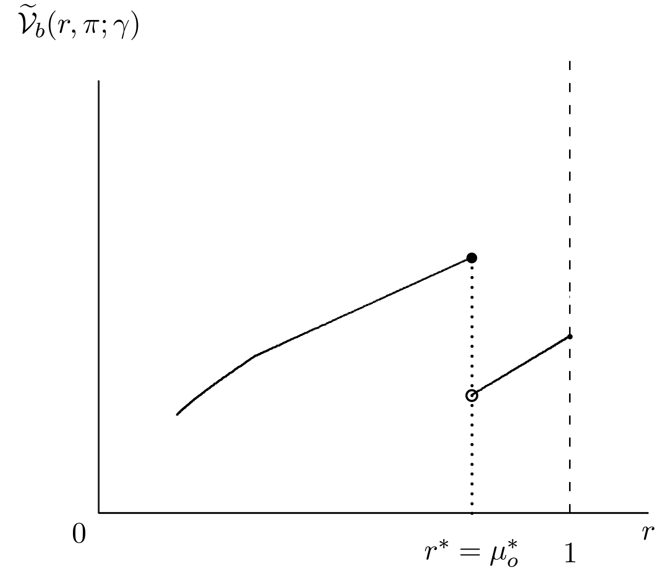

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

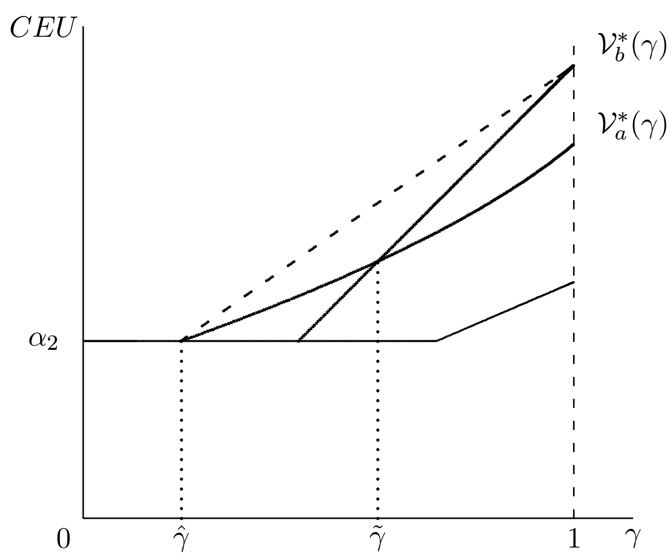

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t,\tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S| \ f(s) \geq t\}$ and $\{s \in S| \ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S| \ f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S| \ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}
\mathcal{V}_a^*(\gamma) :=\ & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\
=\ & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\
& \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.
\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau\in[0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \ + \ (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ, φ). Thus, its decision problem for given (ψ, φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0, *one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period 1, i.e.,* $w = \tau$, *the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \leq \beta_h \cdot \rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell \cdot \rho_1(\tau;r,w_0) \leq \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. *When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for* $\tau < 1$ *the constraint*

$$\beta_\ell \cdot \rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \leq \beta_h \cdot \rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h \cdot \rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h) = 1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell) = 0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) := & \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ = & \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ & \qquad + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \le \rho_2(\pi; r, w_0) \le \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1-\pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \; & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \; & \max\{\mathcal{V}_n^*(\gamma),\; \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1-r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1-r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\, , \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ *is a best reply for* $r = \mu_\ell$ *and* $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

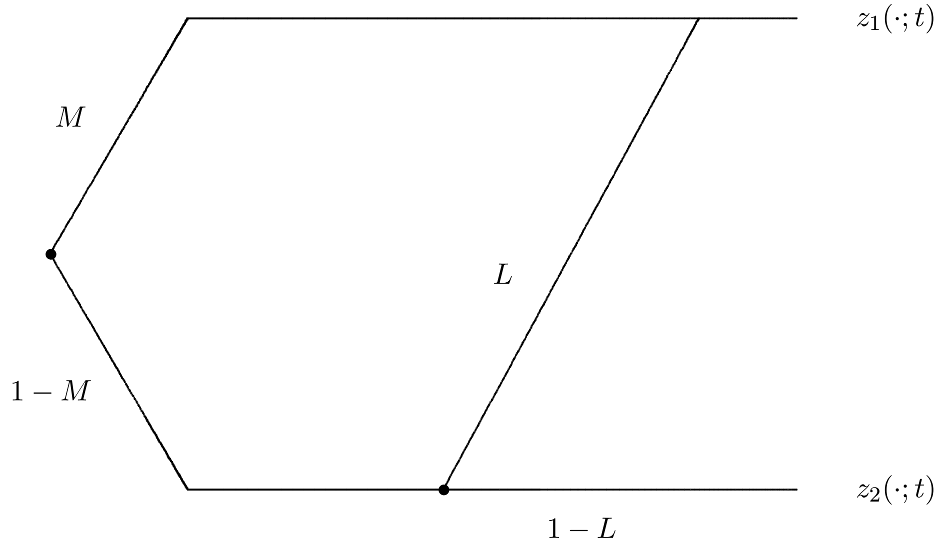

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{array}{rl} z_1(M,L) := & M + \alpha_1 \cdot L \cdot (1-M) \quad \text{and} \\ z_2(M,L) := & \alpha_2 \cdot (1-L) \cdot (1-M). \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

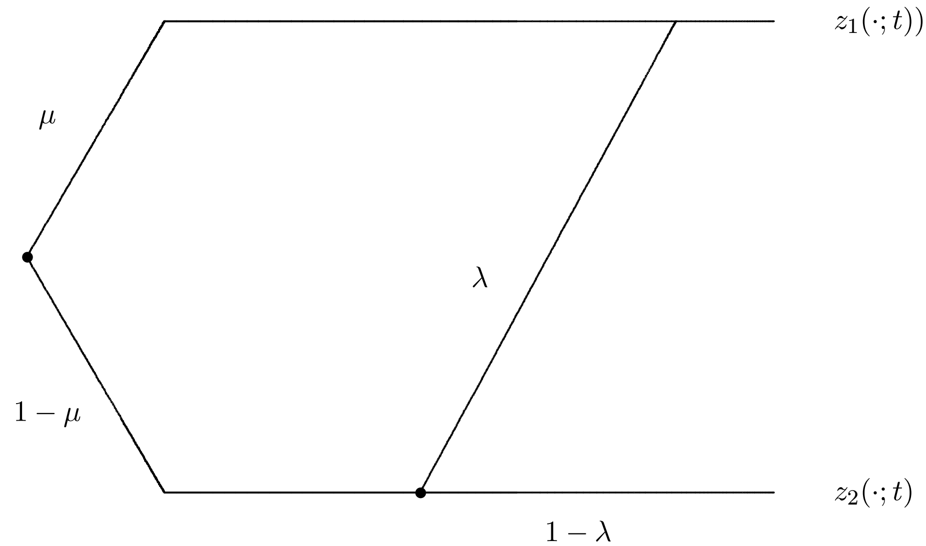

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$
\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}
$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

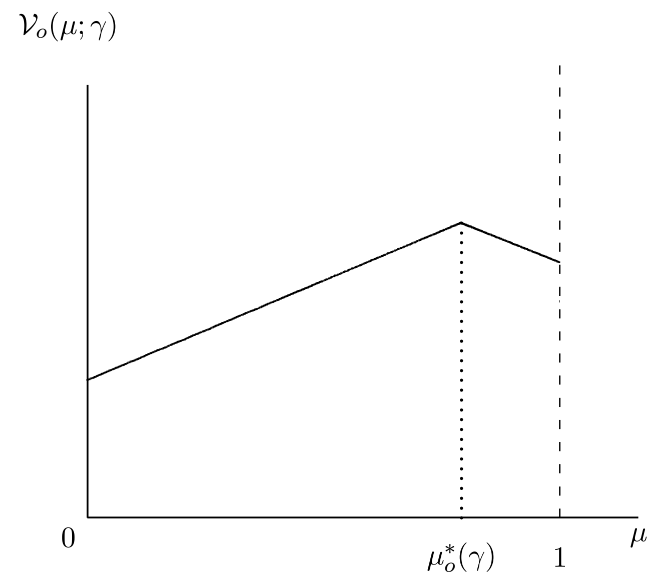

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

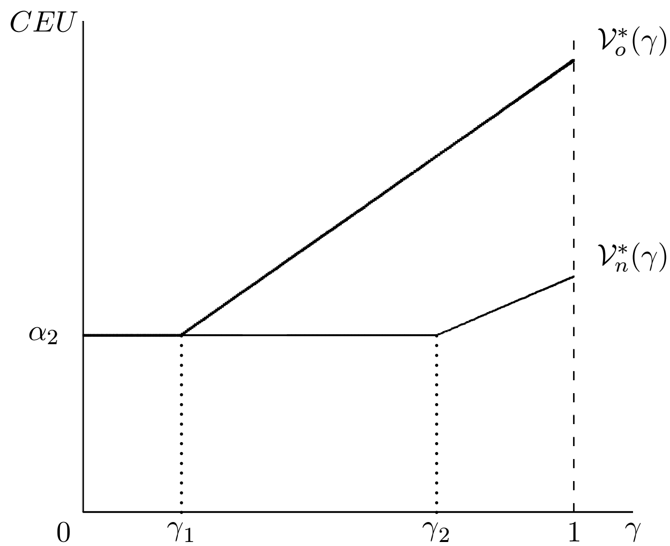

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

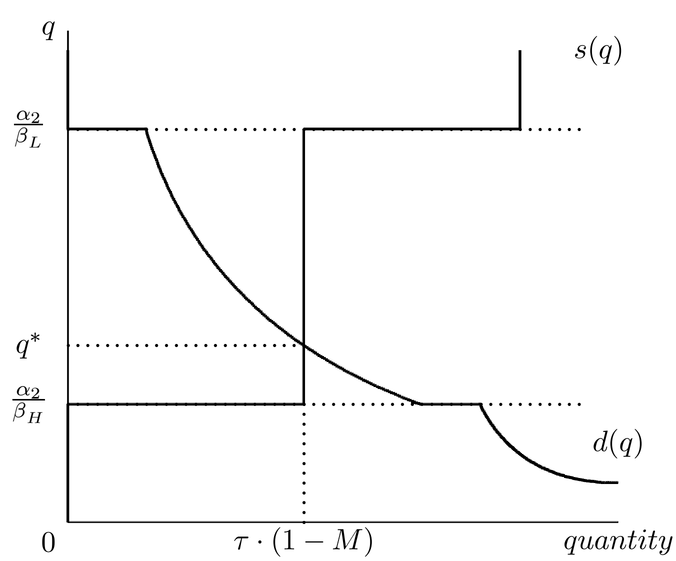

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau, \mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau, \mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

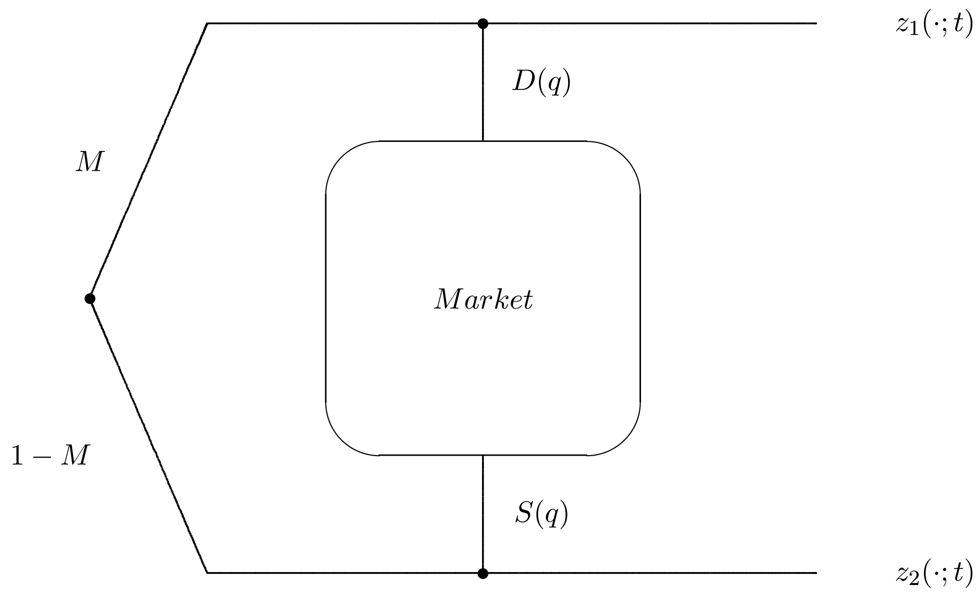

Figure 6: Asset market

Denote by $R^m(q_a; t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a; t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M; \tau, \mu; t)$, from the investment decision M is easily computed:

$$v_a(M; \tau, \mu; t) := M \cdot R^m(q_a^E(\tau, \mu); t) + (1 - M) \cdot R^a(q_a^E(\tau, \mu); t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]}[v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1,$ *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi,$
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1),$ *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi),$
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

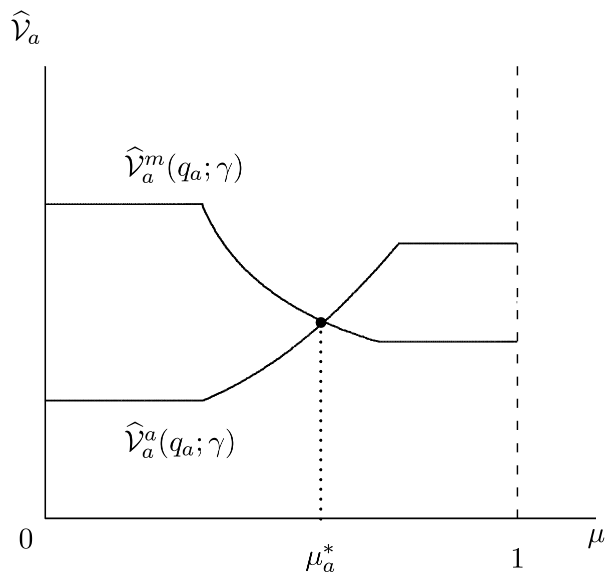

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

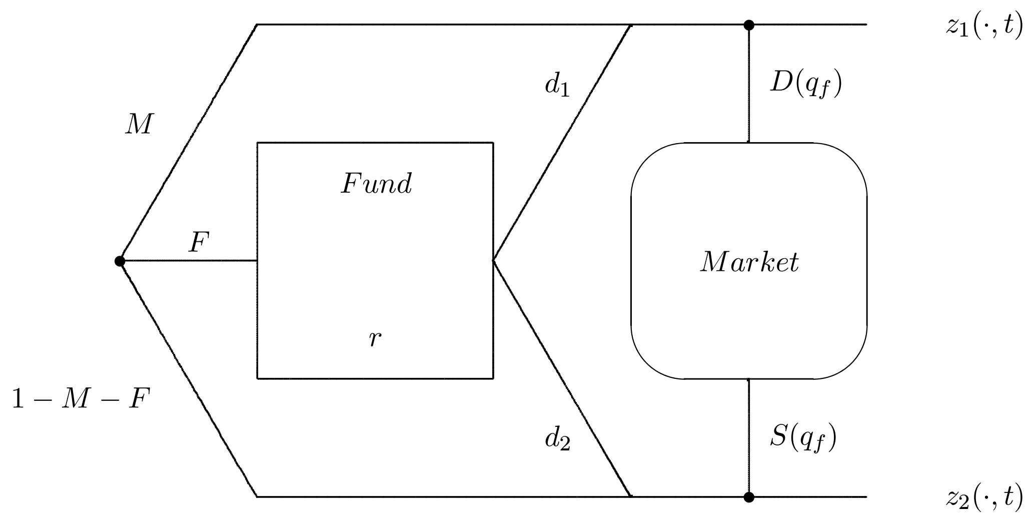

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] \\ +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

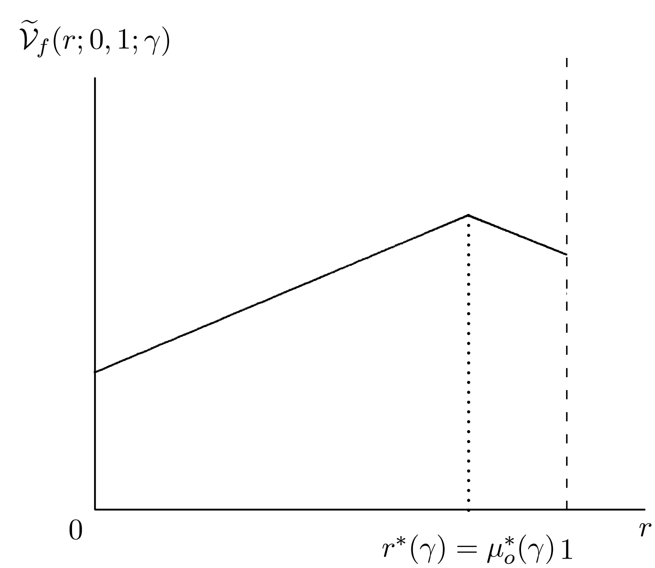

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*
For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

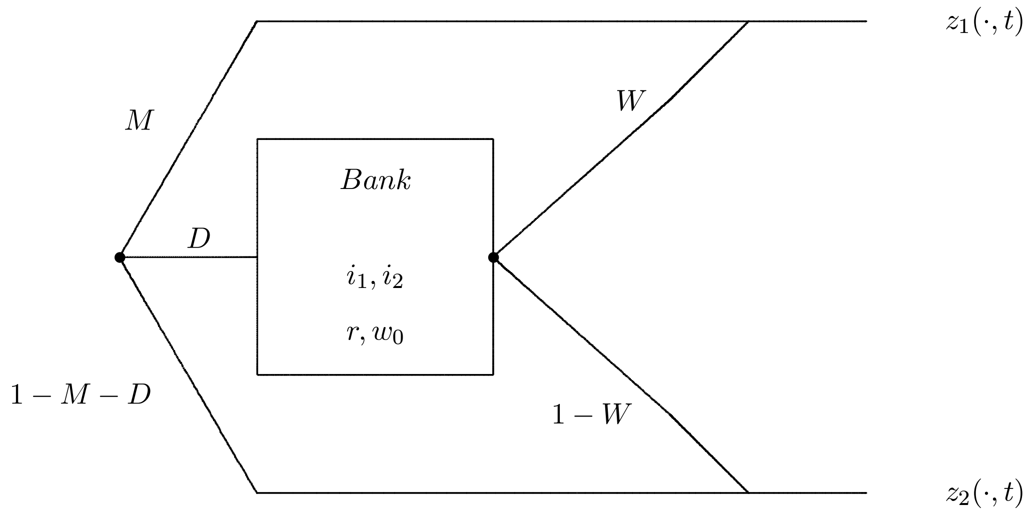

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

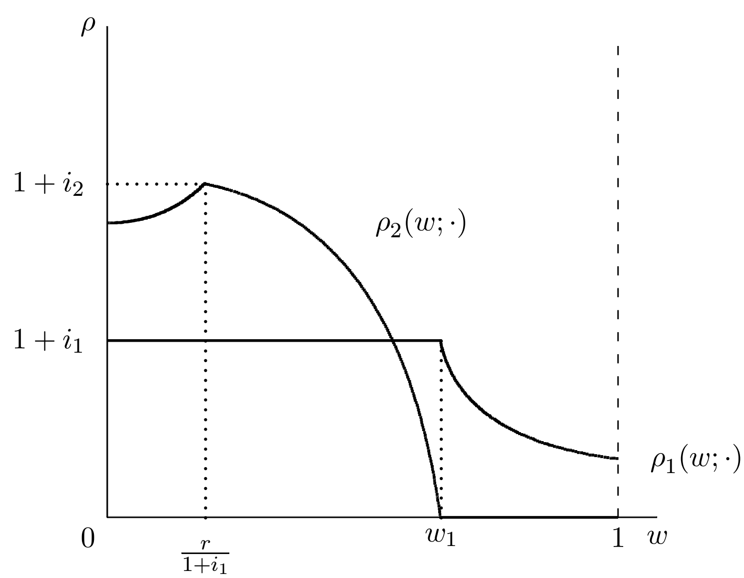

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1 - \tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

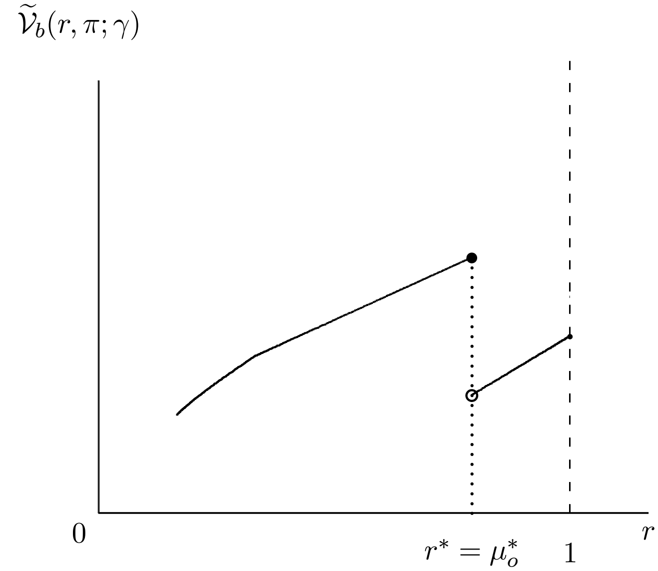

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

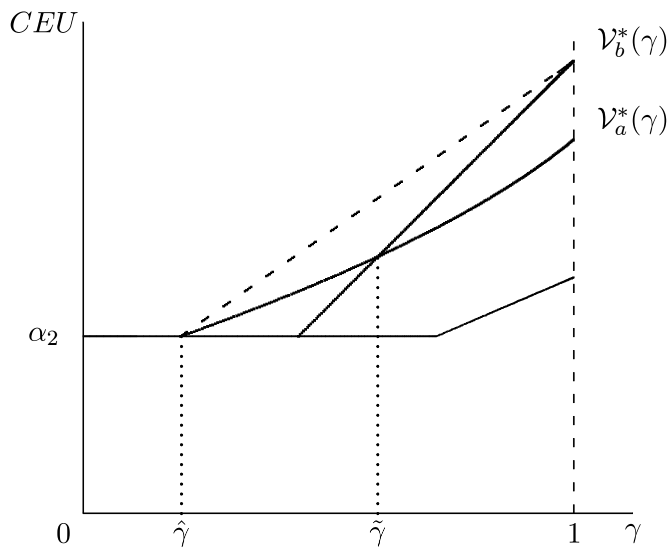

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \; f(s) \geq t\}$ and $\{s \in S | \; f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S | \; f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S | \; f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0,\underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu),1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0,\underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu),1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & + \sqrt{ \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h } \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) := \;& \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\ =\;& \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau\in[0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \ + \ (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$
$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2 \cdot (1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ,φ). Thus, its decision problem for given (ψ,φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w=\tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ the constraint

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution “competing banks” under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau;r,w_0;\ell) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) < \beta_\ell \cdot \rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) \geq \beta_\ell \cdot \rho_1(\tau;r,w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D,M;r,w_0;\gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\ &\qquad\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D,M;r,w_0;\gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r,w_0;\gamma) &:= \max_{\substack{(D,M)\in[0,1]\times[0,1] \\ s.t.\ D+M\leq 1}} \mathcal{V}_b(D,M;r,w_0;\gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi;r,w_0;h) + (1-\pi)\cdot v_b(\pi;r,w_0;\ell)\} \\ &\qquad + (1-\gamma)\cdot \min_{\tau\in[0,1]} v_b(\tau;r,w_0;t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r,w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r,w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$, taking into account the withdrawal behaviour $W(\tau;r,w_0;t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

Appendix B: Proofs

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_h + \tau \cdot \alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_\ell + \tau \cdot \alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\,, \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ *is a best reply for* $r = \mu_\ell$ *and* $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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ABSTRACT. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. INTRODUCTION

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$*, where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

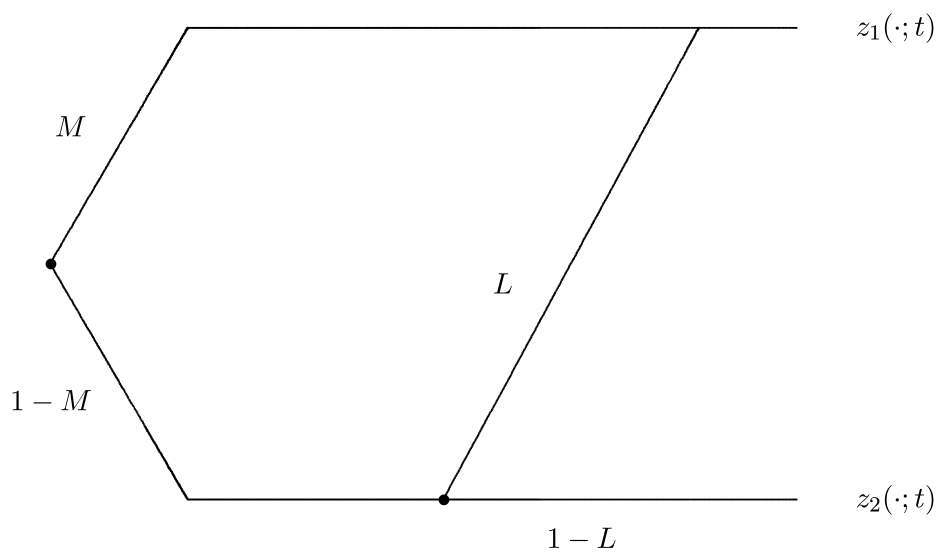

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{array}{ll} z_1(M,L) := & M + \alpha_1 \cdot L \cdot (1-M) \quad \text{and} \\ z_2(M,L) := & \alpha_2 \cdot (1-L) \cdot (1-M). \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1-M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := &\quad \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ &\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = &\quad \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ &\qquad + (1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

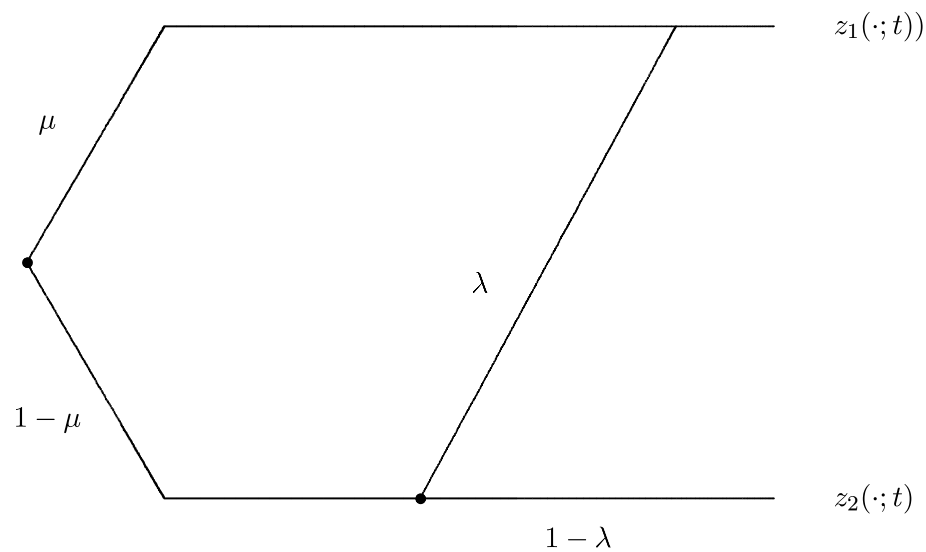

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$
\begin{aligned}
U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\
&:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}].
\end{aligned}
$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{lllll} \max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\ \text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\ & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\ & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\ & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\ & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN} \end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := & \quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

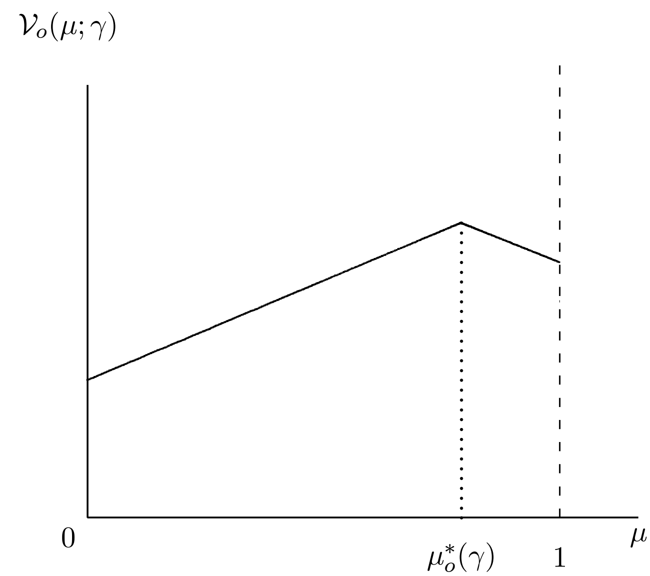

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

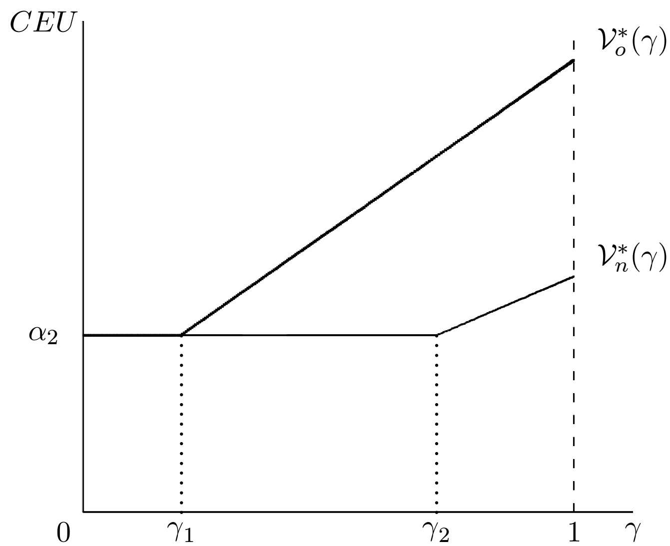

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

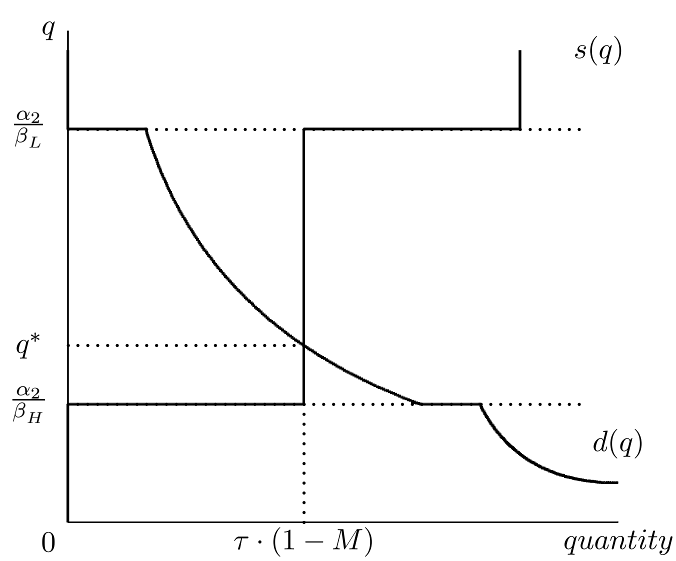

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

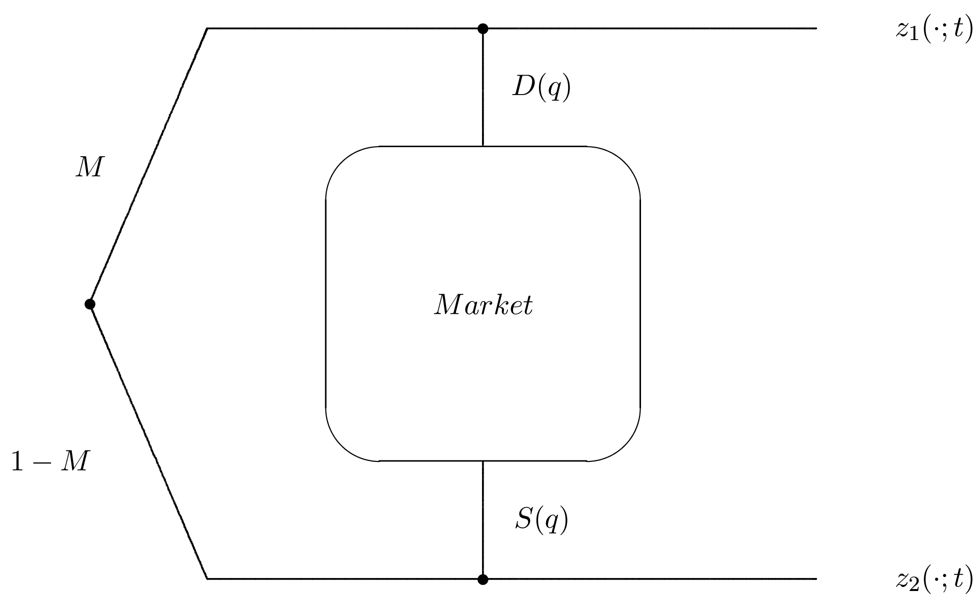

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\mathcal{V}_a(M,\mu;\gamma) := \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

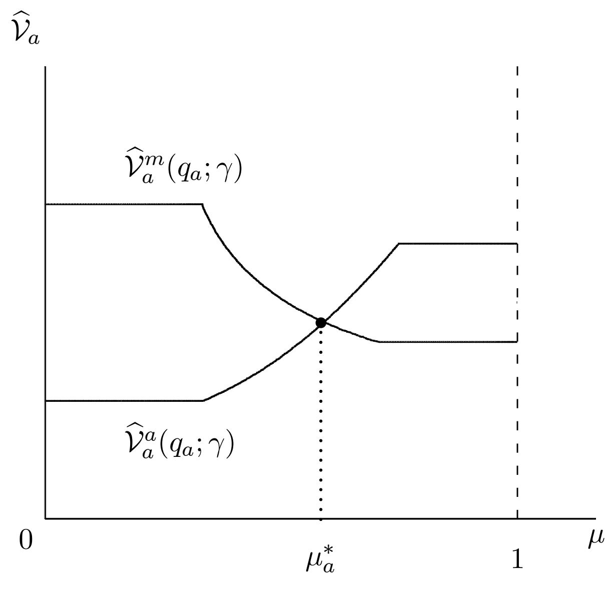

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

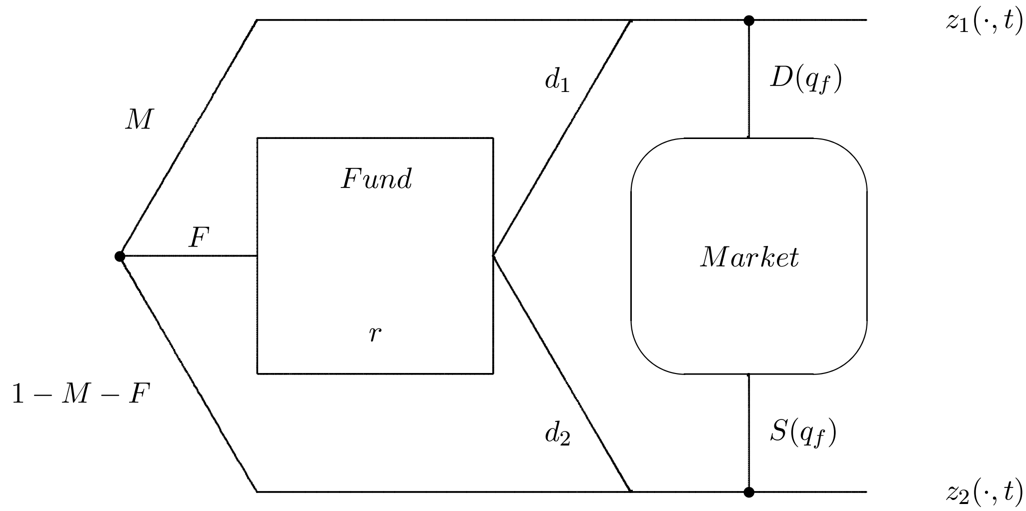

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 :=& \quad r \\ d_2 :=& \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)]$$
$$+(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \quad \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$
$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

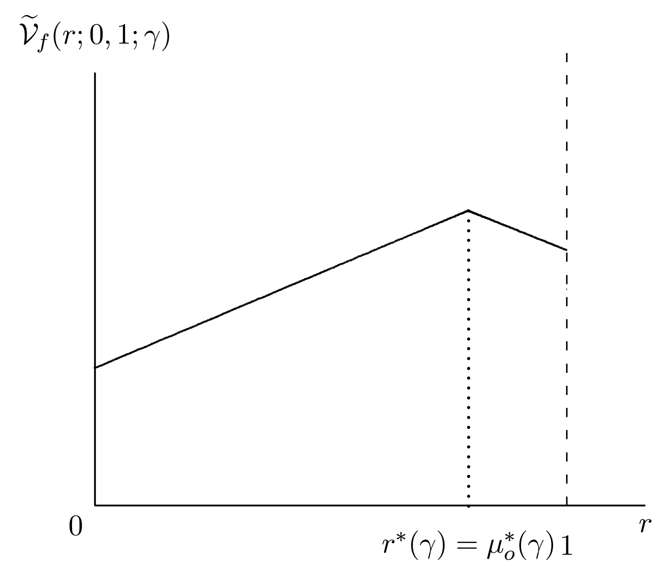

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

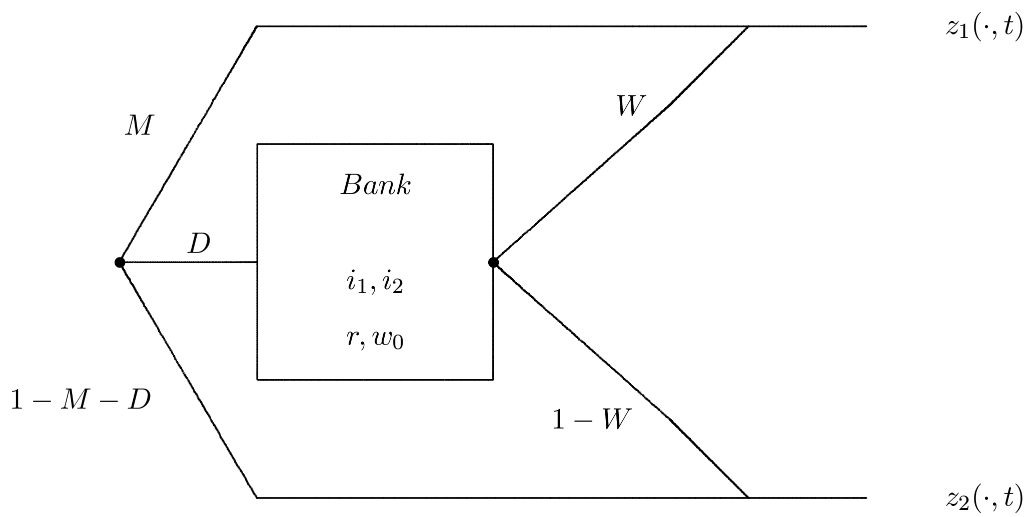

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

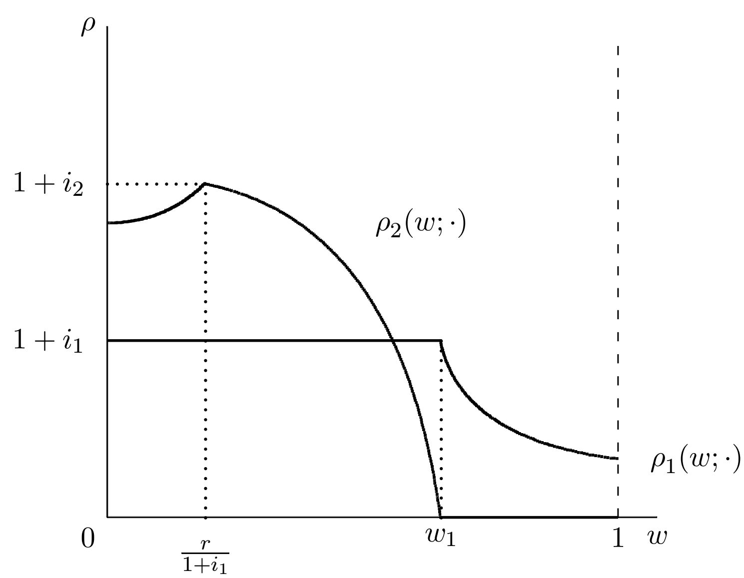

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

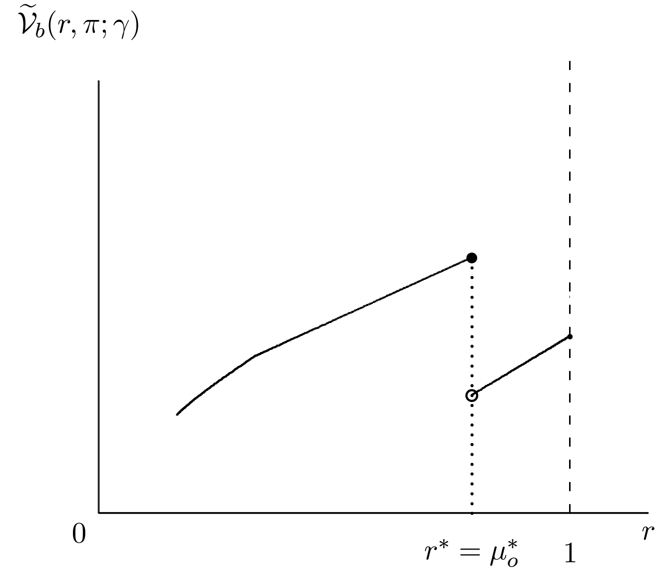

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

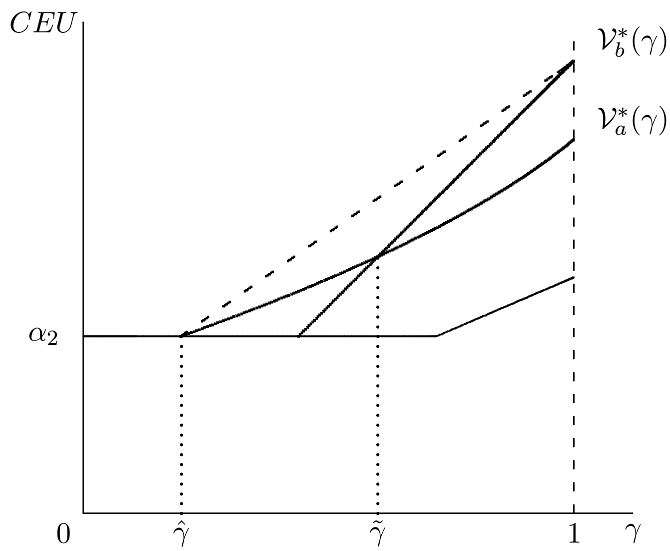

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S |\ f(s) \geq t\}$ and $\{s \in S |\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\int^{C} f \, d\nu = \int_{0}^{\infty} \nu(\{s \in S |\ f(s) \geq t\}) \, dt + \int_{-\infty}^{0} [\nu(\{s \in S |\ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)] + (1-\gamma)\cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_\ell] + (1-\gamma)\cdot \beta_\ell] + (1-M)\cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau)\cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi)\cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ. *This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$, *depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} &\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ &+ \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \,\Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) :=\ & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma)\\ =\ & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell,\\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau \in [0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \, + \, (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ,φ). Thus, its decision problem for given (ψ,φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w; i_1, i_2, r) := \begin{cases} \dfrac{\alpha_2 \cdot (1-r) + r - (1+i_1) \cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1 + i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)} \cdot [(1-r) - \dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1 + (1-\alpha_1) \cdot r}{1+i_1} \right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1 + (1-\alpha_1) \cdot r}{1+i_1}, 1 \right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0*, one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w; r, w_0) := \quad \widehat{\rho}_1(w; i_1(r, w_0), i_2(r, w_0), r),$$

$$\rho_2(w; r, w_0) := \quad \widehat{\rho}_2(w; i_1(r, w_0), i_2(r, w_0), r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period 1, i.e.,* $w = \tau$*, the following self selection constraints must hold:* [36]

$$\rho_2(\tau; r, w_0) \le \beta_h \cdot \rho_1(\tau; r, w_0) \qquad S_h$$

$$\beta_\ell \cdot \rho_1(\tau; r, w_0) \le \rho_2(\tau; r, w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau < 1$ *the constraint*

$$\beta_\ell \cdot \rho_1(\tau; r, w_0) > \rho_2(\tau; r, w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau; r, w_0; h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) \le \beta_h \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) > \beta_h \cdot \rho_1(\tau; r, w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau; h) = 1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau; \ell) = 0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) &:= \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ &\qquad + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \ & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \ & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu\in[\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau; 0, 1; \mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\,, \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ *is a best reply for* $r = \mu_\ell$ *and* $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ is convex

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma)\cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi)\cdot \alpha_2 + (1-\gamma)\cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a}\left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}\right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a &:= 2\cdot \pi \cdot \beta_h \\ b_1 &:= \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 &:= \gamma \cdot (\alpha_2 - \beta_\ell) \\ c &:= 4 \cdot \pi \cdot (1-\pi)\cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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ABSTRACT. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. INTRODUCTION

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

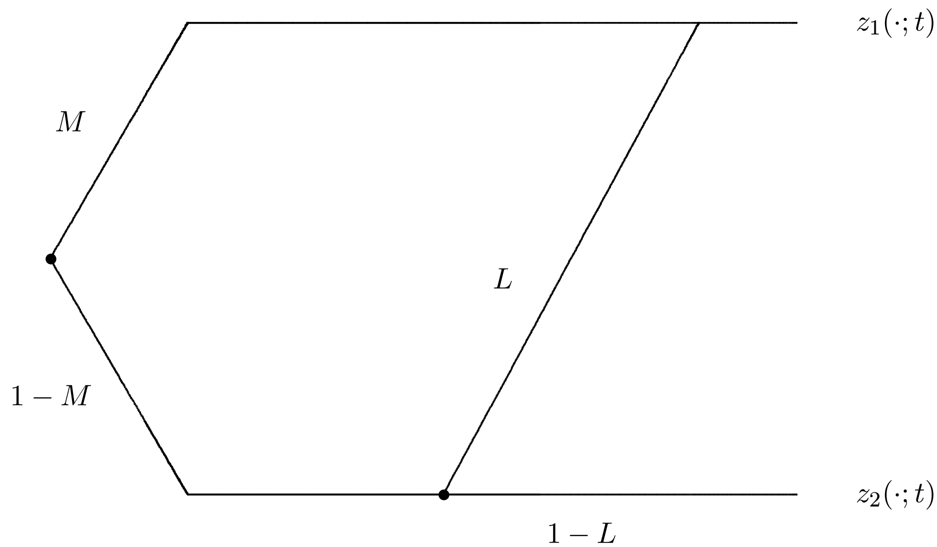

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) :=& \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) :=& \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M; t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

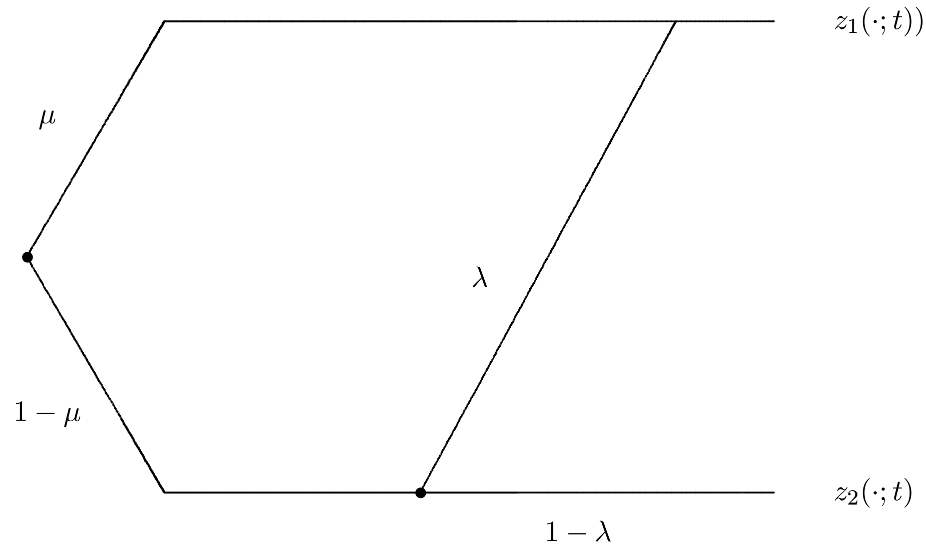

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := \quad & u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = \quad & \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

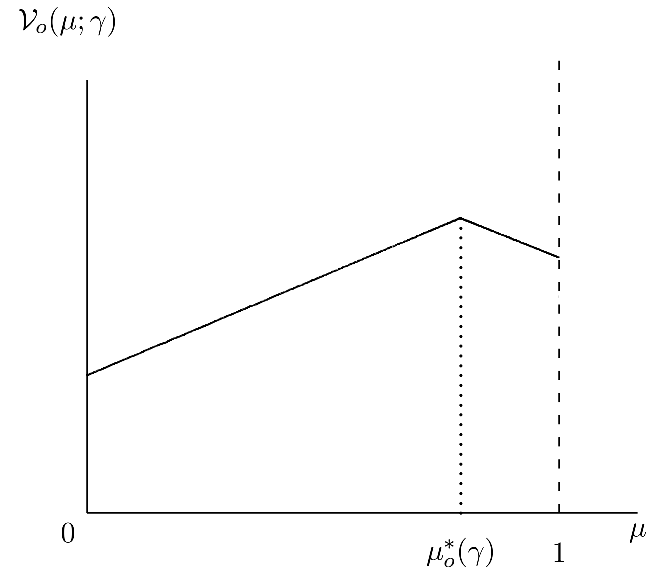

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

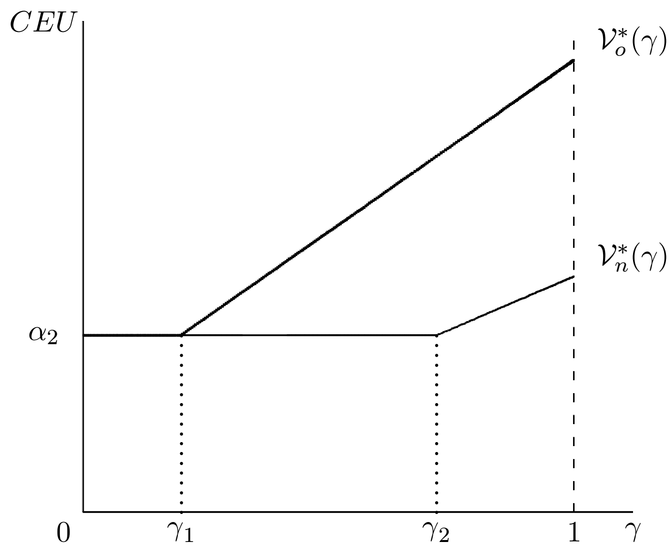

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

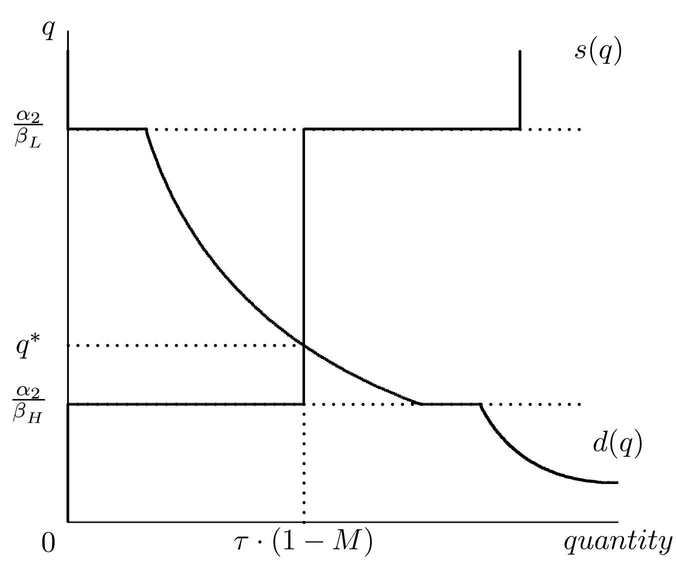

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

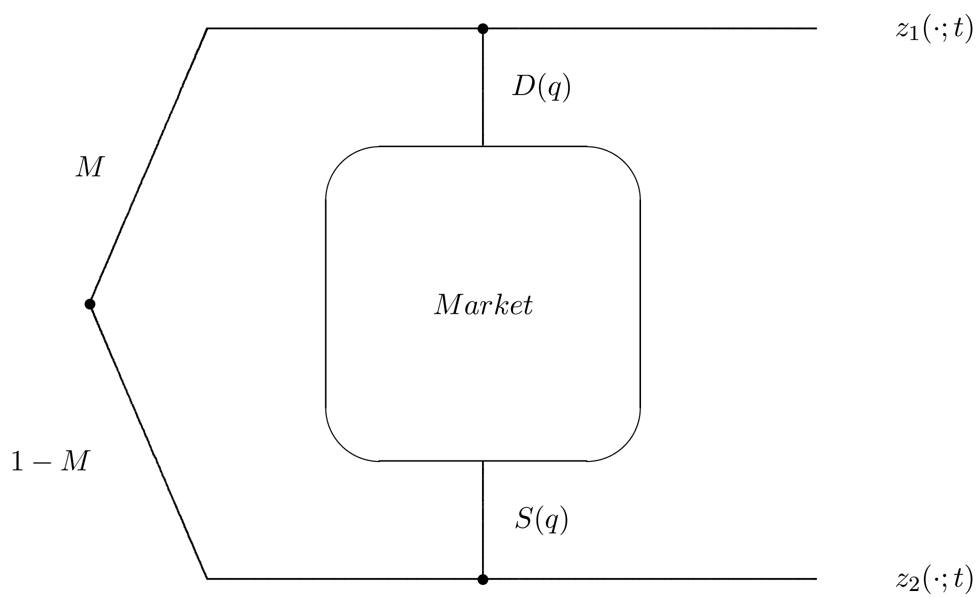

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\mathcal{V}_a(M, \mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1 - \pi) \cdot v_a(M; \pi, \mu; \ell)] \\ + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] \, .$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma), \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

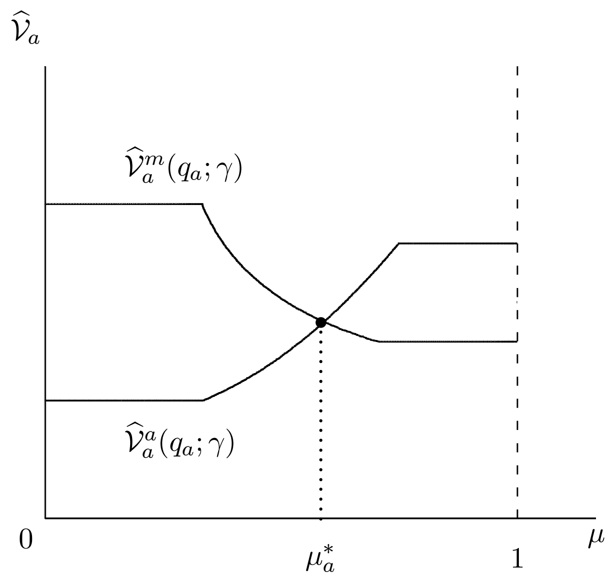

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

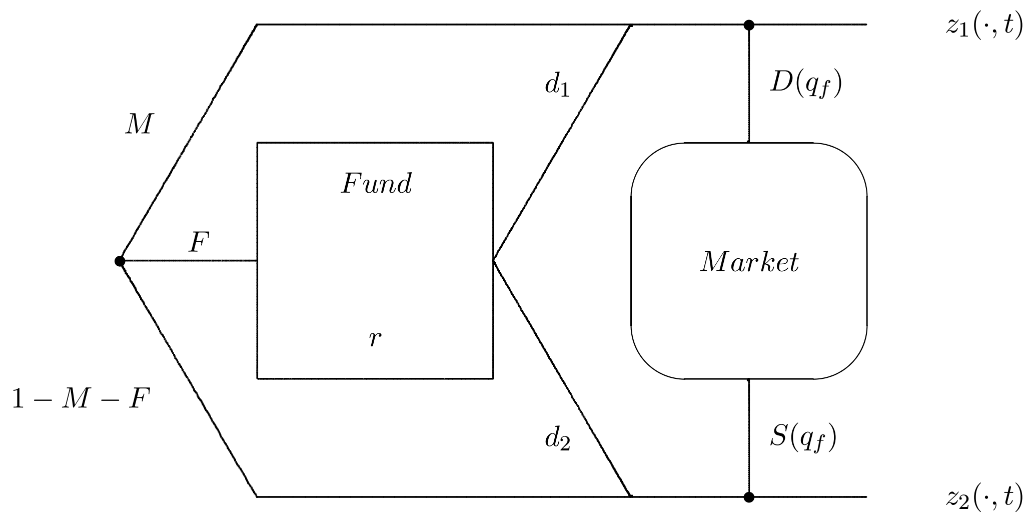

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) \,+\, (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] \\ +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

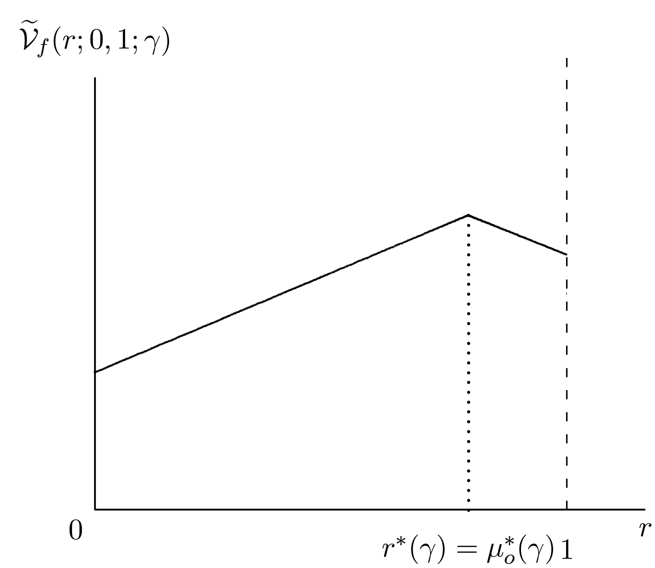

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.
For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.
The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

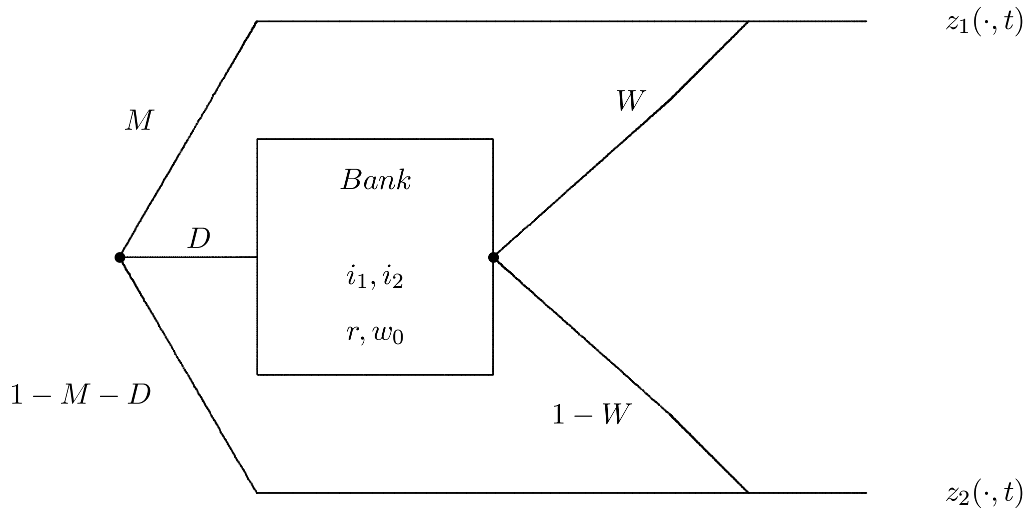

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

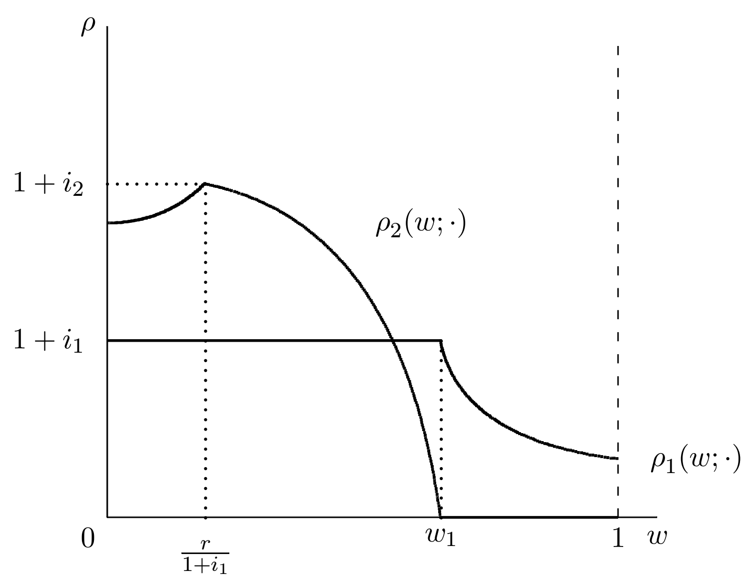

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D,M;r,w_0;\gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\
&\qquad\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D,M;r,w_0;\gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D,M;r,w_0;\gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$.

It remains to determine the deposit contract parameters (r,w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r,\pi;\gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

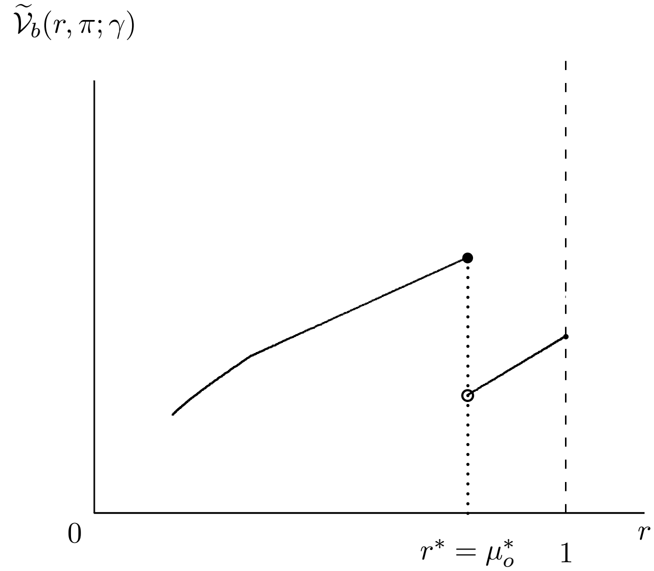

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

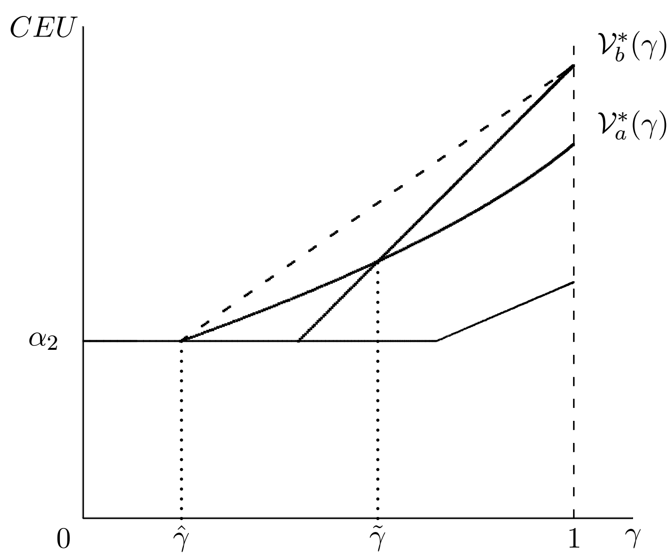

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. CONCLUDING REMARKS

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S |\ f(s) \geq t\}$ and $\{s \in S |\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f\, d\nu = \int_0^\infty \nu(\{s \in S |\ f(s) \geq t\})\, dt + \int_{-\infty}^0 [\nu(\{s \in S |\ f(s) \geq t\}) - 1]\, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$
\begin{aligned}
\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\
&= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2 \\
&= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.
\end{aligned}
$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$
\begin{aligned}
\underline{\tau}(\mu) &:= \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)}, \\
\overline{\tau}(\mu) &:= \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.
\end{aligned}
$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$
z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}
$$

$$
z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}
$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0,\overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu),1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0,\overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu),1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1+\gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \textit{if} \quad \pi \in [0,\underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu),1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ. *This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$, *depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) :=\ & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ =\ & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} &\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\left[- \left(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \right) \right. \\ &\left. + \sqrt{\left(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \right)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \right]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma, \pi, \alpha_2, \beta_h, \beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned} \mathcal{V}_a^*(\gamma) := \quad & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\ = \quad & \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}. \end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau \in [0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$*. When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$*.*

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h*, and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ*.*

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \, + \, (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$*. In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F*. Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$
$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ, φ). Thus, its decision problem for given (ψ, φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \textit{if} \quad w \leq \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \textit{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w=\tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \leq \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \leq \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ the constraint

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \leq \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau;r,w_0;\ell) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) < \beta_\ell \cdot \rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) \geq \beta_\ell \cdot \rho_1(\tau;r,w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D,M;r,w_0;\gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D,M;r,w_0;\gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r,w_0;\gamma) &:= \max_{\substack{(D,M)\in[0,1]\times[0,1] \\ s.t.\ D+M\leq 1}} \mathcal{V}_b(D,M;r,w_0;\gamma) \\ &= \max\{[\gamma\cdot\{\pi\cdot v_b(\pi;r,w_0;h) + (1-\pi)\cdot v_b(\pi;r,w_0;\ell)\} \\ &\qquad + (1-\gamma)\cdot \min_{\tau\in[0,1]} v_b(\tau;r,w_0;t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r,w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r,w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$, taking into account the withdrawal behaviour $W(\tau;r,w_0;t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau=\pi$ and $\tau=1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1-\pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \;& \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \;& \max\{\mathcal{V}_n^*(\gamma),\; \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1-r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1-r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\, , \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ *is a best reply for* $r = \mu_\ell$ *and* $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.

For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ is convex

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a}\left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}\right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a &:= 2 \cdot \pi \cdot \beta_h \\ b_1 &:= \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 &:= \gamma \cdot (\alpha_2 - \beta_\ell) \\ c &:= 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \; & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\
= \; & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\
& -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \; & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. NON-ADDITIVE BELIEFS AND THE CHOQUET INTEGRAL

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

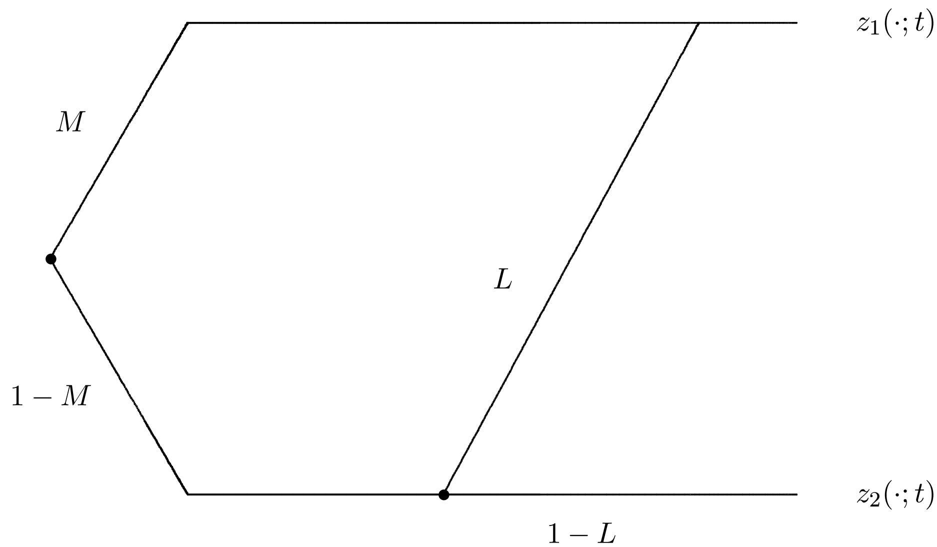

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) :=&\quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) :=&\quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1[\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

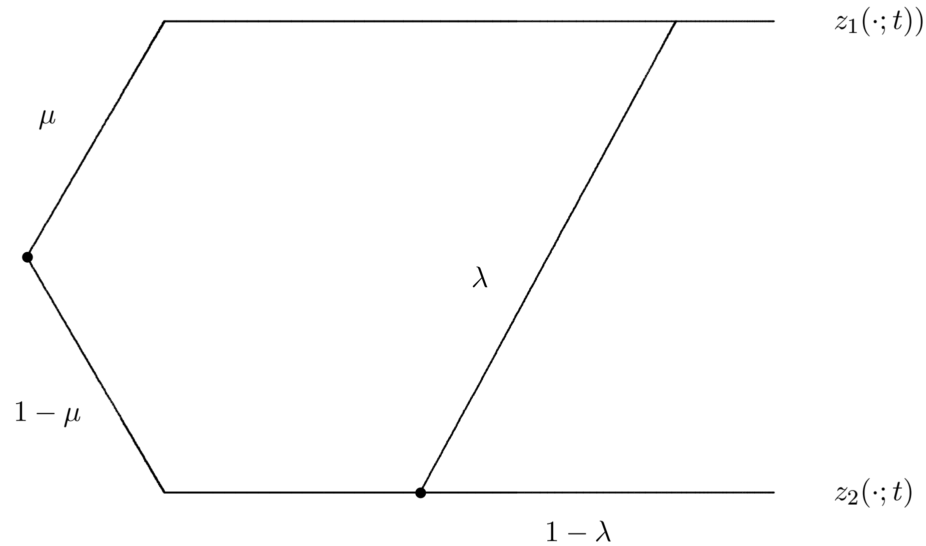

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

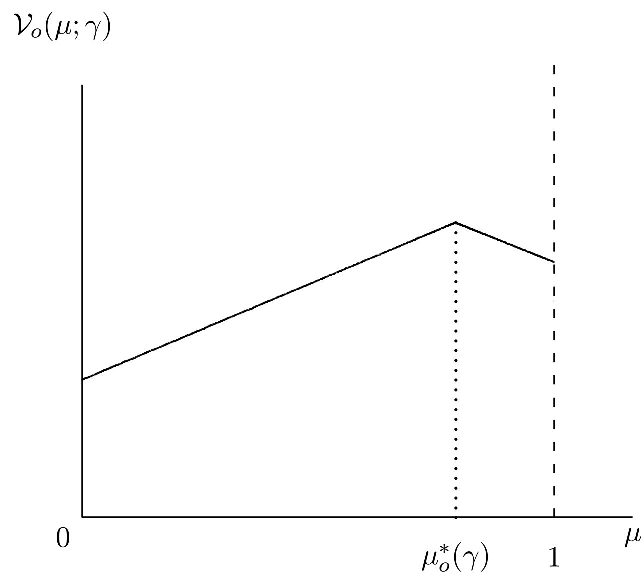

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

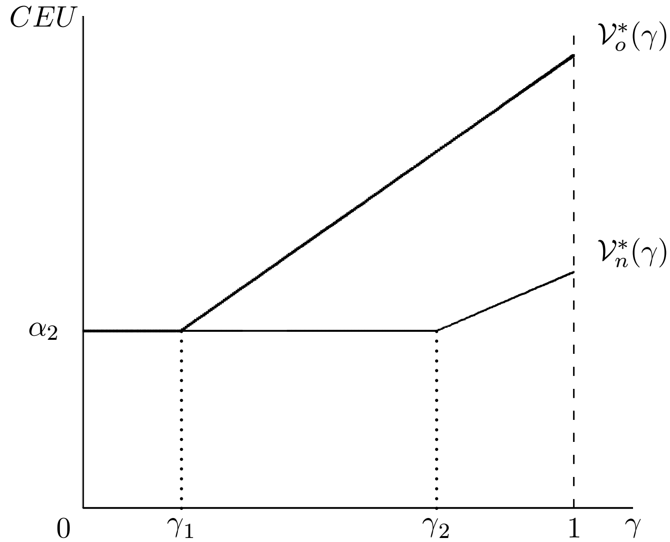

Figure 4: Optimal contract vs. no intermediation

4. ASSET MARKET AND MUTUAL FUND

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

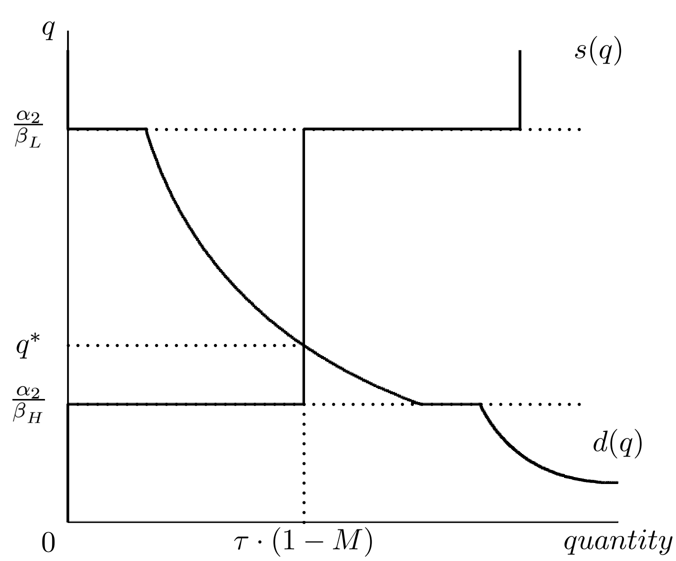

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

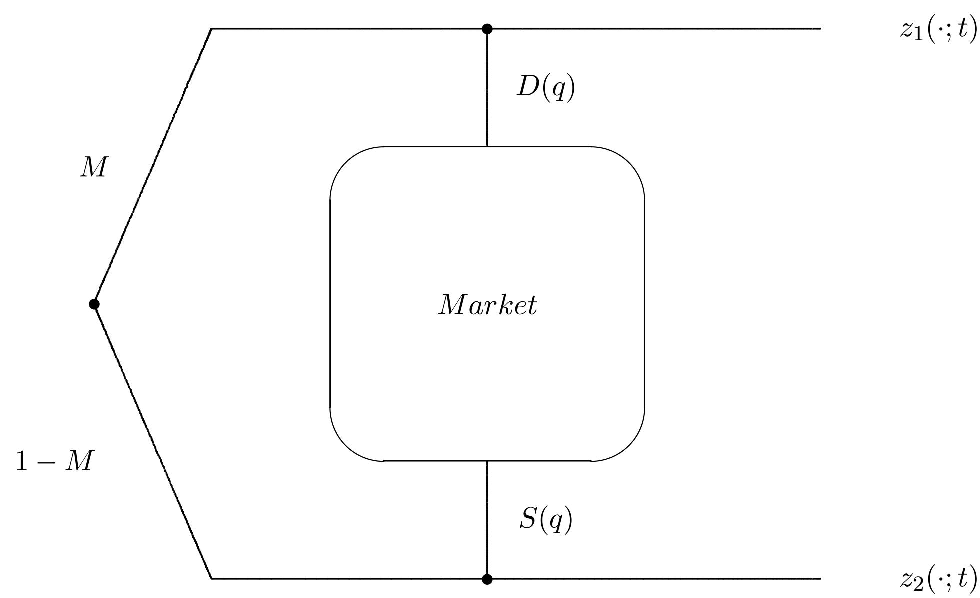

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] .\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma), \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

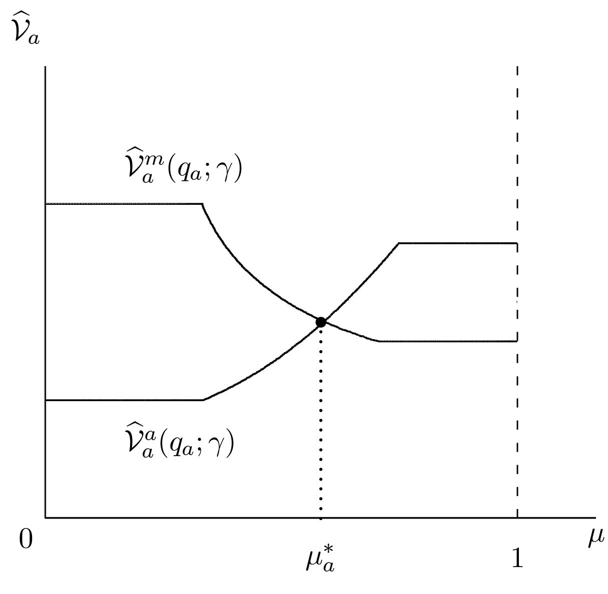

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

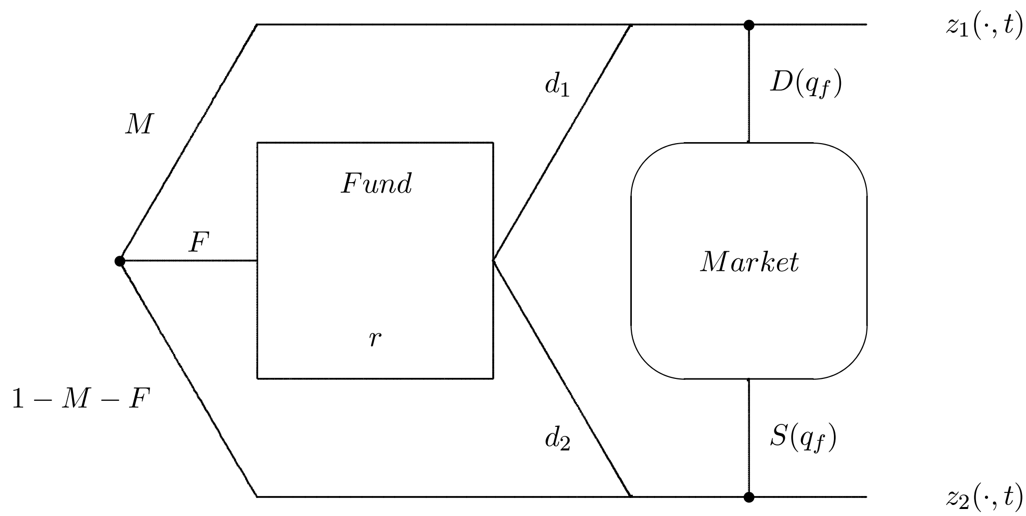

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 :=& \quad r \\ d_2 :=& \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

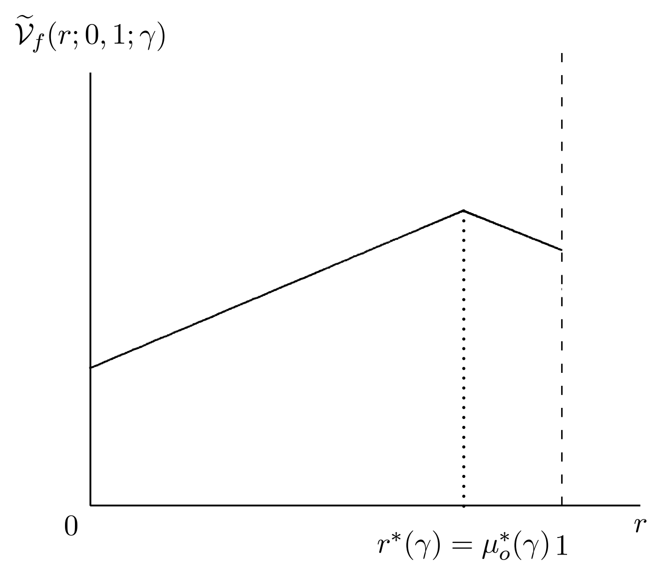

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0, 1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

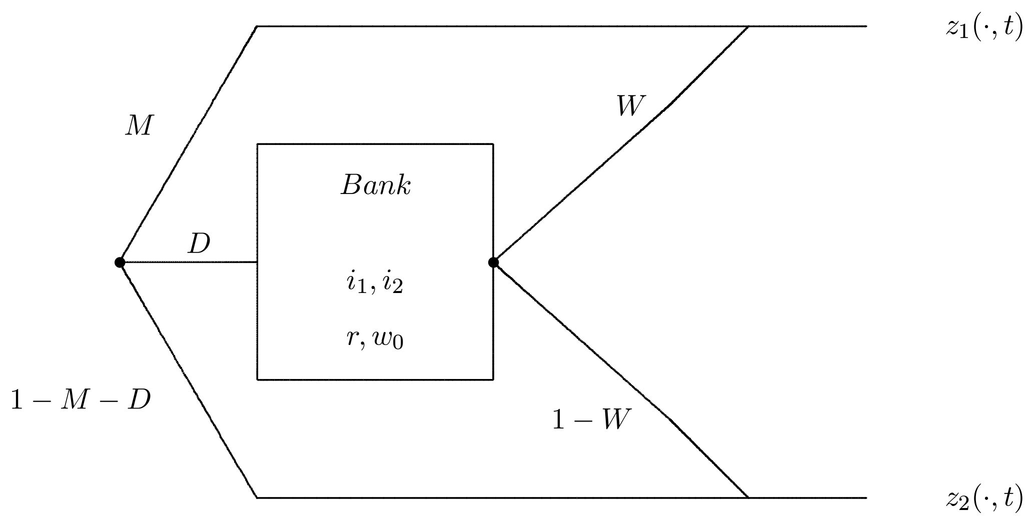

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

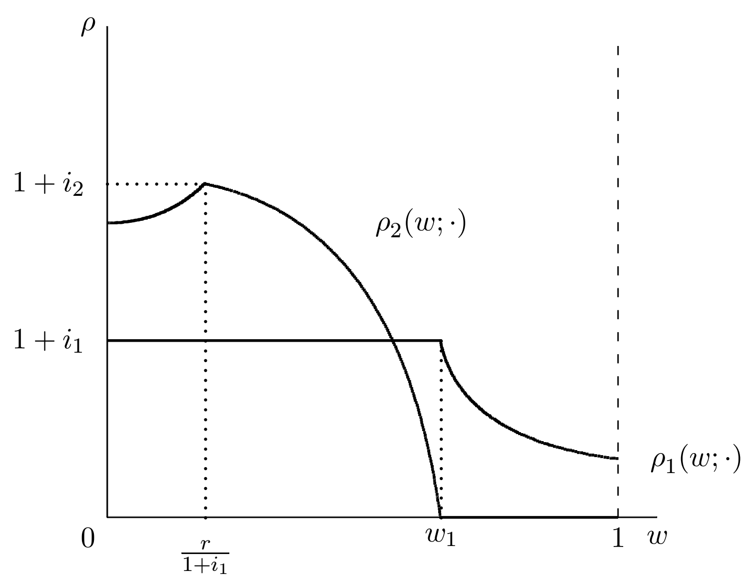

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}&\mathcal{V}_b(D, M; r, w_0; \gamma)\\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau\\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

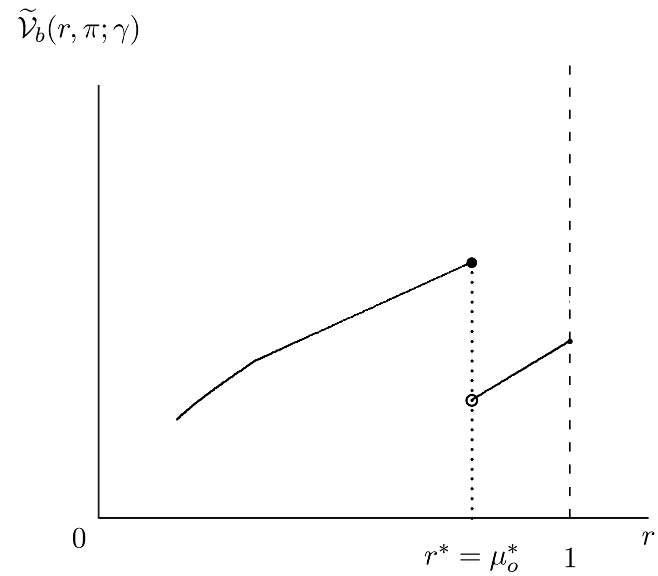

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ =& \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The “worst-case” pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

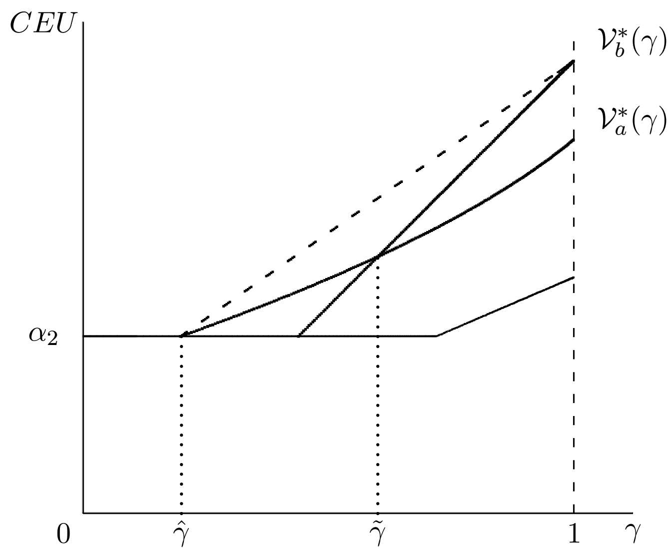

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. CONCLUDING REMARKS

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S| \; f(s) \geq t\}$ and $\{s \in S| \; f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^{\infty} \nu(\{s \in S| \; f(s) \geq t\}) \, dt + \int_{-\infty}^{0} [\nu(\{s \in S| \; f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$
\begin{aligned}
\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)] + (1-\gamma)\cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\
&= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_\ell] + (1-\gamma)\cdot \beta_\ell] + (1-M)\cdot \alpha_2 \\
&= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.
\end{aligned}
$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$
\begin{aligned}
\underline{\tau}(\mu) &:= \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)}, \\
\overline{\tau}(\mu) &:= \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.
\end{aligned}
$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$
z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}
$$

$$
z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}
$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau)\cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \textit{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ. *This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$, *depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market exists and the price*

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma, \pi, \alpha_2, \beta_h, \beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}
\mathcal{V}_a^*(\gamma) :=\ & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\
=\ & \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\
& \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.
\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. *For investing in the illiquid asset it occurs for* $(t,\tau) = (\ell,\tau)$ *for any* $\tau\in[0,1]$. *From this it follows that for an asset price* q_a *the ex-ante Choquet expected utility for a consumer who holds a fraction* M *of his initial one unit of wealth as money is*

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t,\tau) = (\ell, 0)$*. When this is not the case, investment policy* M *may affect the state* (t,τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$*.*

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h*, and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ*.*

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \ + \ (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$*. In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F*. Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$
$$+(1-\gamma)\cdot [r\cdot \beta_\ell + \alpha_2 \cdot (1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ,φ)*. Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$.
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1, i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \leq \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\ \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w; i_1, i_2, r) := \begin{cases} \dfrac{\alpha_2 \cdot (1-r) + r - (1+i_1) \cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1 + i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)} \cdot [(1-r) - \dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1 + (1-\alpha_1) \cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1 + (1-\alpha_1) \cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0*, one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w; r, w_0) := \quad \widehat{\rho}_1(w; i_1(r, w_0), i_2(r, w_0), r),$$

$$\rho_2(w; r, w_0) := \quad \widehat{\rho}_2(w; i_1(r, w_0), i_2(r, w_0), r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period 1, i.e.,* $w = \tau$*, the following self selection constraints must hold:* [36]

$$\rho_2(\tau; r, w_0) \leq \beta_h \cdot \rho_1(\tau; r, w_0) \qquad S_h$$

$$\beta_\ell \cdot \rho_1(\tau; r, w_0) \leq \rho_2(\tau; r, w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau < 1$ *the constraint*

$$\beta_\ell \cdot \rho_1(\tau; r, w_0) > \rho_2(\tau; r, w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau; r, w_0; h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) \leq \beta_h \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) > \beta_h \cdot \rho_1(\tau; r, w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau; h) = 1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau; \ell) = 0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) &:= \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ &\qquad + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1-\pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \ \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ =& \ \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1-r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1-r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

Appendix B: Proofs

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_h + \tau \cdot \alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_\ell + \tau \cdot \alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau), \mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\,, \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := &\quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = &\quad \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$z_{2h}(\tau) := \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right]$$
$$= \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h.$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ is a best reply for $r = \mu_\ell$ and $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell$$
$$= \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell.$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1 \right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ is convex

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a}\left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}\right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a :=& \quad 2 \cdot \pi \cdot \beta_h \\ b_1 :=& \quad \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 :=& \quad \gamma \cdot (\alpha_2 - \beta_\ell) \\ c :=& \quad 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$
\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \; & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\
= \; & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\
& -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \; & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}
$$

As $c = 4 \cdot (1-\pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\widetilde{\gamma}) = \mathcal{V}_a^*(\widetilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a, b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a, b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a, b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1-\mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.
[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$*, where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

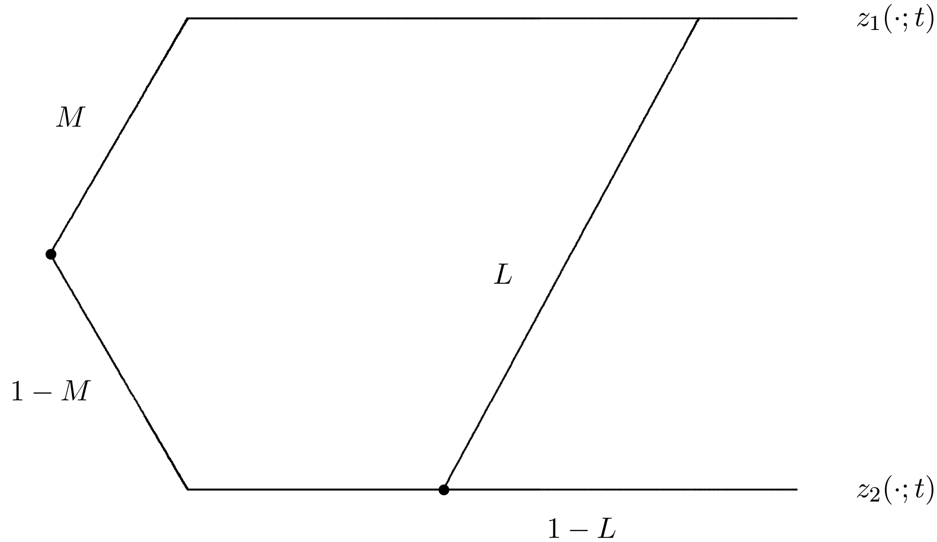

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=& \quad M + \alpha_1 \cdot L \cdot (1-M) \quad \text{and} \\ z_2(M,L) :=& \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1-M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types
The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1-\tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) :=\; & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)] \cdot p(\tau)\, d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ =\; & \gamma \cdot [\mathbb{E}_p\{\tau\} \cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\}) \cdot v_n(M;\ell)] \\ & + (1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) \;=\; \beta_h \cdot M + (1-M)\cdot \alpha_2 \;\geq\; \beta_\ell \cdot M + (1-M)\cdot \alpha_2 \;=\; v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) \;:=\; \mathcal{V}_n(M_n^*(\gamma);\gamma) \;=\; \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

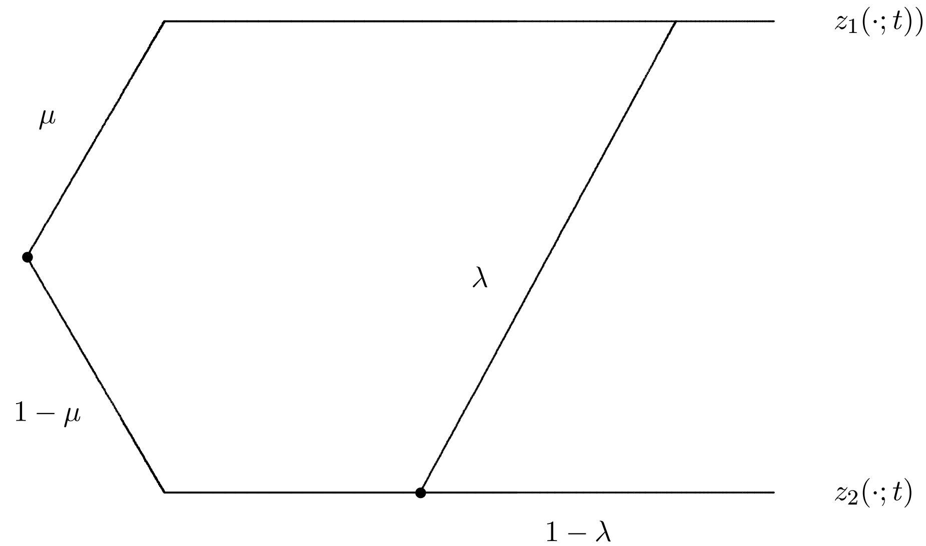

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
 & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
 & \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
 & \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
 & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) :=& \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ =& \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \textit{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \textit{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \textit{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

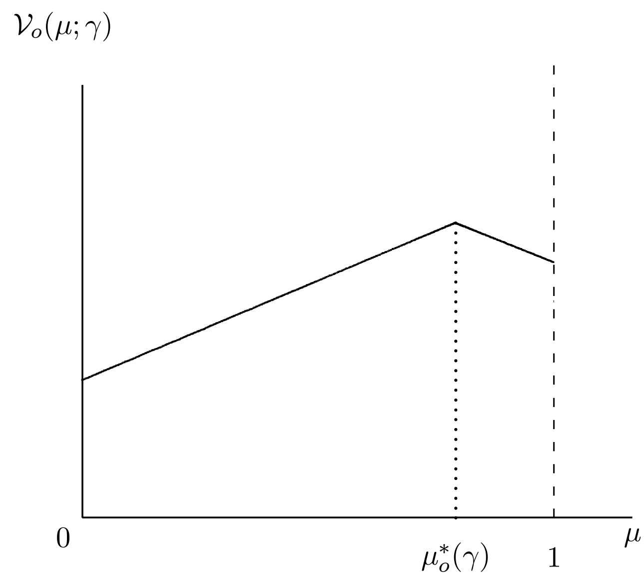

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

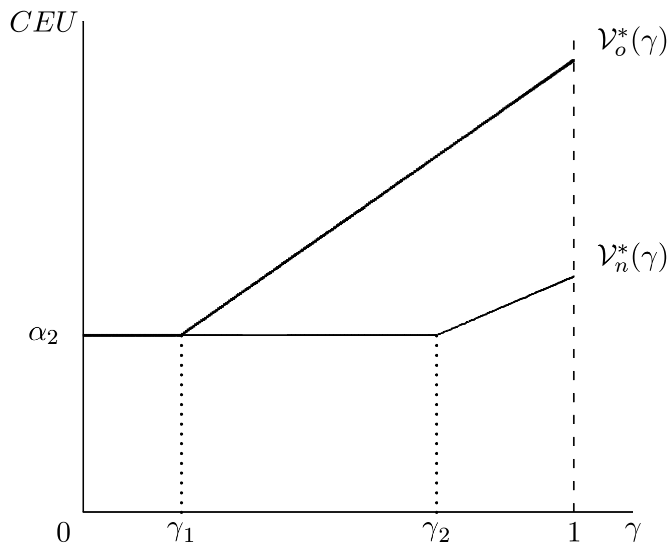

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

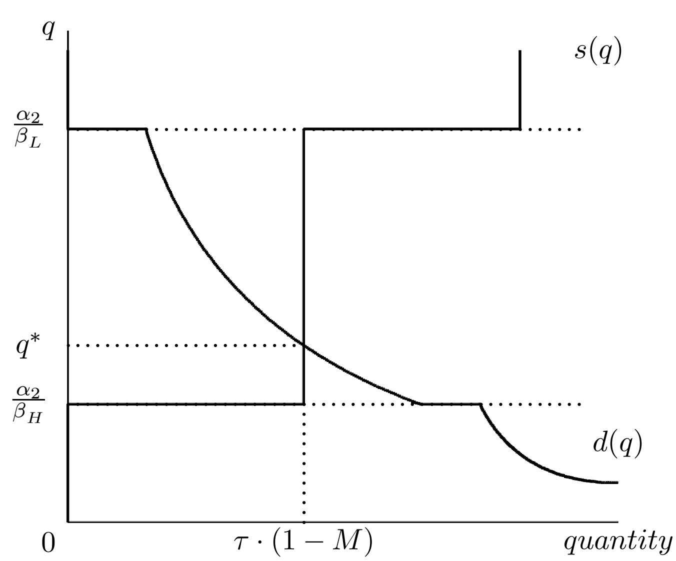

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

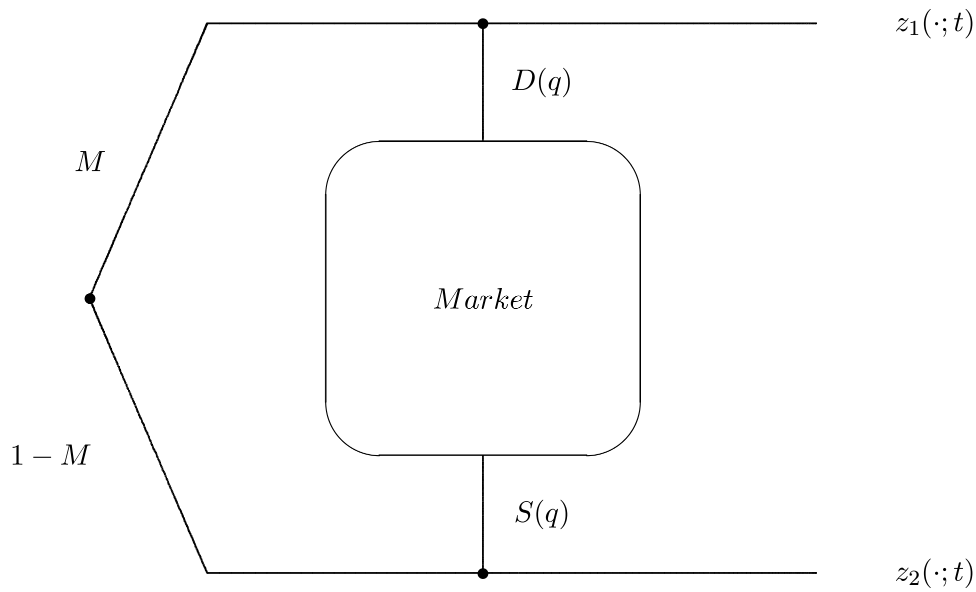

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] .\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma), \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

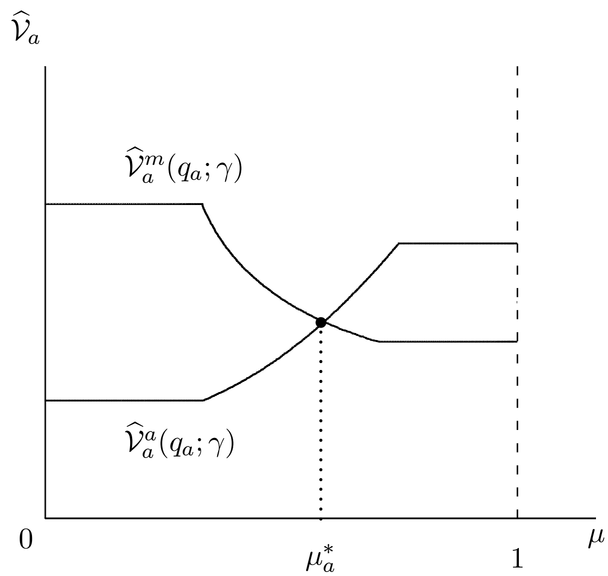

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

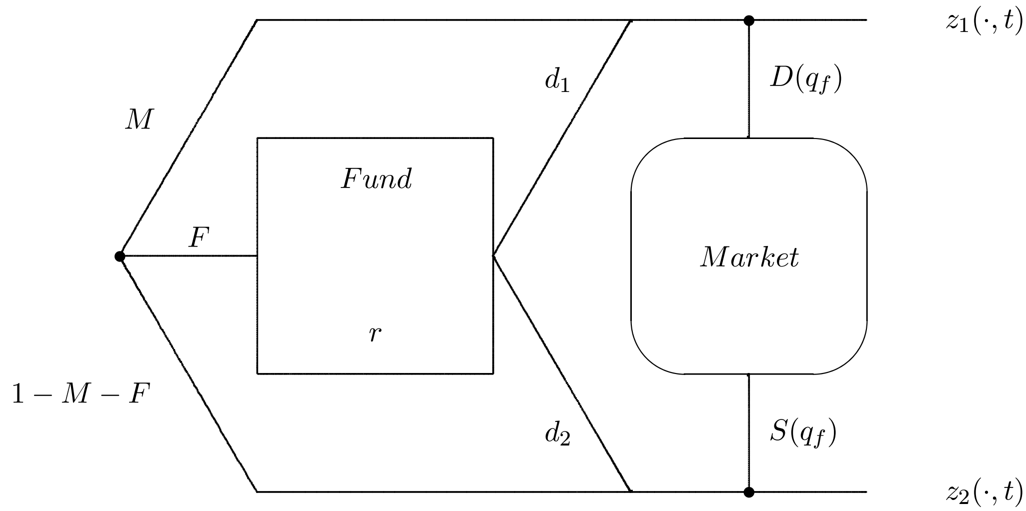

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by[24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

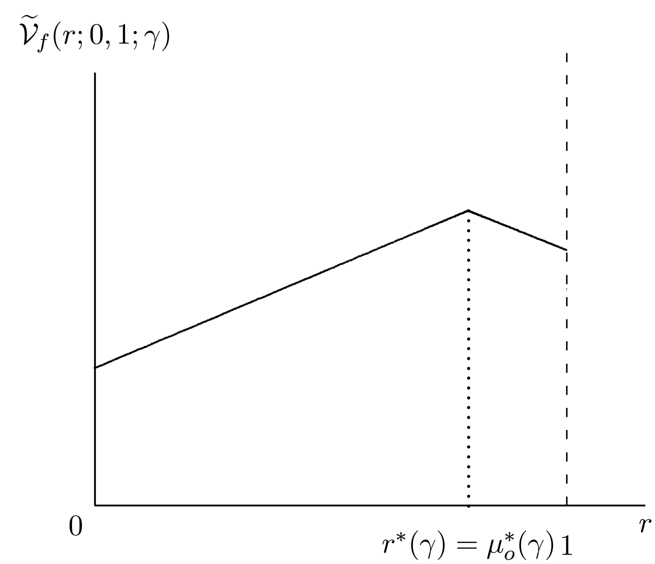

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26]See Appendix A, Remark 10.
[27]See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

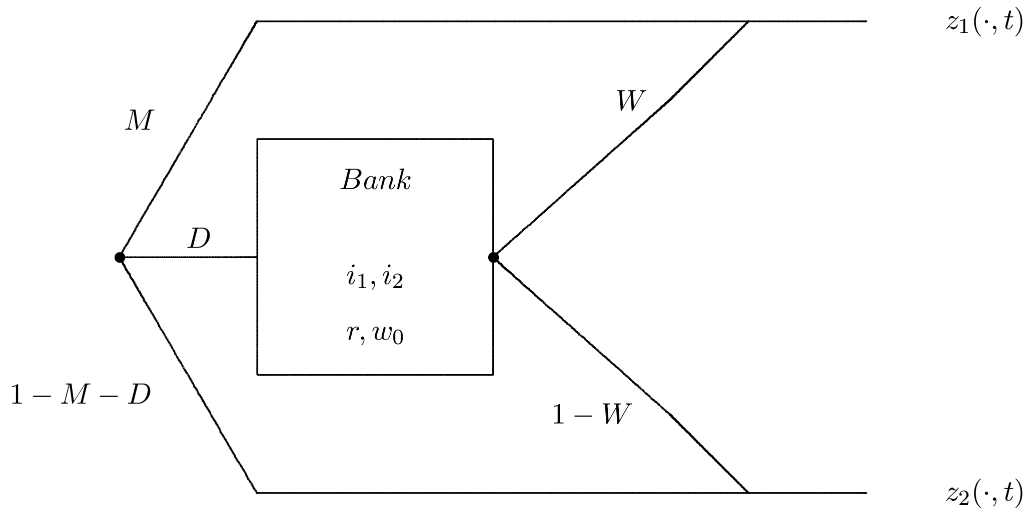

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

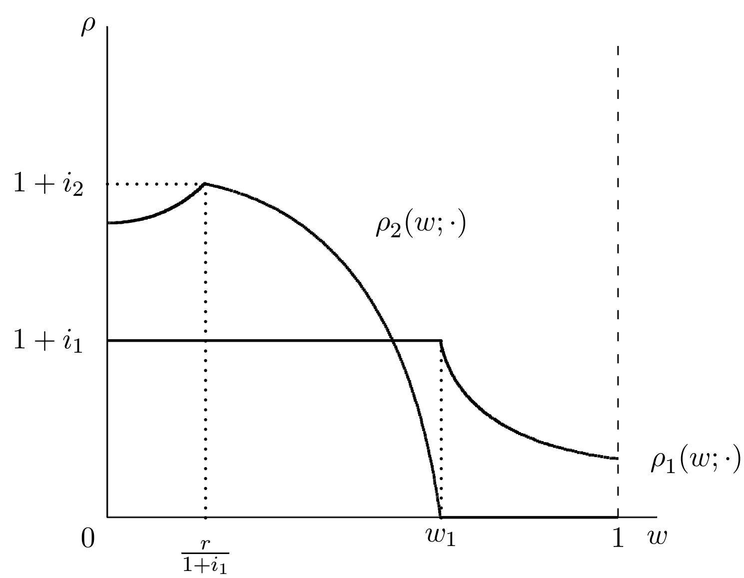

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) :=& \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ =& \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D,M;r,w_0;\gamma)\\
&:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D,M;r,w_0;\gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D,M;r,w_0;\gamma)$$

$$\text{s.t.} \quad D+M \le 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$.

It remains to determine the deposit contract parameters (r,w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau=\pi$ and $\tau=1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r<1$. As a consequence, banks offer deposit contracts on the basis of $w_0=\pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r,\pi;\gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0=0$ and $w_0=1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

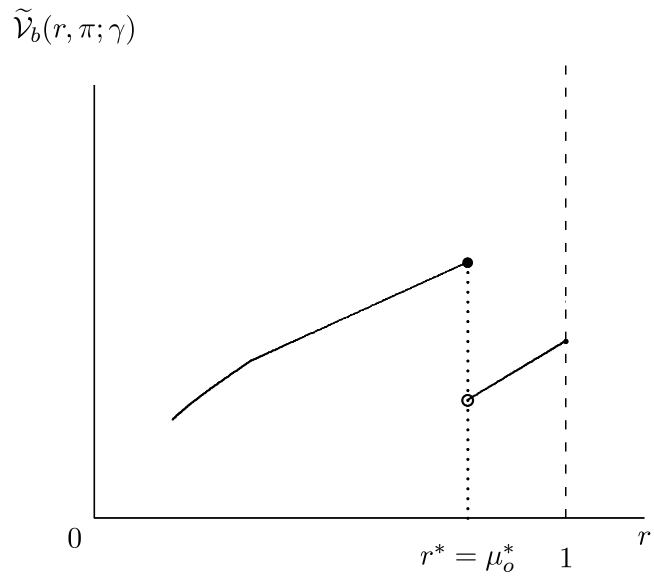

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract (r^*, w_0^*), with $w_0^* = \pi$ and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ such that the equilibrium the aggregate level of deposits is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. BANKS, MUTUAL FUNDS, OR ASSET MARKETS?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

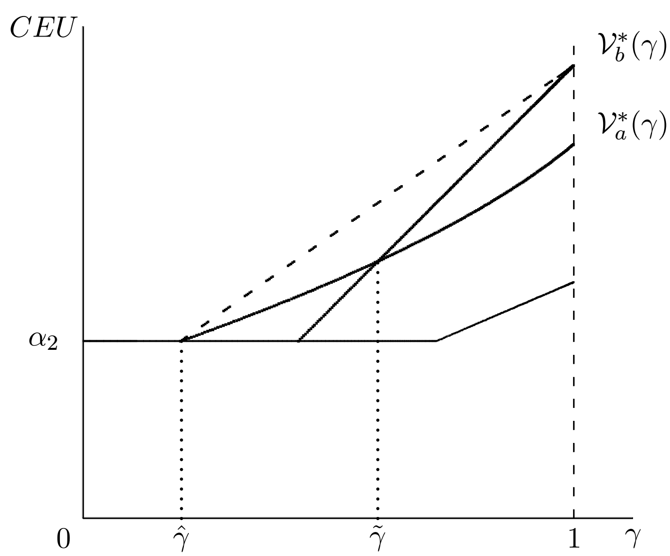

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t,\tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S|\ f(s) \geq t\}$ and $\{s \in S|\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S|\ f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S|\ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0,\overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu),1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0,\overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu),1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0,\underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu),1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ. *This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$, *depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) := \quad & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\ = \quad & \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau \in [0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2\cdot(1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) \;+\; (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ,φ)*. Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$.
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1, i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1+(1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0*, one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w;r,w_0) := \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period* 1*, i.e.,* $w=\tau$*, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 *to lie between the type-contingent valuations of the actual return in Period* 1*. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for* $\tau < 1$ *the constraint*

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau;r,w_0;h) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) &:= \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ &\qquad + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0, 1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := {} & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = {} & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

Appendix B: Proofs

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_h + \tau \cdot \alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_\ell + \tau \cdot \alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \le \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \ge \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\,, \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) :=& \quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ =& \quad \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ is a best reply for $r = \mu_\ell$ and $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1 \right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ is convex

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a} \left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c} \right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a :=& \quad 2 \cdot \pi \cdot \beta_h \\ b_1 :=& \quad \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 :=& \quad \gamma \cdot (\alpha_2 - \beta_\ell) \\ c :=& \quad 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \;& b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\
= \;& (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\
& -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \;& [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\tilde{\gamma}) = \mathcal{V}_a^*(\tilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a, b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a, b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a, b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1-\mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned}\tau \cdot z_{1h}(\tau,\mu) + (1-\tau)\cdot z_{1\ell}(\tau,\mu) &= \mu,\\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu).\end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau}\cdot\frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)} = \mu + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau\cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) = \alpha_2\cdot(1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$.

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S. *The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

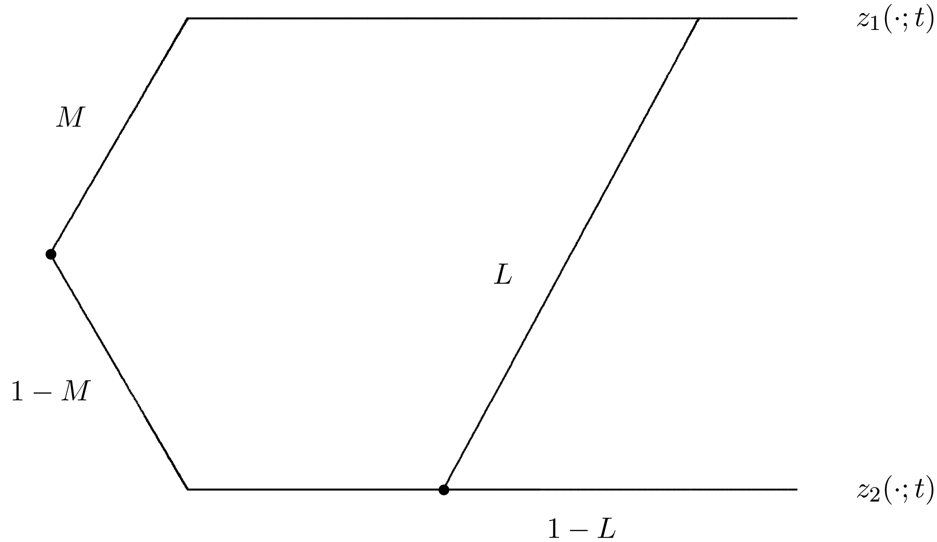

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=& \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) :=& \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M; t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

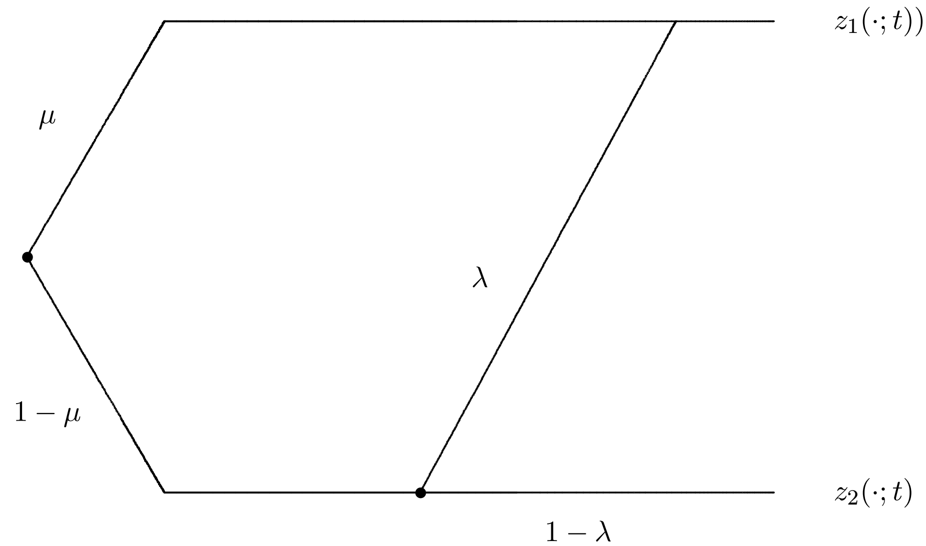

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll} \max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\ \text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\ & \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\ & \tau \cdot z_{1h} + (1-\tau)\cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu)\cdot\lambda & \mathrm{F}_1 \\ & \tau \cdot z_{2h} + (1-\tau)\cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu)\cdot(1-\lambda) & \mathrm{F}_2 \\ & \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN} \end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) :=&\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ =&\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

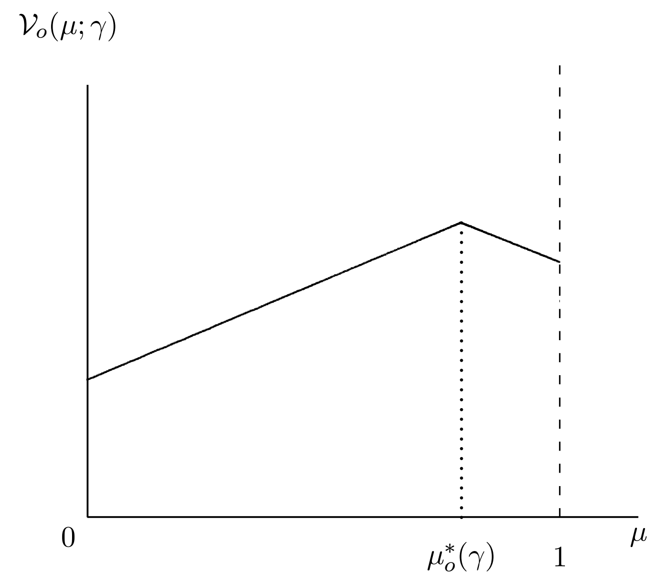

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

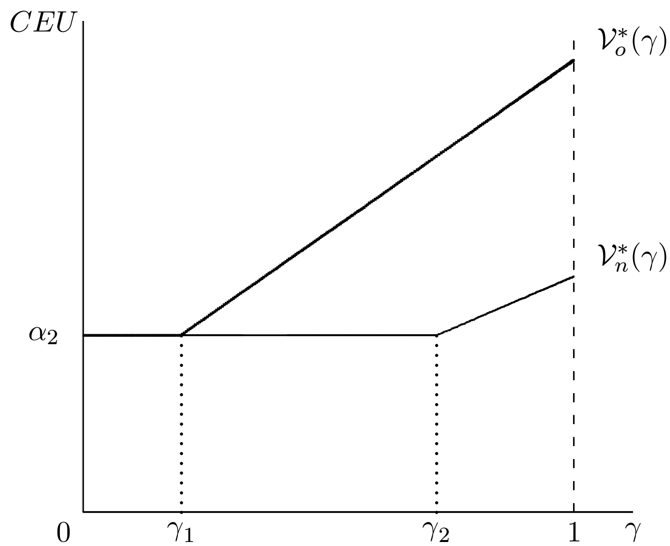

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

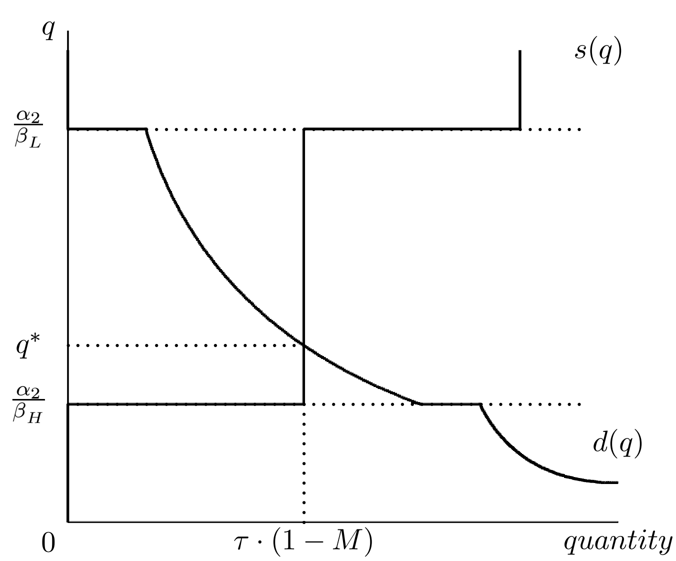

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

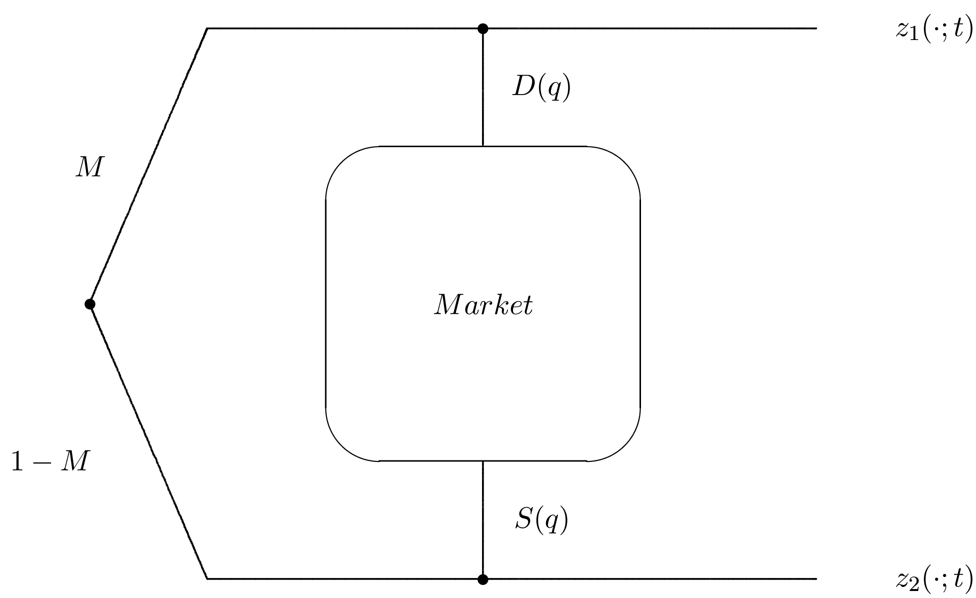

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\mathcal{V}_a(M,\mu;\gamma) := \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)] .$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

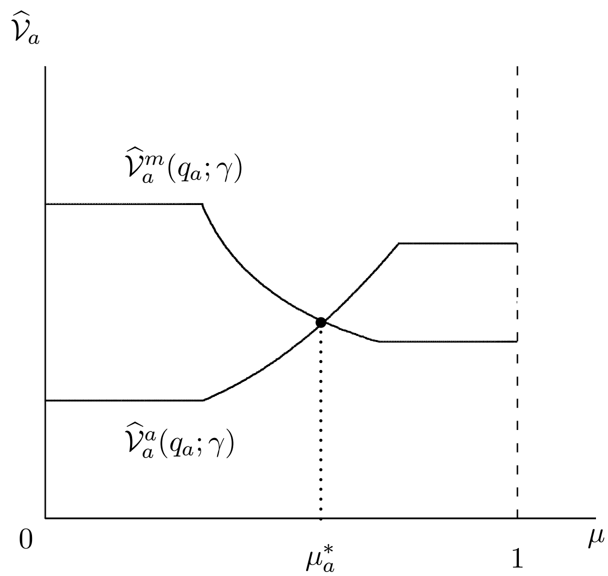

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

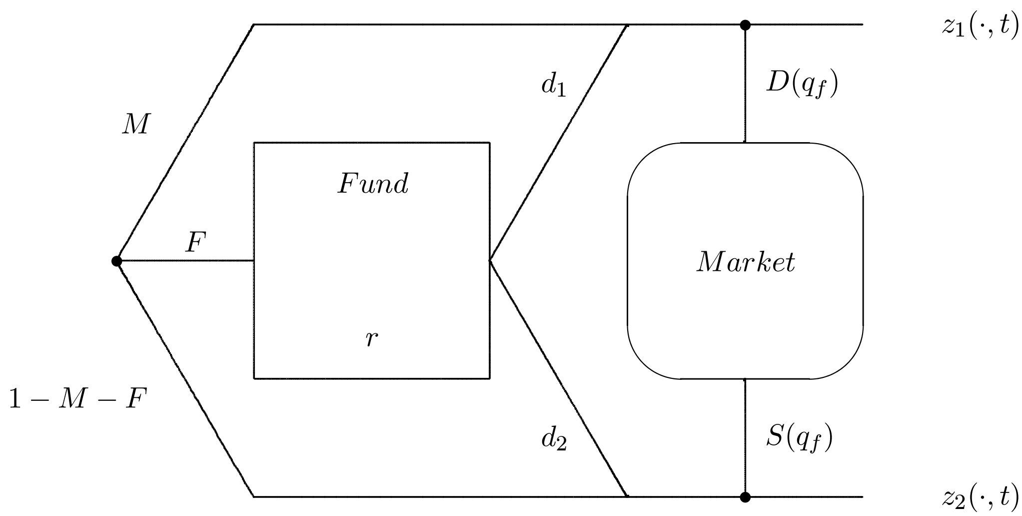

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 :=& \quad r \\ d_2 :=& \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau)\cdot\mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r)\cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma \cdot [\pi \cdot v_f(F,M;\pi,\psi,\varphi,r;h) \, + \, (1-\pi) \cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] \\ +(1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M \leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

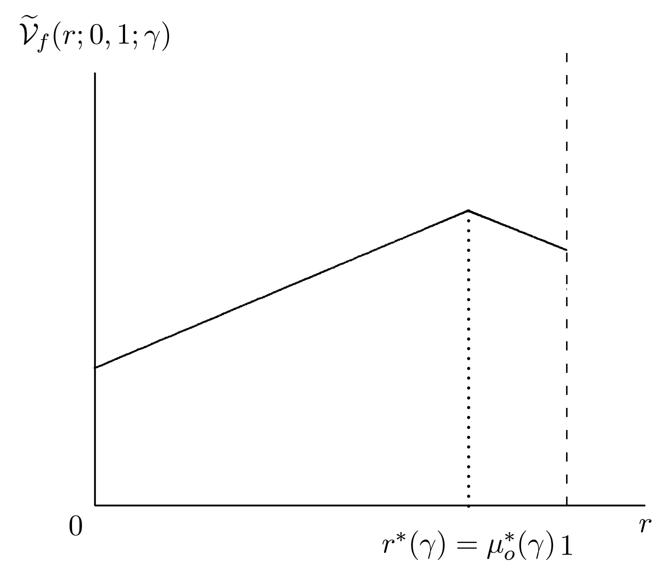

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0, 1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.

[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. BANKS

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

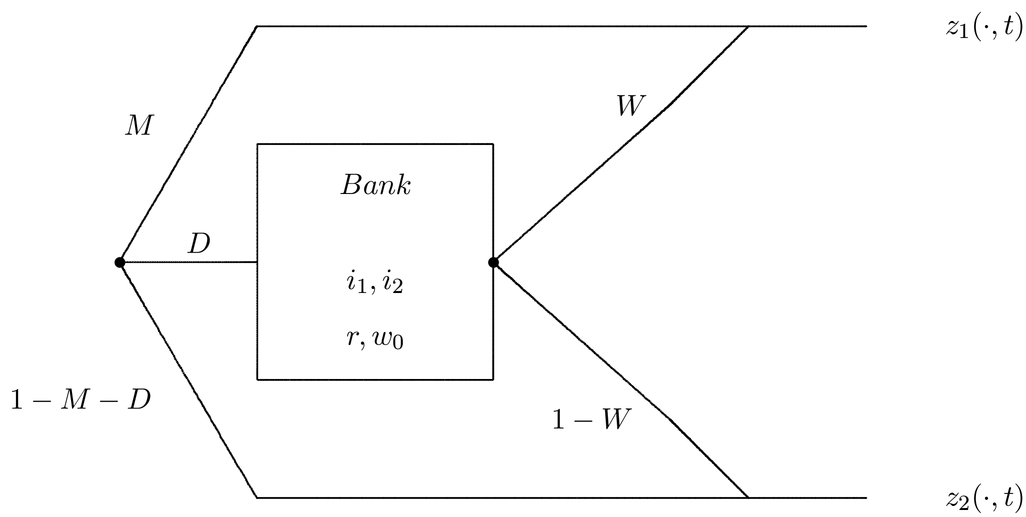

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1+i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

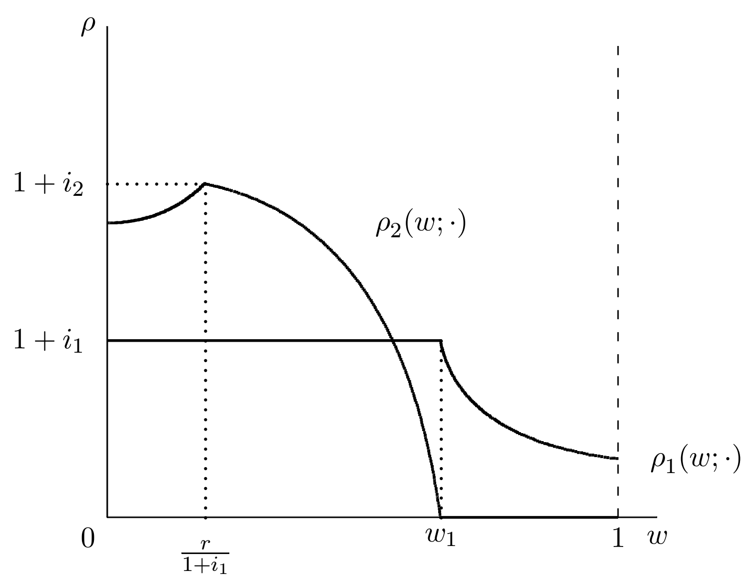

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D,M;r,w_0;\gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\
&\qquad\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D,M;r,w_0;\gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D,M;r,w_0;\gamma)$$

$$\text{s.t.} \quad D+M \le 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$.

It remains to determine the deposit contract parameters (r,w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r,\pi;\gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

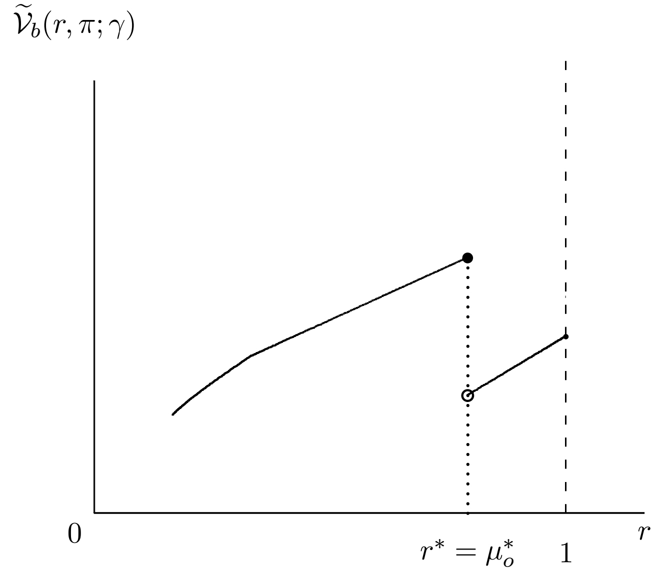

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. BANKS, MUTUAL FUNDS, OR ASSET MARKETS?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

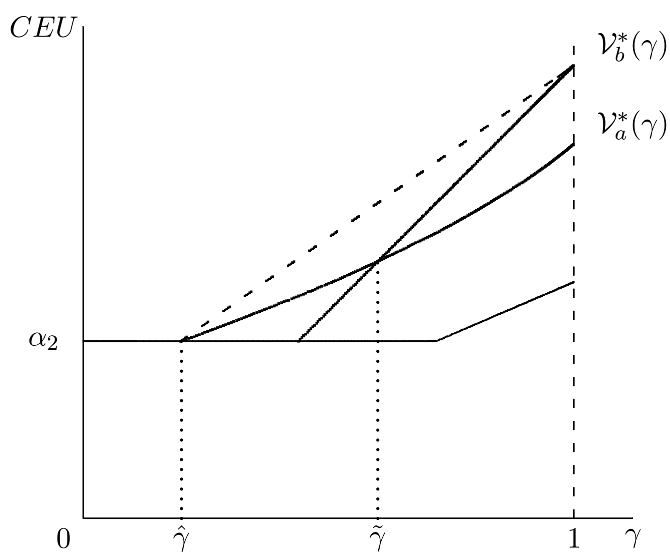

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S|\ f(s) \geq t\}$ and $\{s \in S|\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\int^{C} f\, d\nu = \int_0^{\infty} \nu(\{s \in S|\ f(s) \geq t\})\, dt + \int_{-\infty}^{0} [\nu(\{s \in S|\ f(s) \geq t\}) - 1]\, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)] + (1-\gamma)\cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_\ell] + (1-\gamma)\cdot \beta_\ell] + (1-M)\cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)}, \\ \overline{\tau}(\mu) &:= \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0,\overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu),1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0,\overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu),1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0,\underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu),1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned}\mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}\end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned}&\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\Big[-\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell)\Big) \\ &+ \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell)\Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h}\,\Big].\end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi$,
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi)$,
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) := \quad & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\ = \quad & \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma),\alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau\in[0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t,\tau) = (\ell, 0)$. When this is not the case, investment policy M may affect the state (t,τ) for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of $\widehat{\mathcal{V}}_a^m$ and $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and invest the remaining fraction $(1-F-M)$ in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, and

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of $h-$types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ consumers, the worst case occurs when $q_f^*(\tau; \psi, \varphi, r)$ attains its upper bound at $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. In this case a consumer's utility equals

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M and in F. Consumers of type h can always obtain a utility level at least as high as the consumers of type ℓ by copying the latter's behaviour.

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma\cdot[\pi\cdot\beta_h\cdot(r+q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot\alpha_2\cdot[\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)}+(1-r)]$$
$$+(1-\gamma)\cdot[r\cdot\beta_\ell+\alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ,φ). Thus, its decision problem for given (ψ,φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^*=0$ or $r^*=1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^*\cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1,i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1+(1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w=\tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ the constraint

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau;r,w_0;\ell) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) < \beta_\ell \cdot \rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) \geq \beta_\ell \cdot \rho_1(\tau;r,w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D,M;r,w_0;\gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau) \cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D,M;r,w_0;\gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r,w_0;\gamma) &:= \max_{\substack{(D,M)\in[0,1]\times[0,1] \\ s.t.\ D+M\leq 1}} \mathcal{V}_b(D,M;r,w_0;\gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi;r,w_0;h) + (1-\pi) \cdot v_b(\pi;r,w_0;\ell)\} \\ &\qquad + (1-\gamma) \cdot \min_{\tau\in[0,1]} v_b(\tau;r,w_0;t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r,w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r,w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$, taking into account the withdrawal behaviour $W(\tau;r,w_0;t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \;& \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \;& \max\{\mathcal{V}_n^*(\gamma),\; \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \le \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \ge \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell), \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := &\quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = &\quad \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$z_{2h}(\tau) := \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right]$$
$$= \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h.$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ is a best reply for $r = \mu_\ell$ and $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell$$
$$= \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell.$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1 \right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ *is convex*

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a}\left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c} \right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a &:= 2 \cdot \pi \cdot \beta_h \\ b_1 &:= \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 &:= \gamma \cdot (\alpha_2 - \beta_\ell) \\ c &:= 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \; & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + b_2{}^2 + c \cdot \gamma^2 \\
= \; & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + b_2{}^2 \\
& -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \; & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\tilde{\gamma}) = \mathcal{V}_a^*(\tilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a, b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a, b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a, b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1-\mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned} \tau \cdot z_{1h}(\tau,\mu) + (1-\tau) \cdot z_{1\ell}(\tau,\mu) &= \mu, \\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu). \end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)} = \mu + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$.

(iii) $\tau \in [\overline{\tau}, 1]$

For these values of τ there are "too many" $h-$types in the economy. So it is optimal to choose $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$. Even for this choice, however, the proportion of $h-$types is so high, that for $z_{2h}(\tau,\mu) = 0$ the self selection constraint S_h is violated. The payout $z_{2h}(\tau,\mu)$ obtains its lowest value when S_h binds and its highest value when S_ℓ binds. The self selection constraint S_h reads

$$\beta_h \cdot z_{1h}(\tau,\mu) + z_{2h}(\tau,\mu) \geq \beta_h \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu)$$

and after substitution and rearranging we obtain

$$z_{2h}(\tau,\mu) = z_{2\ell}(\tau,\mu) - \beta_h \cdot \frac{\mu}{\tau}.$$

Using $z_{2\ell}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)]$ we find $z_{2h}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)] - \beta_h \cdot \frac{\mu}{\tau}$, from which

$$z_{2h}(\tau,\mu) = \alpha_2 \cdot (1-\mu) - \frac{(1-\tau)}{\tau} \cdot \beta_h \cdot \mu = \alpha_2 - \frac{\mu}{\tau} \cdot (\tau \cdot \alpha_2 + (1-\tau) \cdot \beta_h)$$

and

$$z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu) + \beta_h \cdot \mu$$

follow.

■

Proof of Lemma 14

The structure of this proof is as follows. First the average interim utility for the investment policy μ is determined for $\tau = \pi$. Then the interim utility of the worst case is determined and both are combined to calculate the ex-ante Choquet expected indirect utility of investment policy μ.

(i) Average interim utility

For any $\mu \in [0,1]$ let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13. For any investment policy μ and for $\tau = \pi$ the average interim utility is

$$\widetilde{U}(\pi,\mu) := \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu).$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6]For a definition of capacities see Appendix A, Definition 2.

[7]For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S. *The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

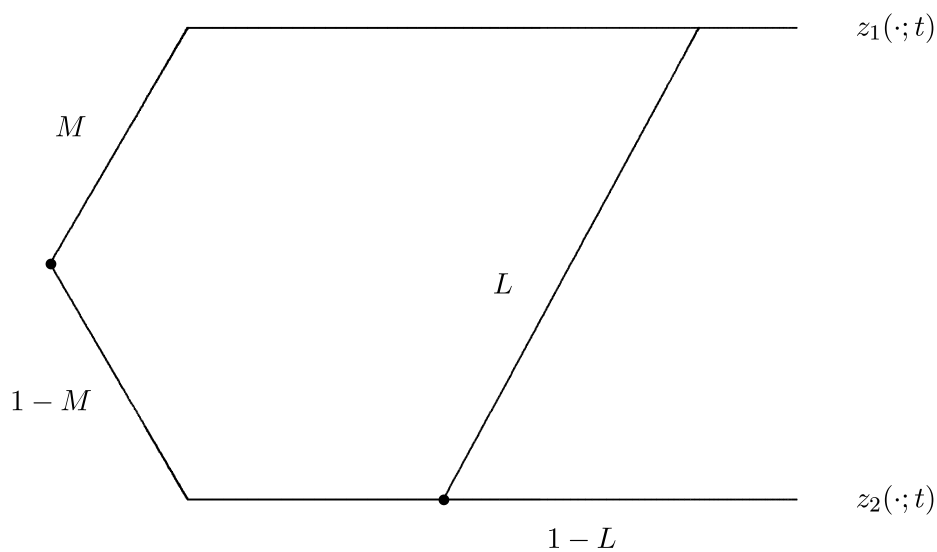

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0, 1]$, and a liquidation decision in Period 1, $L \in [0, 1]$, the following consumption levels result:

$$\begin{aligned} z_1(M, L) &:= \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M, L) &:= \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0, 1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) :=\ & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ &\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ =\ & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ &\qquad + (1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \ \geq\ \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

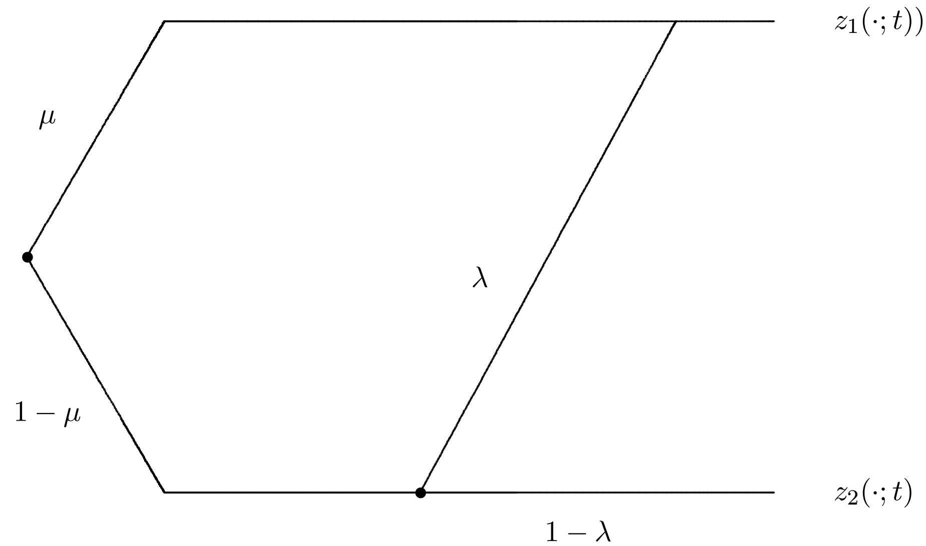

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := \quad & u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = \quad & \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

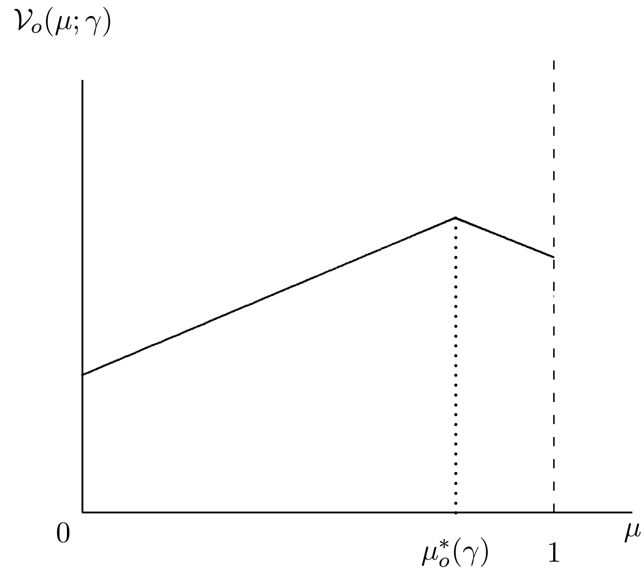

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

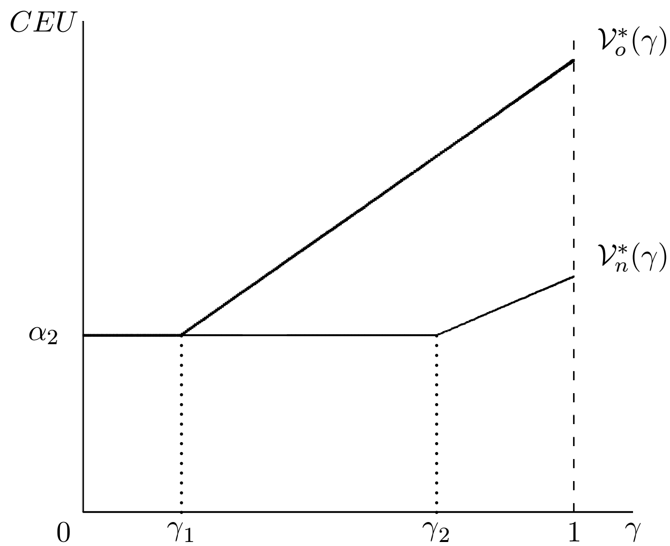

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

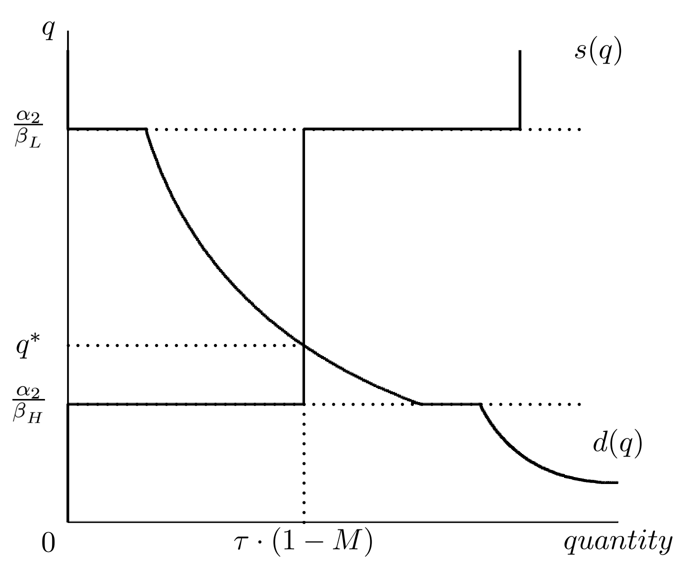

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

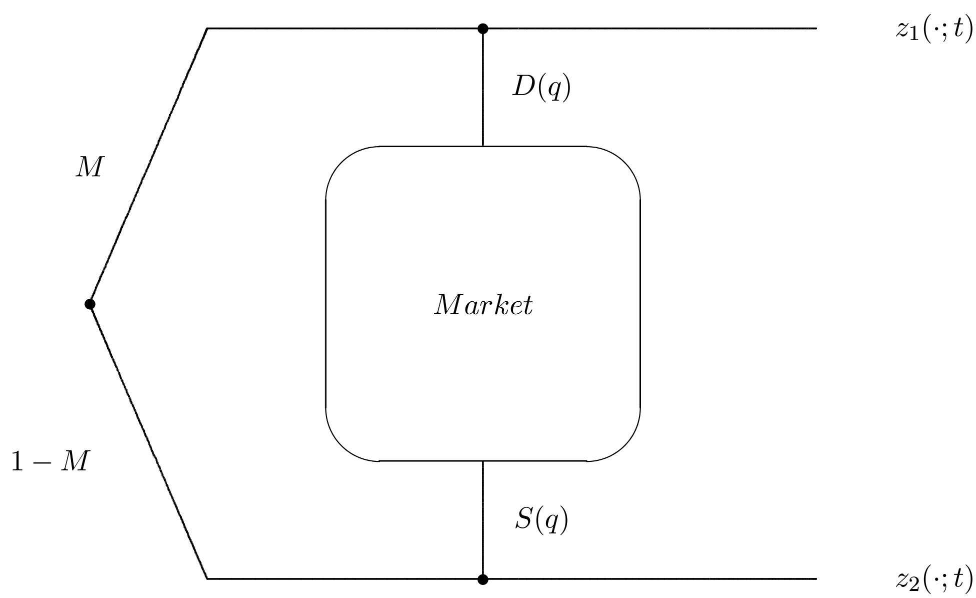

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\mathcal{V}_a(M,\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

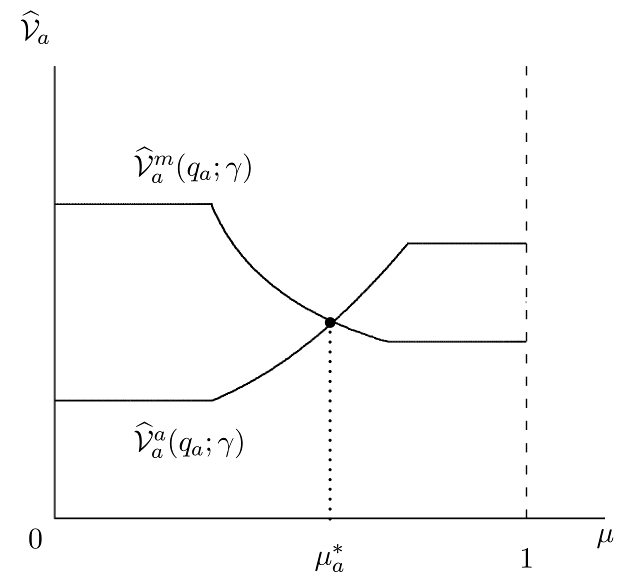

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

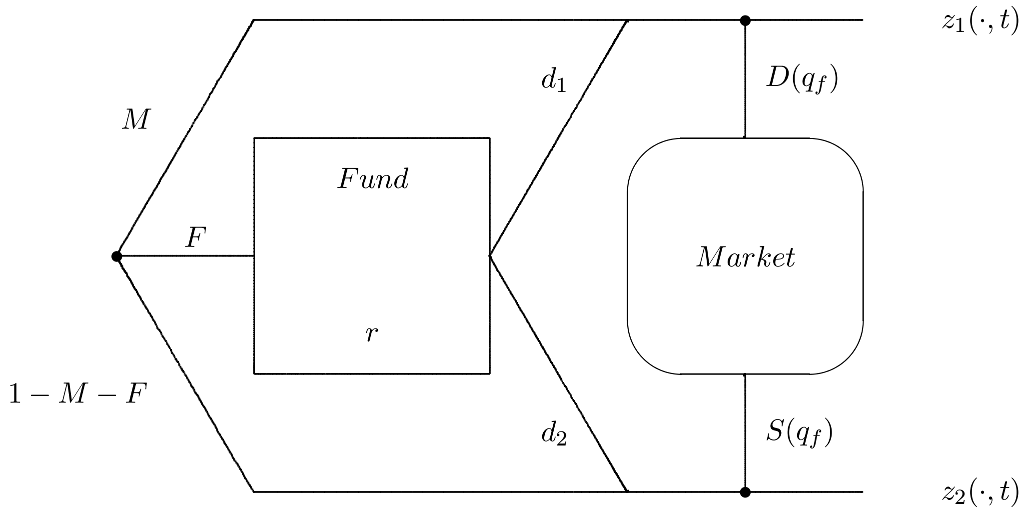

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

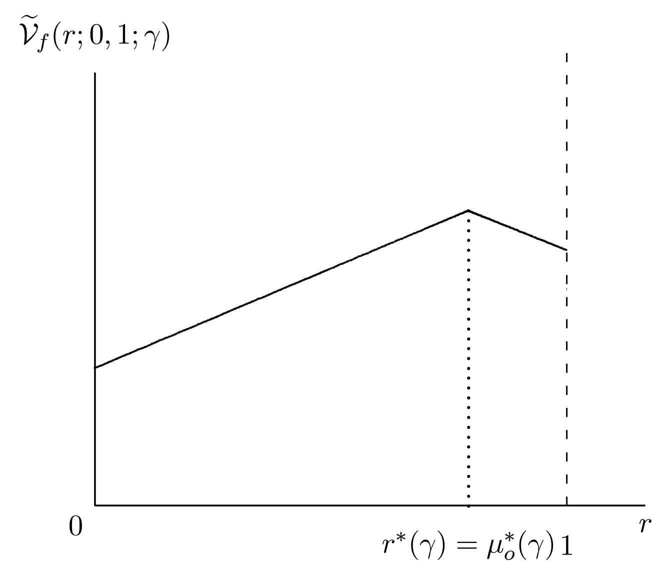

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0,1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.

[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

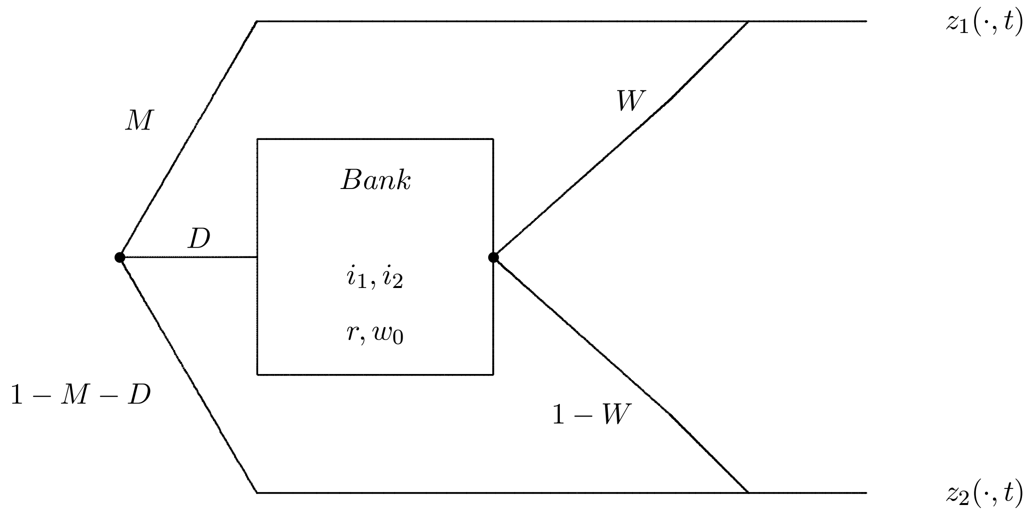

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

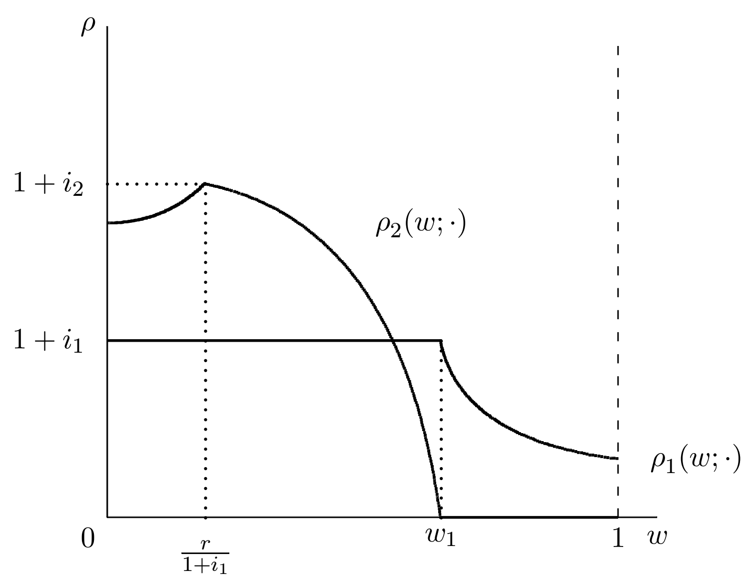

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1 - \tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$
$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

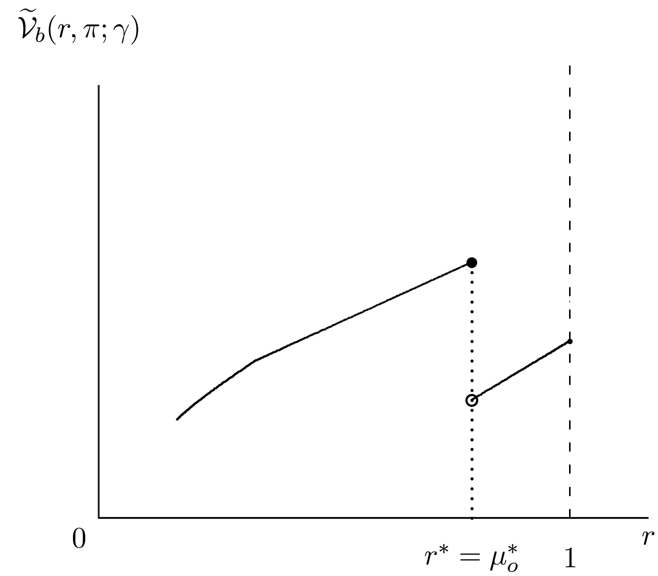

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ =& \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

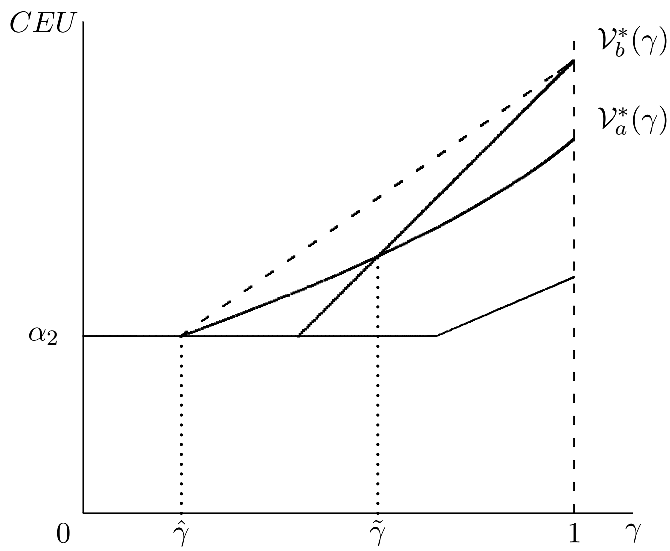

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \; f(s) \geq t\}$ and $\{s \in S | \; f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S | \; f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S | \; f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2\\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},\\ \overline{\tau}(\mu) &:= \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi) \cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ*. This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$*, depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned}\mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}\end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned}&\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ &+ \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big].\end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market exists and the price*

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi$,
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi)$,
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}
\mathcal{V}_a^*(\gamma) := \quad & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\
= \quad & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\
& \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.
\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. *For investing in the illiquid asset it occurs for* $(t,\tau) = (\ell,\tau)$ *for any* $\tau \in [0,1]$. *From this it follows that for an asset price* q_a *the ex-ante Choquet expected utility for a consumer who holds a fraction* M *of his initial one unit of wealth as money is*

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$*. When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$*.*

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h*, and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ*.*

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M) \in [0,1] \times [0,1] \\ s.t.\ F+M \leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+ (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$*. In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F*. Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ, φ)*. Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$*.*
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$*.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1, i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w=\tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau < 1$ the constraint

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for ℓ−types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0, 1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) &:= \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ &\qquad + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0, 1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \ \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ =& \ \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau), \mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau; 0, 1; \mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell), \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) :=& \quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ =& \quad \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$z_{2h}(\tau) := \quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right]$$

$$= \quad \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h.$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ *is a best reply for* $r = \mu_\ell$ *and* $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ *is convex*

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a}\left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}\right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a &:= 2 \cdot \pi \cdot \beta_h \\ b_1 &:= \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 &:= \gamma \cdot (\alpha_2 - \beta_\ell) \\ c &:= 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \; & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\
= \; & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\
& -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \; & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\widetilde{\gamma}) = \mathcal{V}_a^*(\widetilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a,b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a,b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a,b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1-\mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned} \tau \cdot z_{1h}(\tau,\mu) + (1-\tau) \cdot z_{1\ell}(\tau,\mu) &= \mu, \\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu). \end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)} = \mu + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$.

(iii) $\tau \in [\overline{\tau}, 1]$

For these values of τ there are "too many" $h-$types in the economy. So it is optimal to choose $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$. Even for this choice, however, the proportion of $h-$types is so high, that for $z_{2h}(\tau,\mu) = 0$ the self selection constraint S_h is violated. The payout $z_{2h}(\tau,\mu)$ obtains its lowest value when S_h binds and its highest value when S_ℓ binds. The self selection constraint S_h reads

$$\beta_h \cdot z_{1h}(\tau,\mu) + z_{2h}(\tau,\mu) \geq \beta_h \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu)$$

and after substitution and rearranging we obtain

$$z_{2h}(\tau,\mu) = z_{2\ell}(\tau,\mu) - \beta_h \cdot \frac{\mu}{\tau}.$$

Using $z_{2\ell}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)]$ we find $z_{2h}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)] - \beta_h \cdot \frac{\mu}{\tau}$, from which

$$z_{2h}(\tau,\mu) = \alpha_2 \cdot (1-\mu) - \frac{(1-\tau)}{\tau} \cdot \beta_h \cdot \mu = \alpha_2 - \frac{\mu}{\tau} \cdot (\tau \cdot \alpha_2 + (1-\tau) \cdot \beta_h)$$

and

$$z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu) + \beta_h \cdot \mu$$

follow.

■

Proof of Lemma 14

The structure of this proof is as follows. First the average interim utility for the investment policy μ is determined for $\tau = \pi$. Then the interim utility of the worst case is determined and both are combined to calculate the ex-ante Choquet expected indirect utility of investment policy μ.

(i) Average interim utility

For any $\mu \in [0,1]$ let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13. For any investment policy μ and for $\tau = \pi$ the average interim utility is

$$\widetilde{U}(\pi,\mu) := \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu).$$

From Lemma 12 we have

$$\pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu) = \alpha_2 \cdot (1-\mu)$$

substitution of which yields

$$\widetilde{U}(\pi,\mu) = \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu).$$

(ia) $\pi \in [0, \underline{\tau}(\mu))$

Using Lemma 12 we obtain

$$\begin{aligned} \widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot \left(\frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu \right) \\ &\quad + (1-\pi) \cdot \beta_\ell \cdot \left(\mu - \frac{\pi}{(1-\pi)} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} \right) + \alpha_2 \cdot (1-\mu) \\ &= \left(1 + (\frac{\beta_h}{\beta_\ell} - 1) \cdot \pi \right) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu) \,. \end{aligned}$$

(ib) $\pi \in [\underline{\tau}(\mu), 1]$

Using Lemma 12 we obtain

$$\begin{aligned} \widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu) \\ &= \pi \cdot \beta_h \cdot \frac{\mu}{\pi} + \alpha_2 \cdot (1-\mu) = \alpha_2 + (\beta_h - \alpha_2) \cdot \mu, \end{aligned}$$

which complete this part of the proof.

(ii) The worst case

The interim utility a type h consumer obtains from payouts (z_h, z_ℓ) that satisfy the self selection constrains is as least as last as the corresponding utility of a type ℓ consumer since $\beta_h > \beta_\ell$. To determine the worst case, we can therefore restrict our attention to a consumer with a low preference for liquidity. The utility of $\ell-$types from payouts that satisfy the self selection constraints for τ is increasing with τ for given μ.

Therefore, for any value of μ the utility of $\ell-$types obtains its minimum value for $\tau = 0$. Using this, we obtain

$$\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\mu,\tau;t) = \beta_\ell \cdot z_{1\ell}(0,\mu) + z_{2\ell}(0,\mu) = \alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu.$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. NON-ADDITIVE BELIEFS AND THE CHOQUET INTEGRAL

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

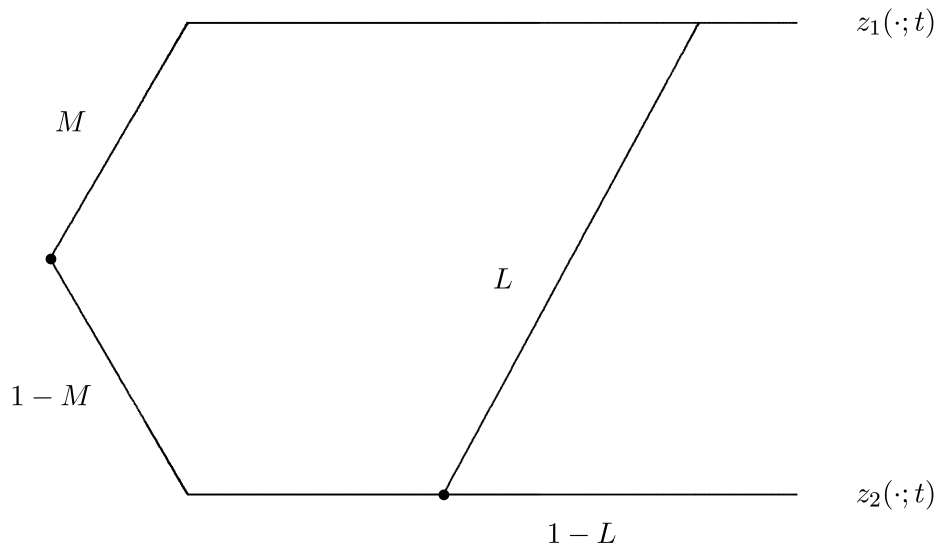

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1 - M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1 - L) \cdot (1 - M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2 \,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

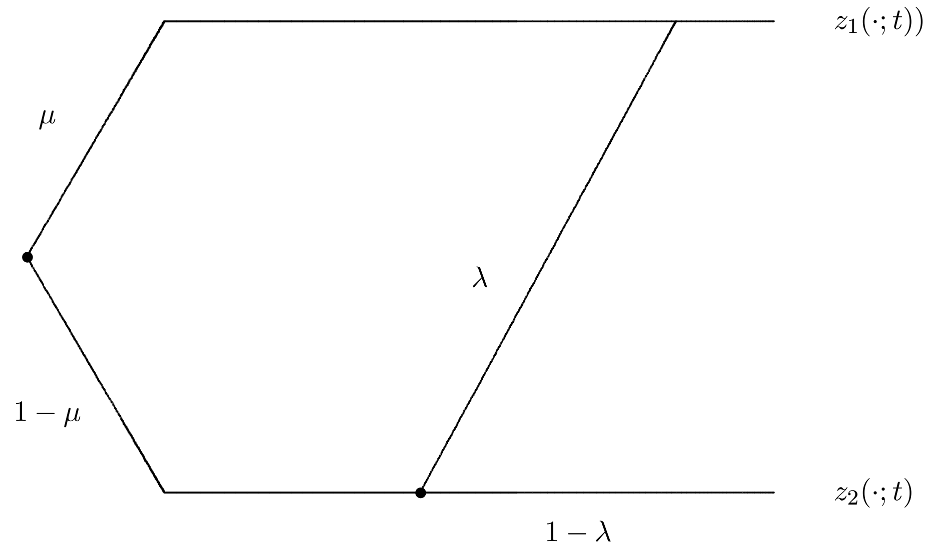

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := \quad & \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := \quad & \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau)\cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau)\cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu)\cdot(1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := \quad & u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = \quad & \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1-\pi) \cdot v_o(\pi, \mu; \ell)] + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

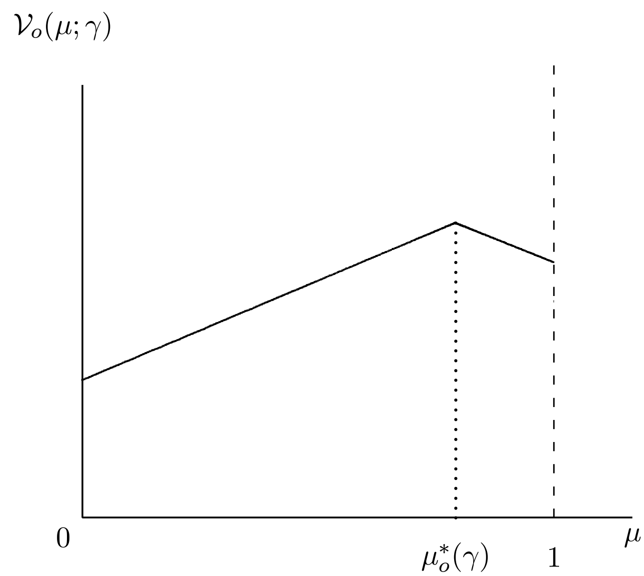

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

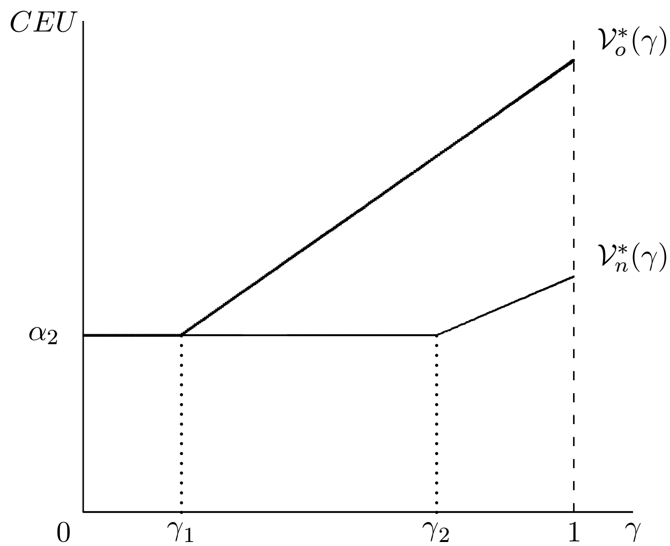

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

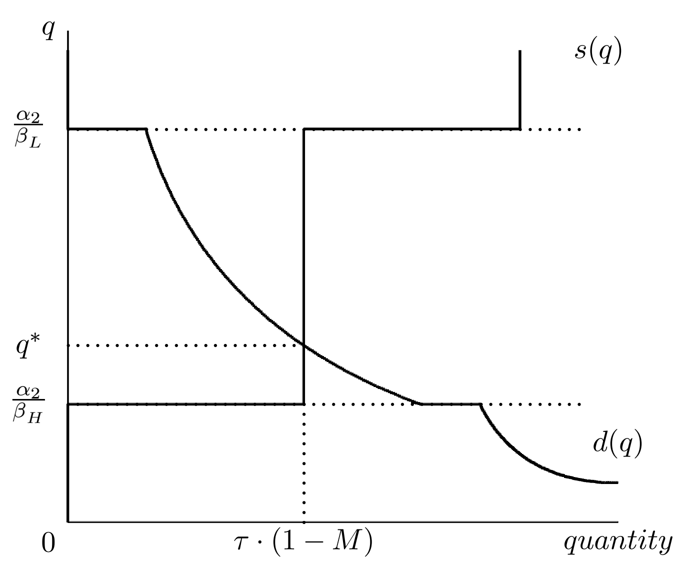

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

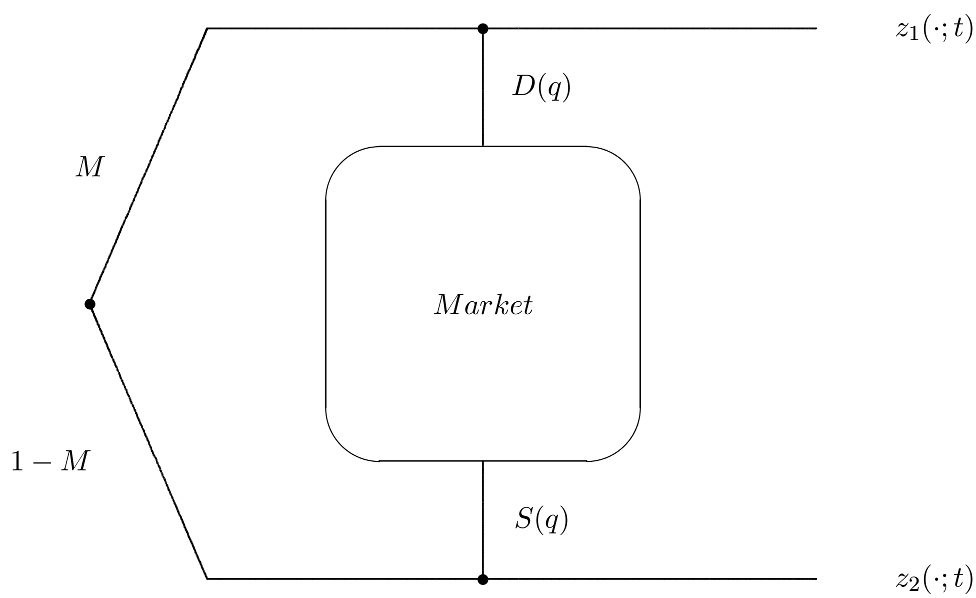

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] \,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

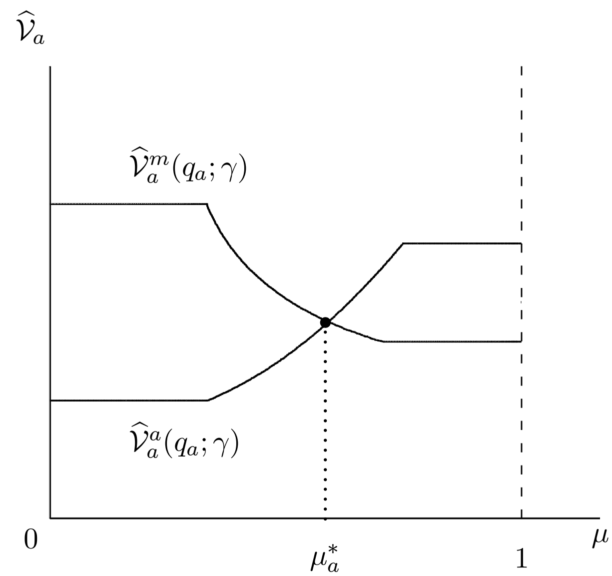

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

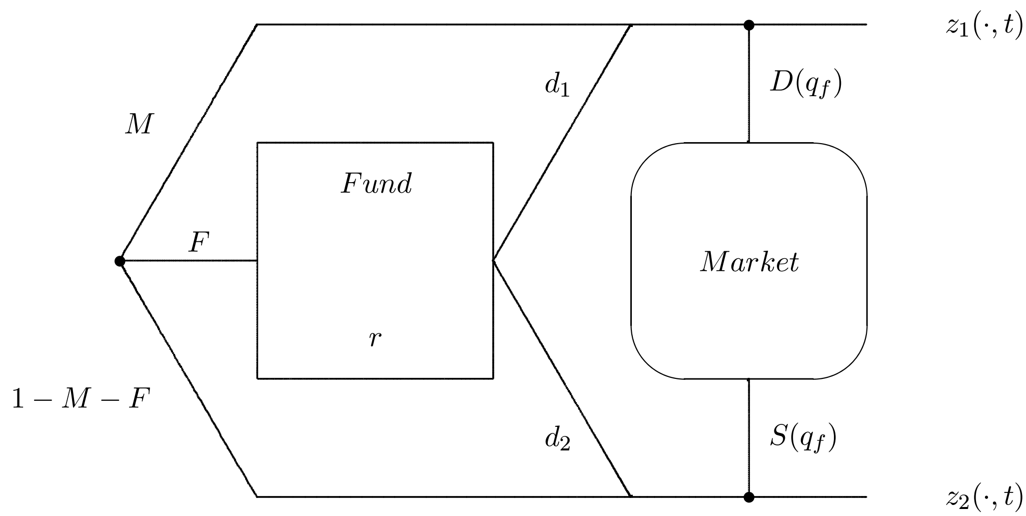

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

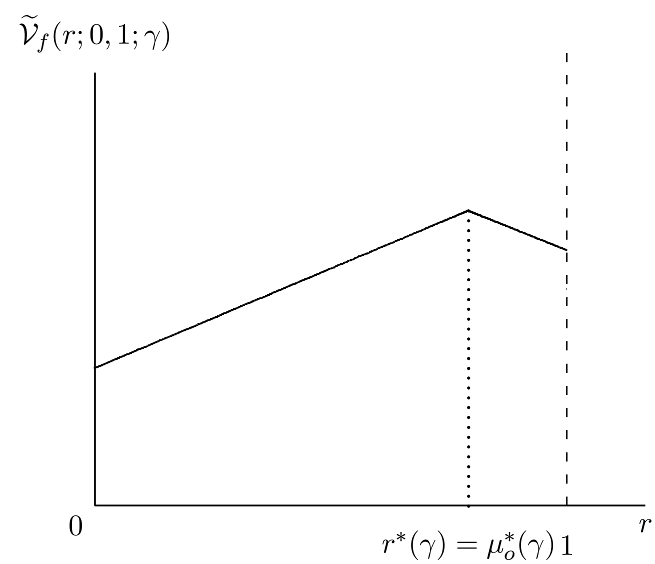

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0, 1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

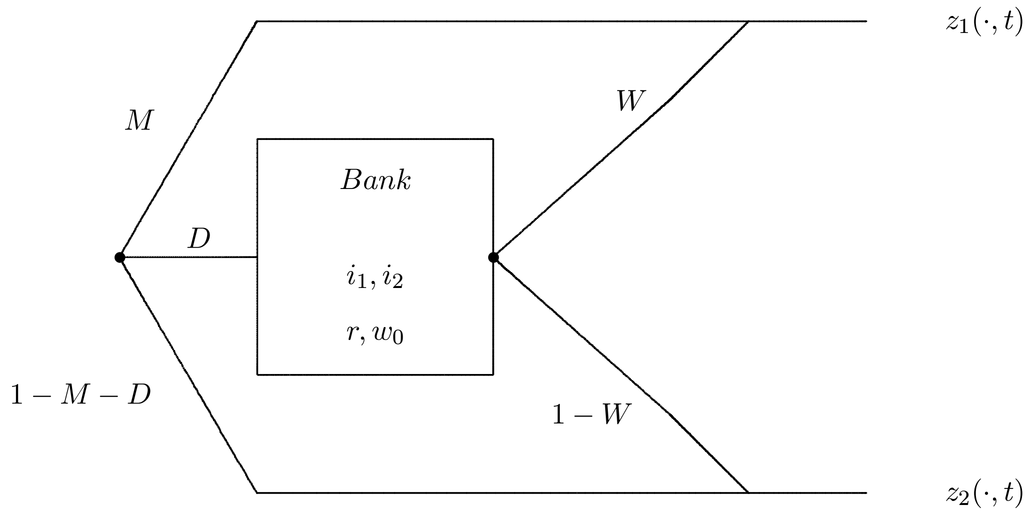

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

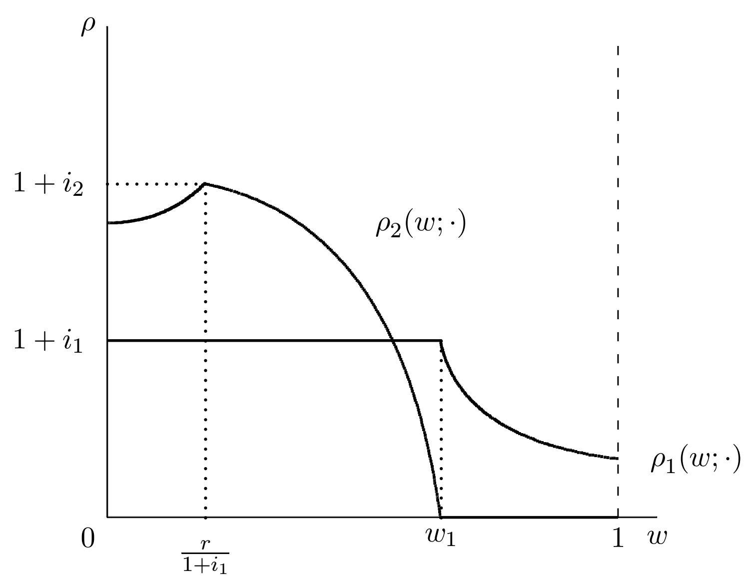

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\begin{aligned}
\max_{(D,M) \in [0,1] \times [0,1]} \quad & \mathcal{V}_b(D, M; r, w_0; \gamma) \\
\text{s.t.} \quad & D + M \leq 1,
\end{aligned}$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

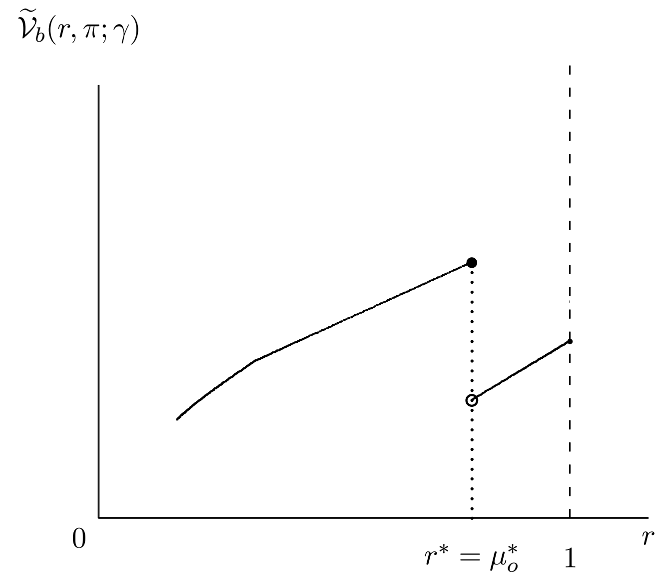

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \; \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \; \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. BANKS, MUTUAL FUNDS, OR ASSET MARKETS?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

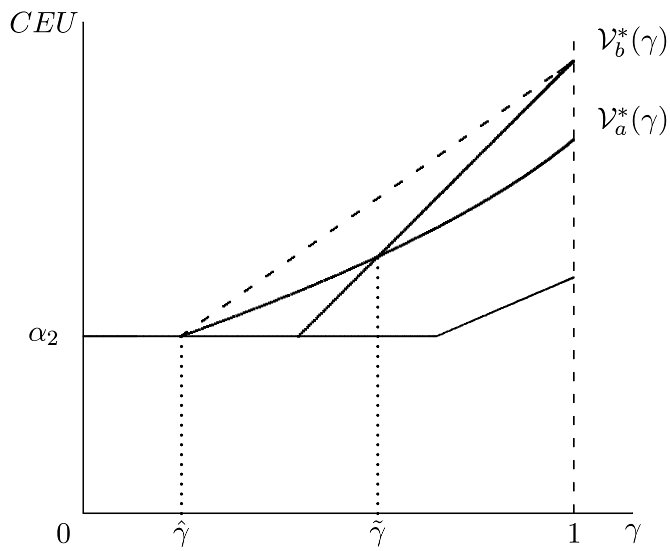

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \ f(s) \geq t\}$ and $\{s \in S | \ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S | \ f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S | \ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi) \cdot v_n(M;\ell)] + (1-\gamma) \cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell] + (1-M) \cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)}, \\ \overline{\tau}(\mu) &:= \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \textit{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) :=& \quad \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ =& \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} &\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\quad \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ &\quad + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) := \;& \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\ = \;& \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma),\alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau\in[0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M) \in [0,1] \times [0,1] \\ s.t.\ F+M \leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \ + \ (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$
$$+(1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$
$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ,φ)*. Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$.
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1, i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0*, one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period 1, i.e.,* $w=\tau$*, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ *the constraint*

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) &:= \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ &\qquad + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0, 1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \; \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ =& \; \max\{\mathcal{V}_n^*(\gamma), \; \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau) \cdot \beta_h + \tau \cdot \alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau) \cdot \beta_\ell + \tau \cdot \alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau; \psi, \varphi; r) := \begin{cases} \frac{\alpha_2}{\beta_h} \cdot (1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau) \cdot \beta_h + \tau \cdot \alpha_2} \\ \frac{(1-\tau)}{\tau} \cdot \frac{\mu}{1-\mu} \cdot (1-r) & \text{for} \quad \mu \in [\mu_h(\tau), \mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell} \cdot (1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau) \cdot \beta_\ell + \tau \cdot \alpha_2}, \end{cases}$$

where $\mu := r \cdot \varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau; 0, 1; \mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell} \cdot (1-r) = \frac{\alpha_2}{\beta_\ell} \cdot \left(\frac{(1-\pi) \cdot \beta_\ell}{(1-\pi) \cdot \beta_\ell + \pi \cdot \alpha_2} \right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi \cdot \alpha_2}{(1-\pi) \cdot \beta_\ell + \pi \cdot \alpha_2} + \frac{(1-\pi) \cdot \alpha_2}{(1-\pi) \cdot \beta_\ell + \pi \cdot \alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau \cdot \frac{\alpha_2}{\beta_\ell} \cdot (1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\,,\, \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) :=& \quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ =& \quad \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ is a best reply for $r = \mu_\ell$ and $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1 \right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi\cdot\gamma\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \le \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ is convex

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma\cdot q_a^*(\gamma)\cdot\pi\cdot\beta_h + \gamma\cdot(1-\pi)\cdot\alpha_2 + (1-\gamma)\cdot\alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma\cdot q^*(\gamma) &= \gamma\cdot\frac{1}{a}\left(-(b_1 + b_2\cdot\frac{1}{\gamma}) + \sqrt{(b_1 + b_2\cdot\frac{1}{\gamma})^2 + c}\right) \\ &\sim -(b_1\cdot\gamma + b_2) + \gamma\cdot\sqrt{(b_1 + b_2\cdot\frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a &:= 2\cdot\pi\cdot\beta_h \\ b_1 &:= \alpha_2 - \pi\cdot(\alpha_2 + \beta_h) \\ b_2 &:= \gamma\cdot(\alpha_2 - \beta_\ell) \\ c &:= 4\cdot\pi\cdot(1-\pi)\cdot\alpha_2\cdot\beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \; & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\
= \; & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\
& -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \; & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\widetilde{\gamma}) = \mathcal{V}_a^*(\widetilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a, b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a, b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a, b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1 - \mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1 - \mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i),* for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1 - \mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1 - \mu)}{1 - \tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned} \tau \cdot z_{1h}(\tau,\mu) + (1-\tau)\cdot z_{1\ell}(\tau,\mu) &= \mu, \\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu). \end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau}\cdot\frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)} = \mu + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau\cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) = \alpha_2\cdot(1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$.

(iii) $\tau \in [\overline{\tau}, 1]$

For these values of τ there are "too many" $h-$types in the economy. So it is optimal to choose $z_{1h}(\tau, \mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau, \mu) = 0$. Even for this choice, however, the proportion of $h-$types is so high, that for $z_{2h}(\tau, \mu) = 0$ the self selection constraint S_h is violated. The payout $z_{2h}(\tau, \mu)$ obtains its lowest value when S_h binds and its highest value when S_ℓ binds. The self selection constraint S_h reads

$$\beta_h \cdot z_{1h}(\tau, \mu) + z_{2h}(\tau, \mu) \geq \beta_h \cdot z_{1\ell}(\tau, \mu) + z_{2\ell}(\tau, \mu)$$

and after substitution and rearranging we obtain

$$z_{2h}(\tau, \mu) = z_{2\ell}(\tau, \mu) - \beta_h \cdot \frac{\mu}{\tau}.$$

Using $z_{2\ell}(\tau, \mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau, \mu)]$ we find $z_{2h}(\tau, \mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau, \mu)] - \beta_h \cdot \frac{\mu}{\tau}$, from which

$$z_{2h}(\tau, \mu) = \alpha_2 \cdot (1-\mu) - \frac{(1-\tau)}{\tau} \cdot \beta_h \cdot \mu = \alpha_2 - \frac{\mu}{\tau} \cdot (\tau \cdot \alpha_2 + (1-\tau) \cdot \beta_h)$$

and

$$z_{2\ell}(\tau, \mu) = \alpha_2 \cdot (1-\mu) + \beta_h \cdot \mu$$

follow.

■

Proof of Lemma 14

The structure of this proof is as follows. First the average interim utility for the investment policy μ is determined for $\tau = \pi$. Then the interim utility of the worst case is determined and both are combined to calculate the ex-ante Choquet expected indirect utility of investment policy μ.

(i) Average interim utility

For any $\mu \in [0,1]$ let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13. For any investment policy μ and for $\tau = \pi$ the average interim utility is

$$\widetilde{U}(\pi, \mu) := \pi \cdot \beta_h \cdot z_{1h}(\pi, \mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi, \mu) + \pi \cdot z_{2h}(\pi, \mu) + (1-\pi) \cdot z_{2\ell}(\pi, \mu).$$

From Lemma 12 we have

$$\pi \cdot z_{2h}(\pi,\mu) + (1-\pi)\cdot z_{2\ell}(\pi,\mu) = \alpha_2 \cdot (1-\mu)$$

substitution of which yields

$$\widetilde{U}(\pi,\mu) = \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi)\cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu).$$

(ia) $\pi \in [0, \underline{\tau}(\mu))$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot \left(\frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu\right) \\
&\quad +(1-\pi)\cdot \beta_\ell \cdot \left(\mu - \frac{\pi}{(1-\pi)} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) + \alpha_2 \cdot (1-\mu) \\
&= \left(1 + (\frac{\beta_h}{\beta_\ell} - 1)\cdot \pi\right) \cdot (\alpha_2 + (\beta_\ell - \alpha_2)\cdot \mu)\,.
\end{aligned}$$

(ib) $\pi \in [\underline{\tau}(\mu), 1]$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi)\cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu) \\
&= \pi \cdot \beta_h \cdot \frac{\mu}{\pi} + \alpha_2 \cdot (1-\mu) = \alpha_2 + (\beta_h - \alpha_2)\cdot \mu,
\end{aligned}$$

which complete this part of the proof.

(ii) The worst case

The interim utility a type h consumer obtains from payouts (z_h, z_ℓ) that satisfy the self selection constrains is as least as last as the corresponding utility of a type ℓ consumer since $\beta_h > \beta_\ell$. To determine the worst case, we can therefore restrict our attention to a consumer with a low preference for liquidity. The utility of $\ell-$types from payouts that satisfy the self selection constraints for τ is increasing with τ for given μ.

Therefore, for any value of μ the utility of $\ell-$types obtains its minimum value for $\tau = 0$. Using this, we obtain

$$\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\mu,\tau;t) = \beta_\ell \cdot z_{1\ell}(0,\mu) + z_{2\ell}(0,\mu) = \alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu.$$

(iii) The ex-ante Choquet expected utility

After rearranging terms, from $\pi = \underline{\tau}(\mu)$ we obtain

$$\mu = \frac{\pi \cdot \alpha_2}{(1-\pi)\cdot\beta_\ell + \pi\cdot\alpha_2} = \mu_\ell.$$

For given investment policy μ and given level of confidence γ, substitution of the results of part *(i)* and part *(ii)* of this proof yields

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) &:= \gamma\cdot\widetilde{U}(\pi,\mu) + (1-\gamma)\cdot\widetilde{U}(0,\mu) \\ &= \begin{cases} (1+\gamma\cdot\left(\frac{\beta_h}{\beta_\ell}-1\right)\cdot\pi)\cdot(\alpha_2+(\beta_\ell-\alpha_2)\cdot\mu) & \text{if} \quad \mu>\mu_\ell \\ \\ \alpha_2+(\gamma\cdot\beta_h+(1-\gamma)\cdot\beta_\ell-\alpha_2)\cdot\mu & \text{if} \quad \mu\le\mu_\ell, \end{cases} \end{aligned}$$

which completes the proof.

■

Proof of Proposition 16

Denote

$$\hat{v}_a(M;q;t) := M\cdot R^m(q;t) + (1-M)\cdot R^a(q;t).$$

For all $q\in[\frac{\alpha_2}{\beta h},\frac{\alpha_2}{\beta_\ell}]$ we have $\hat{v}_a(M;q;h) > \hat{v}_a(M;q;\ell)$. Furthermore, $\hat{v}_a(M;q;\ell)$ is decreasing in q. From $q_a^E(\tau,\mu)\le\frac{\alpha_2}{\beta_\ell}$ it now follows that

$$\begin{aligned} &\min_{(t,\tau)\in\{h,\ell\}\times[0,1]}\left[\hat{v}_a(M;q_a^E(\tau,\mu);t)\right] \\ =\ &\min_{q\in[\frac{\alpha_2}{\beta_h},\frac{\alpha_2}{\beta_\ell}]}\hat{v}_a(M;q;\ell) = M\cdot\beta_\ell + (1-M)\cdot\alpha_2. \end{aligned}$$

To simplify notation, we denote $q_a := q_a^E(\pi,\mu)$. From Remark 7 it follows that in an equilibrium with $\mu\in(0,1)$, the asset price $q_a\in(\frac{\alpha_2}{\beta_h},\frac{\alpha_2}{\beta_\ell})$ must satisfy

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

(i) $q_a = \frac{\alpha_2}{\beta_\ell}$

In this case consumers anticipate the asset price to be at its upper bound for $\tau=\pi$. By deriving a contradiction, we show this cannot occur in equilibrium. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma\cdot[\pi\cdot\beta_h + (1-\pi)\cdot\beta_\ell] + (1-\gamma)\cdot\beta_\ell = \gamma\cdot\pi\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A **simple capacity** based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^{C} f \, d\nu := \gamma \cdot \int_{S} f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1-M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

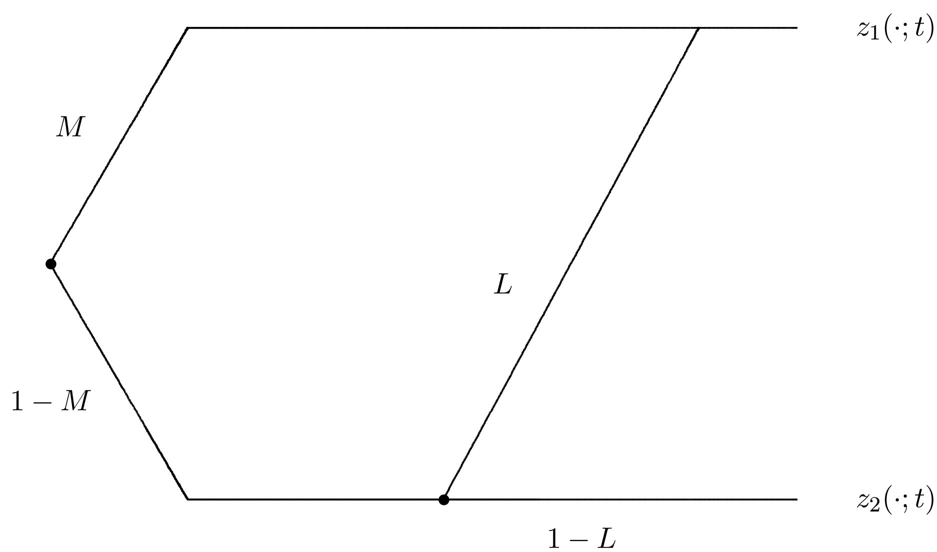

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & +(1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\} \cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\}) \cdot v_n(M;\ell)] \\ & +(1-\gamma) \cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M) \cdot \alpha_2 \geq \beta_\ell \cdot M + (1-M) \cdot \alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2 \,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

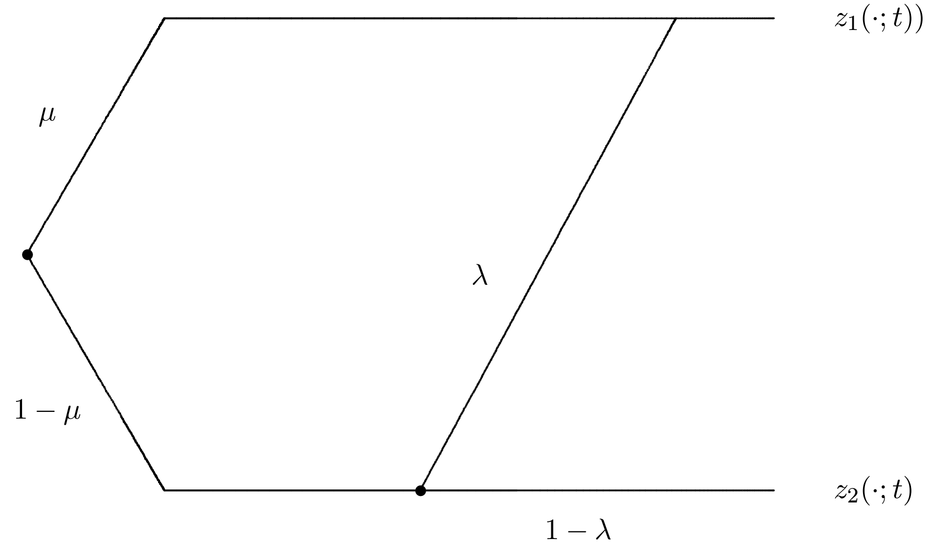

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) := \quad & \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ := \quad & \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \text{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \text{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \text{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \text{F}_2 \\
& \multicolumn{3}{l}{\lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0.} & \text{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := &\quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = &\quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

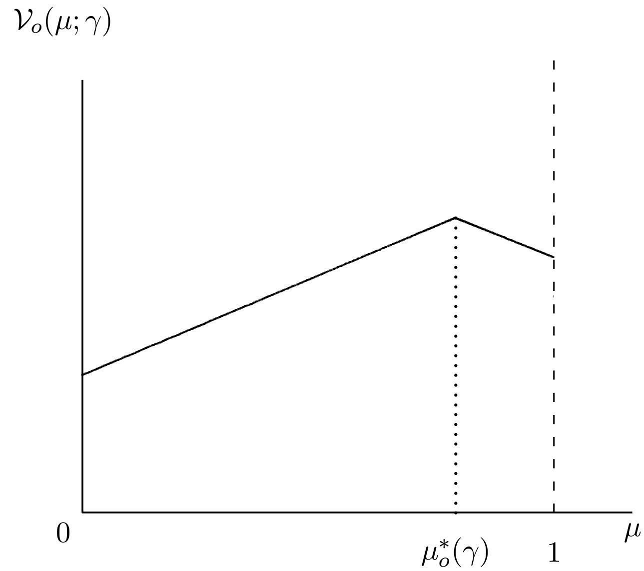

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

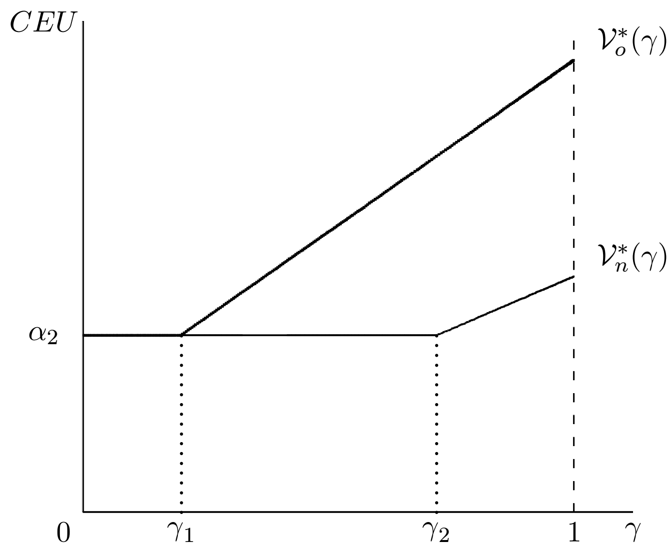

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

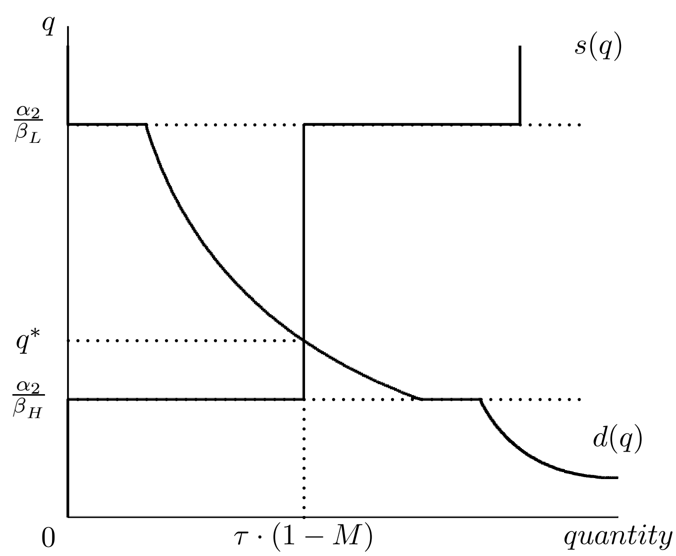

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

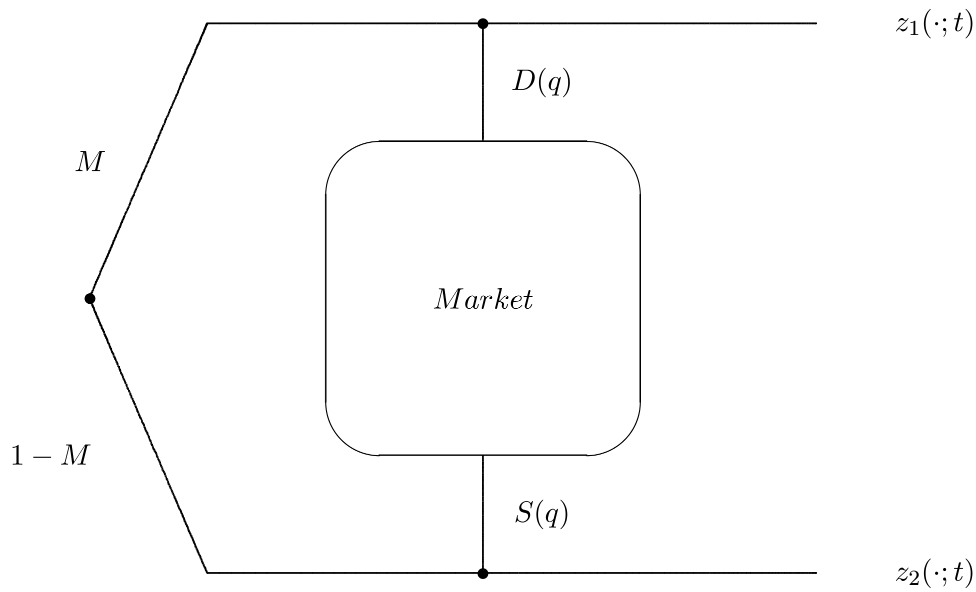

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M, \mu; \gamma) := \quad & \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] .\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0, 1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma), \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma = 1$, *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi$,
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1)$, *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi)$,
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}]$, *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

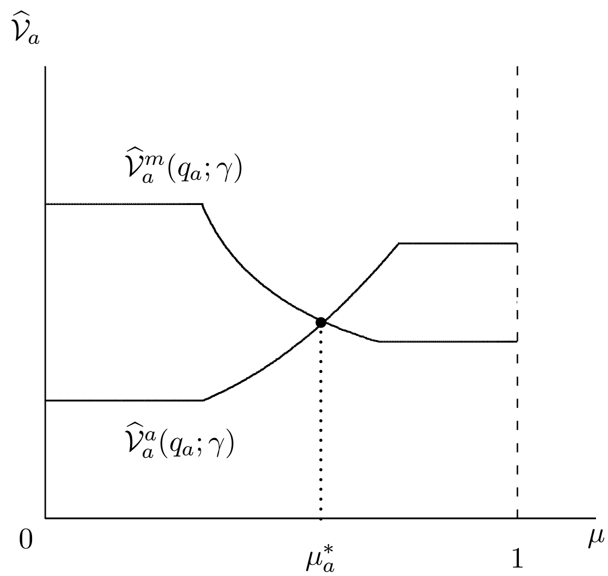

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

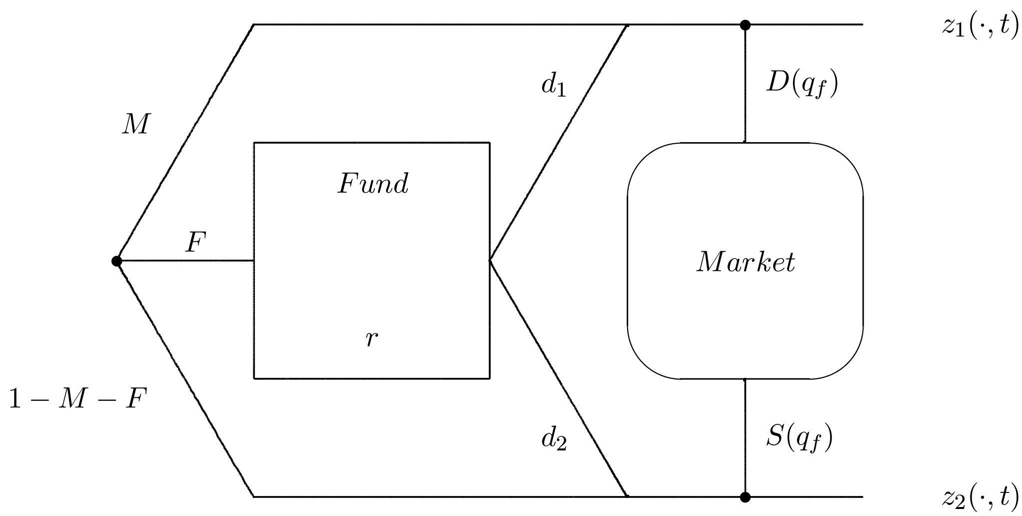

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22]As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

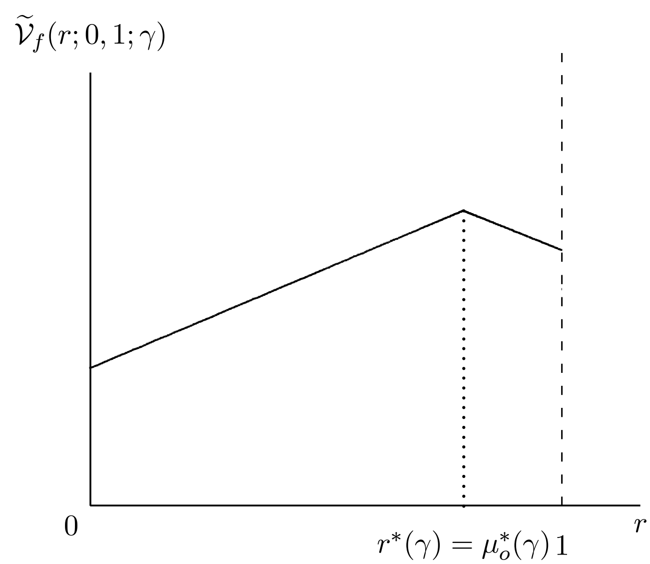

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0, 1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.
2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

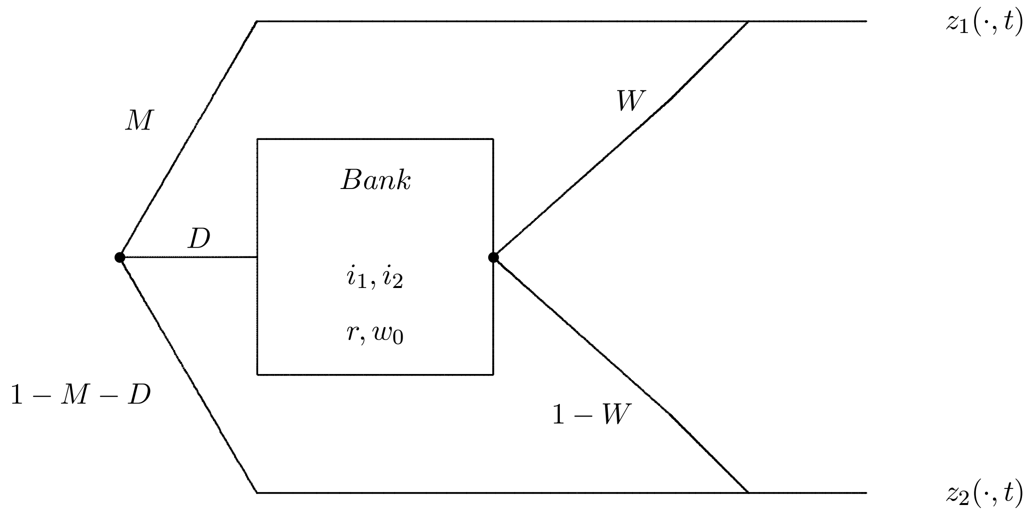

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

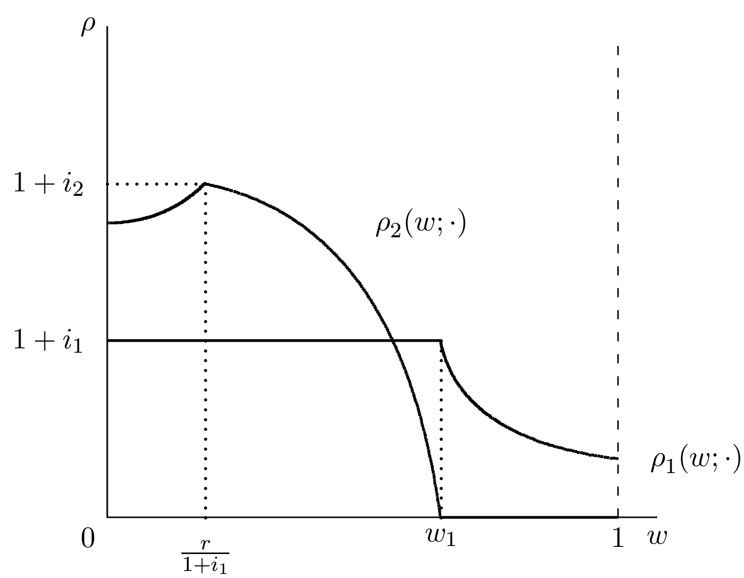

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

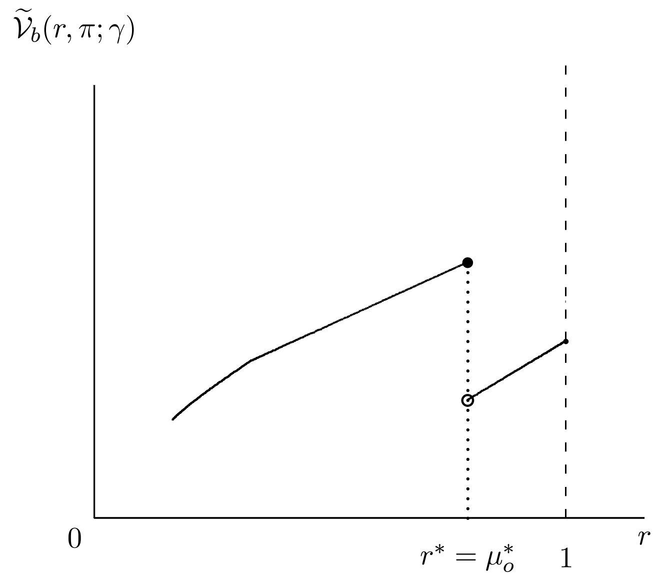

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

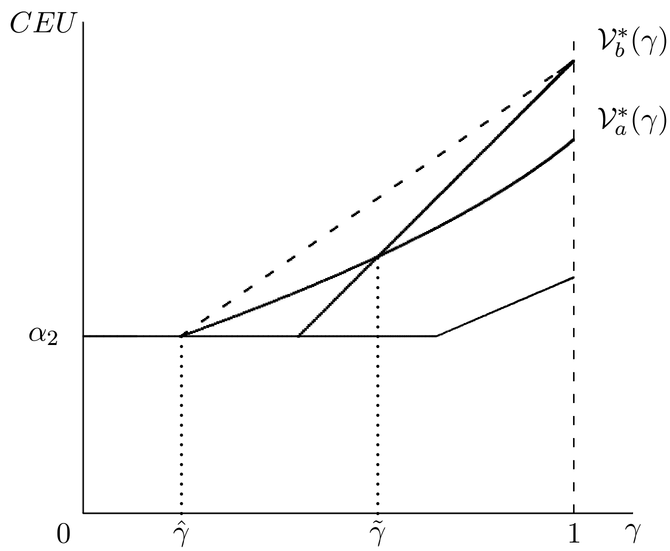

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. CONCLUDING REMARKS

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \; f(s) \geq t\}$ and $\{s \in S | \; f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^{\infty} \nu(\{s \in S | \; f(s) \geq t\}) \, dt + \int_{-\infty}^{0} [\nu(\{s \in S | \; f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma\cdot[\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)]+(1-\gamma)\cdot\min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M\cdot[\gamma\cdot[\pi\cdot\beta_h+(1-\pi)\cdot\beta_\ell]+(1-\gamma)\cdot\beta_\ell]+(1-M)\cdot\alpha_2\\ &= M\cdot[\gamma\cdot\pi\cdot\beta_h+(1-\gamma\cdot\pi)\cdot\beta_\ell]+(1-M)\cdot\alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu\in[0,1]$ and $\tau\in[0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu)=0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell\cdot\mu}{\beta_\ell\cdot\mu+\alpha_2\cdot(1-\mu)},\\ \overline{\tau}(\mu) &:= \frac{\beta_h\cdot\mu}{\beta_h\cdot\mu+\alpha_2\cdot(1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu)=\begin{cases}\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}+\mu & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ \dfrac{\mu}{\tau} & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{1\ell}(\tau,\mu)=\begin{cases}\mu-\left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ 0 & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if } \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \text{if } \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) :=& \quad \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ =& \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} &\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\quad \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ &\quad + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma, \pi, \alpha_2, \beta_h, \beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned} \mathcal{V}_a^*(\gamma) := \quad & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\ = \quad & \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}. \end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a; \gamma) = \gamma\cdot[\pi\cdot R^m(q_a; h) + (1-\pi)\cdot R^m(q_a; \ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a; \gamma) = \gamma\cdot[\pi\cdot R^a(q_a; h) + (1-\pi)\cdot R^a(q_a; \ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t, \tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t, \tau) = (\ell, \tau)$ for any $\tau \in [0, 1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a; \gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a; \gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$*. When this is not the case, investment policy* M *may affect the state* (t, τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$*.*

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h*, and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ*.*

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) \; + \; (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$*. In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F*. Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$
$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ,φ). Thus, its decision problem for given (ψ,φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w; i_1, i_2, r) := \begin{cases} \dfrac{\alpha_2 \cdot (1-r) + r - (1+i_1) \cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1 + i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)} \cdot [(1-r) - \dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1 + (1-\alpha_1) \cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1 + (1-\alpha_1) \cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w; r, w_0) := \quad \widehat{\rho}_1(w; i_1(r, w_0), i_2(r, w_0), r),$$

$$\rho_2(w; r, w_0) := \quad \widehat{\rho}_2(w; i_1(r, w_0), i_2(r, w_0), r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w = \tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau; r, w_0) \le \beta_h \cdot \rho_1(\tau; r, w_0) \qquad S_h$$

$$\beta_\ell \cdot \rho_1(\tau; r, w_0) \le \rho_2(\tau; r, w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau < 1$ the constraint

$$\beta_\ell \cdot \rho_1(\tau; r, w_0) > \rho_2(\tau; r, w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau; r, w_0; h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) \le \beta_h \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) > \beta_h \cdot \rho_1(\tau; r, w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau; h) = 1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau; \ell) = 0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0, 1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) &:= \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ &\qquad + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1-\pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := {} & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = {} & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1-r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1-r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

Appendix B: Proofs

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_h + \tau \cdot \alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_\ell + \tau \cdot \alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau), \mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \le \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \ge \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell + \pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell + \pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell + \pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell), \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ *is a best reply for* $r = \mu_\ell$ *and* $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ *is convex*

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a} \left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c} \right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a &:= 2 \cdot \pi \cdot \beta_h \\ b_1 &:= \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 &:= \gamma \cdot (\alpha_2 - \beta_\ell) \\ c &:= 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \; & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\
= \; & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\
& -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \; & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\widetilde{\gamma}) = \mathcal{V}_a^*(\widetilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a,b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a,b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a,b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1-\mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned} \tau \cdot z_{1h}(\tau,\mu) + (1-\tau) \cdot z_{1\ell}(\tau,\mu) &= \mu, \\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu). \end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)} = \mu + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$.

(iii) $\tau \in [\overline{\tau}, 1]$

For these values of τ there are "too many" $h-$types in the economy. So it is optimal to choose $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$. Even for this choice, however, the proportion of $h-$types is so high, that for $z_{2h}(\tau,\mu) = 0$ the self selection constraint S_h is violated. The payout $z_{2h}(\tau,\mu)$ obtains its lowest value when S_h binds and its highest value when S_ℓ binds. The self selection constraint S_h reads

$$\beta_h \cdot z_{1h}(\tau,\mu) + z_{2h}(\tau,\mu) \geq \beta_h \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu)$$

and after substitution and rearranging we obtain

$$z_{2h}(\tau,\mu) = z_{2\ell}(\tau,\mu) - \beta_h \cdot \frac{\mu}{\tau}.$$

Using $z_{2\ell}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)]$ we find $z_{2h}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)] - \beta_h \cdot \frac{\mu}{\tau}$, from which

$$z_{2h}(\tau,\mu) = \alpha_2 \cdot (1-\mu) - \frac{(1-\tau)}{\tau} \cdot \beta_h \cdot \mu = \alpha_2 - \frac{\mu}{\tau} \cdot (\tau \cdot \alpha_2 + (1-\tau) \cdot \beta_h)$$

and

$$z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu) + \beta_h \cdot \mu$$

follow.

■

Proof of Lemma 14

The structure of this proof is as follows. First the average interim utility for the investment policy μ is determined for $\tau = \pi$. Then the interim utility of the worst case is determined and both are combined to calculate the ex-ante Choquet expected indirect utility of investment policy μ.

(i) Average interim utility

For any $\mu \in [0,1]$ let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13. For any investment policy μ and for $\tau = \pi$ the average interim utility is

$$\widetilde{U}(\pi,\mu) := \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu).$$

From Lemma 12 we have

$$\pi \cdot z_{2h}(\pi,\mu) + (1-\pi)\cdot z_{2\ell}(\pi,\mu) = \alpha_2 \cdot (1-\mu)$$

substitution of which yields

$$\widetilde{U}(\pi,\mu) = \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi)\cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu).$$

(ia) $\pi \in [0, \underline{\tau}(\mu))$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot \left(\frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu\right) \\
&\quad + (1-\pi)\cdot \beta_\ell \cdot \left(\mu - \frac{\pi}{(1-\pi)} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) + \alpha_2 \cdot (1-\mu) \\
&= \left(1 + (\frac{\beta_h}{\beta_\ell} - 1)\cdot \pi\right) \cdot (\alpha_2 + (\beta_\ell - \alpha_2)\cdot \mu)\,.
\end{aligned}$$

(ib) $\pi \in [\underline{\tau}(\mu), 1]$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi)\cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu) \\
&= \pi \cdot \beta_h \cdot \frac{\mu}{\pi} + \alpha_2 \cdot (1-\mu) = \alpha_2 + (\beta_h - \alpha_2)\cdot \mu,
\end{aligned}$$

which complete this part of the proof.

(ii) The worst case

The interim utility a type h consumer obtains from payouts (z_h, z_ℓ) that satisfy the self selection constrains is as least as last as the corresponding utility of a type ℓ consumer since $\beta_h > \beta_\ell$. To determine the worst case, we can therefore restrict our attention to a consumer with a low preference for liquidity. The utility of $\ell-$types from payouts that satisfy the self selection constraints for τ is increasing with τ for given μ.

Therefore, for any value of μ the utility of $\ell-$types obtains its minimum value for $\tau = 0$. Using this, we obtain

$$\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\mu,\tau;t) = \beta_\ell \cdot z_{1\ell}(0,\mu) + z_{2\ell}(0,\mu) = \alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu.$$

(iii) The ex-ante Choquet expected utility

After rearranging terms, from $\pi = \underline{\tau}(\mu)$ we obtain

$$\mu = \frac{\pi \cdot \alpha_2}{(1-\pi) \cdot \beta_\ell + \pi \cdot \alpha_2} = \mu_\ell.$$

For given investment policy μ and given level of confidence γ, substitution of the results of part *(i)* and part *(ii)* of this proof yields

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) &:= \gamma \cdot \widetilde{U}(\pi,\mu) + (1-\gamma) \cdot \widetilde{U}(0,\mu) \\ &= \begin{cases} (1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu) & \text{if} \quad \mu > \mu_\ell \\ \alpha_2 + (\gamma \cdot \beta_h + (1-\gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu & \text{if} \quad \mu \le \mu_\ell, \end{cases} \end{aligned}$$

which completes the proof.

■

Proof of Proposition 16

Denote

$$\hat{v}_a(M;q;t) := M \cdot R^m(q;t) + (1-M) \cdot R^a(q;t).$$

For all $q \in [\frac{\alpha_2}{\beta h}, \frac{\alpha_2}{\beta_\ell}]$ we have $\hat{v}_a(M;q;h) > \hat{v}_a(M;q;\ell)$. Furthermore, $\hat{v}_a(M;q;\ell)$ is decreasing in q. From $q_a^E(\tau,\mu) \le \frac{\alpha_2}{\beta_\ell}$ it now follows that

$$\begin{aligned} &\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \left[\hat{v}_a(M;q_a^E(\tau,\mu);t)\right] \\ = &\min_{q\in[\frac{\alpha_2}{\beta_h},\frac{\alpha_2}{\beta_\ell}]} \hat{v}_a(M;q;\ell) = M \cdot \beta_\ell + (1-M) \cdot \alpha_2. \end{aligned}$$

To simplify notation, we denote $q_a := q_a^E(\pi,\mu)$. From Remark 7 it follows that in an equilibrium with $\mu \in (0,1)$, the asset price $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$ must satisfy

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

(i) $q_a = \frac{\alpha_2}{\beta_\ell}$

In this case consumers anticipate the asset price to be at its upper bound for $\tau = \pi$. By deriving a contradiction, we show this cannot occur in equilibrium. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell = \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma \cdot [\pi \cdot \frac{\alpha_2}{\beta_\ell} \cdot \beta_h + (1-\pi) \cdot \alpha_2] + (1-\gamma) \cdot \alpha_2 = \gamma \cdot \pi \cdot \frac{\alpha_2}{\beta_\ell} \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \alpha_2.$$

This implies $\widehat{\mathcal{V}}_a^m(q_a;\gamma) < \widehat{\mathcal{V}}_a^a(q_a;\gamma)$. In this case, any consumer would invest all his wealth in the illiquid asset, i.e. $\mu = 0$. This aggregate investment policy, however, leads to the asset price $q_a = \frac{\alpha_2}{\beta_h}$, which contradicts assumption *(i)*.

(ii) $q_a = \frac{\alpha_2}{\beta_h}$

In this case, the consumers anticipate the asset price to obtain its lower bound for $\tau = \pi$. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_h] + (1-\gamma) \cdot \beta_\ell = \gamma \cdot \beta_h + (1-\gamma) \cdot \beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma \cdot [\pi \cdot \alpha_2 + (1-\pi) \cdot \alpha_2] + (1-\gamma) \cdot \alpha_2 = \alpha_2.$$

This lower bound for the asset price will only be obtained in equilibrium if

$$\alpha_2 \geq \gamma \cdot \beta_h + (1-\gamma) \cdot \beta_\ell.$$

Rearranging terms, this inequality leads to $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. From Proposition 10 it follows that for these levels of confidence financial institutions are superfluous form the ex-ante point of view.

(iii) $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$

For any such q_a to be the anticipated equilibrium price in Period 1 for $\tau = \pi$, we must have

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Substitution of $R^m(q_a;h) = \beta_h$, $R^m(q_a;\ell) = \frac{\alpha_2}{q_a}$, $R^a(q_a;h) = \beta_h \cdot q_a$ and $R^a(q_a;\ell) = \alpha_2$ leads to

$$\begin{aligned} & \gamma \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \frac{1}{q_a} \cdot \alpha_2 + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h \cdot q_a + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2. \end{aligned}$$

Rearranging yields

$$(1-\gamma) \cdot [\alpha_2 - \beta_\ell] = \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot \frac{1}{q_a}\right] \cdot \gamma \cdot (1-q_a).$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. NON-ADDITIVE BELIEFS AND THE CHOQUET INTEGRAL

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.
[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.

[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.

[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

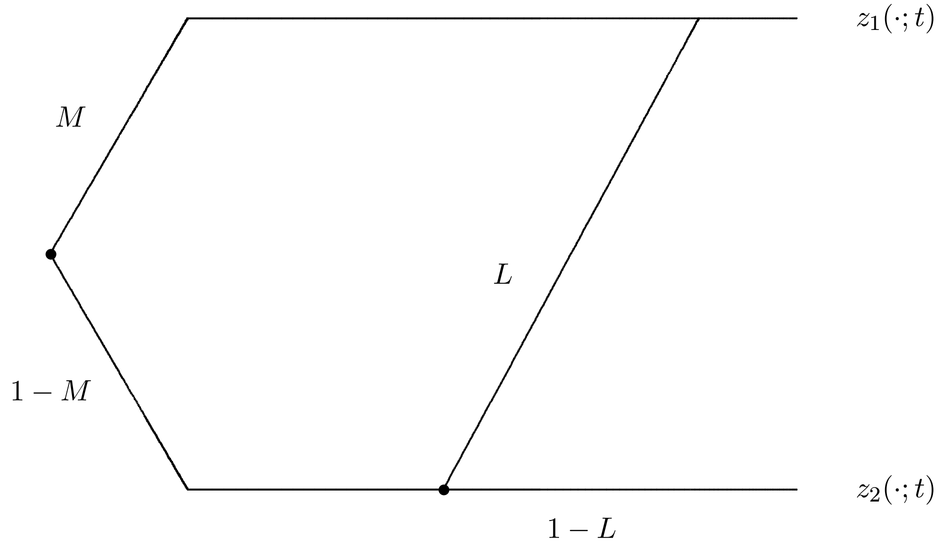

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) :=&\quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) :=&\quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1-M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) :=\ & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ =\ & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

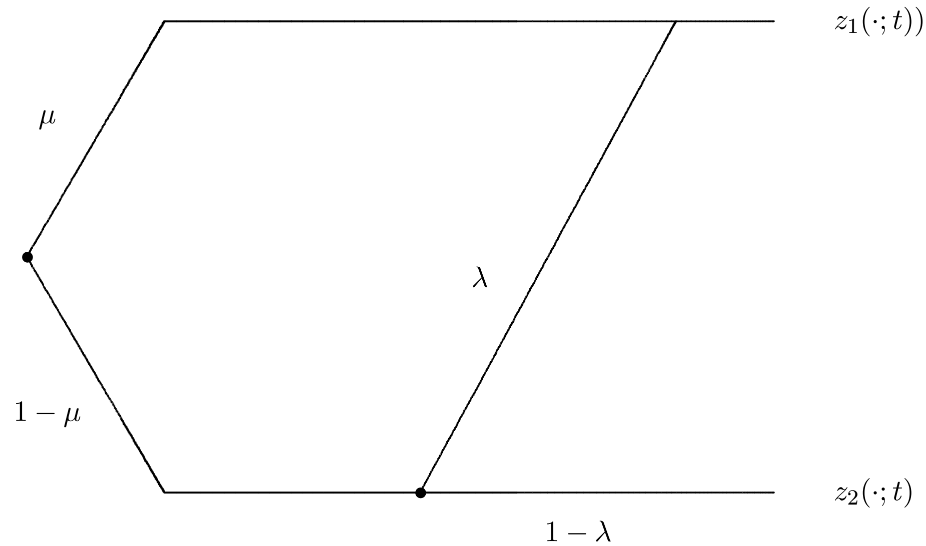

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) := &\quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ = &\quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.

[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

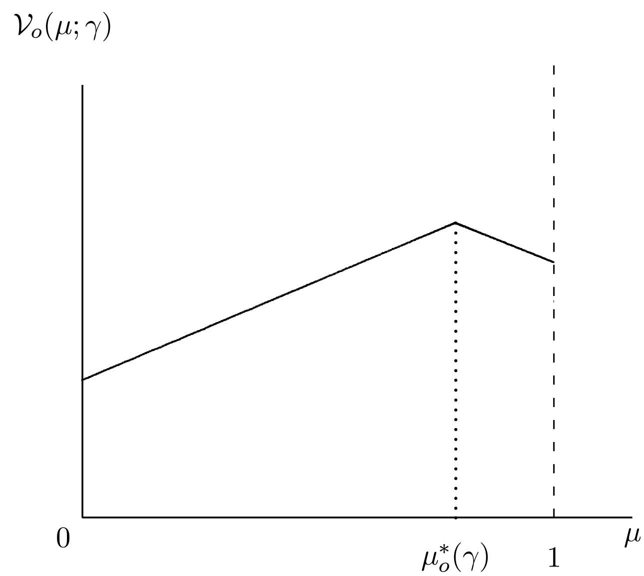

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

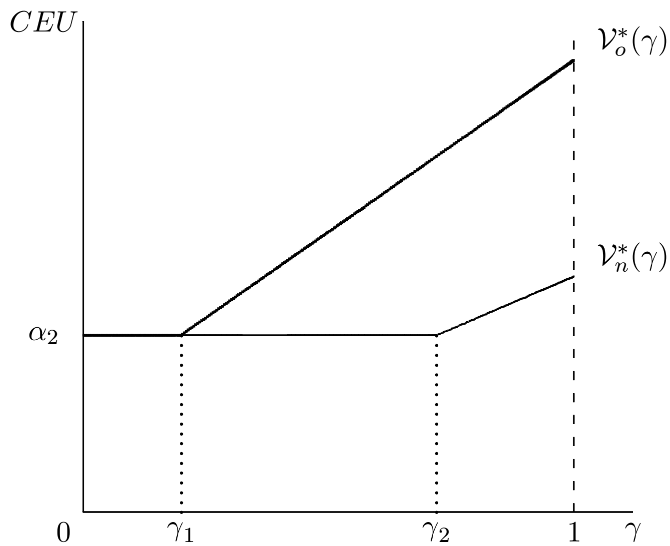

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

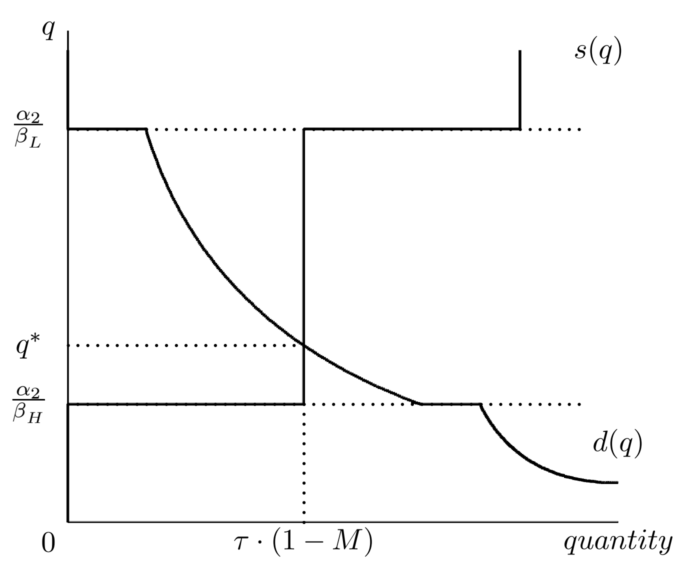

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

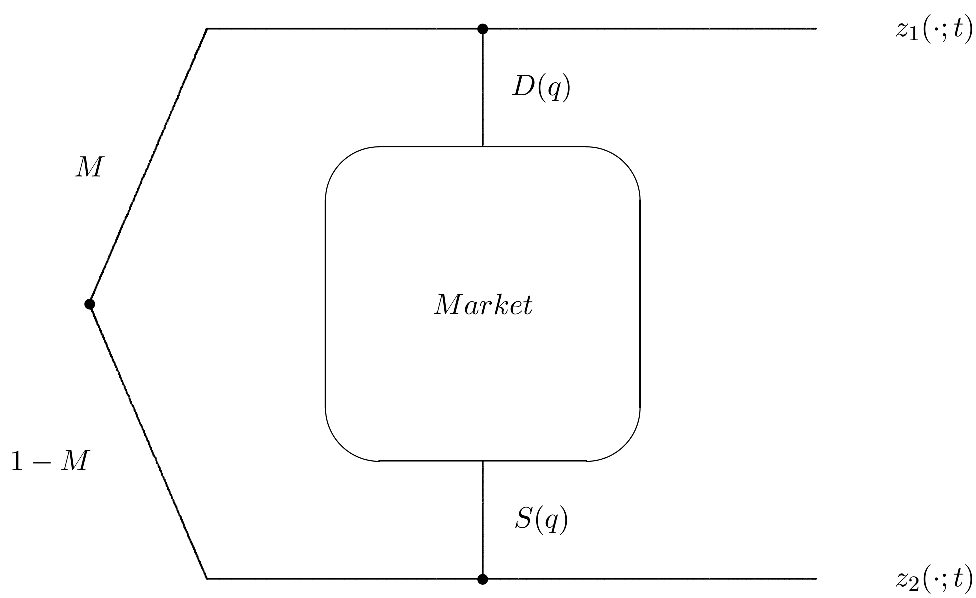

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

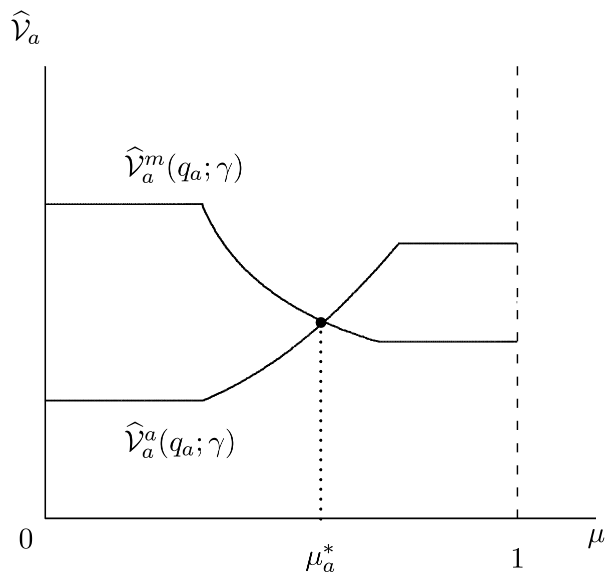

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

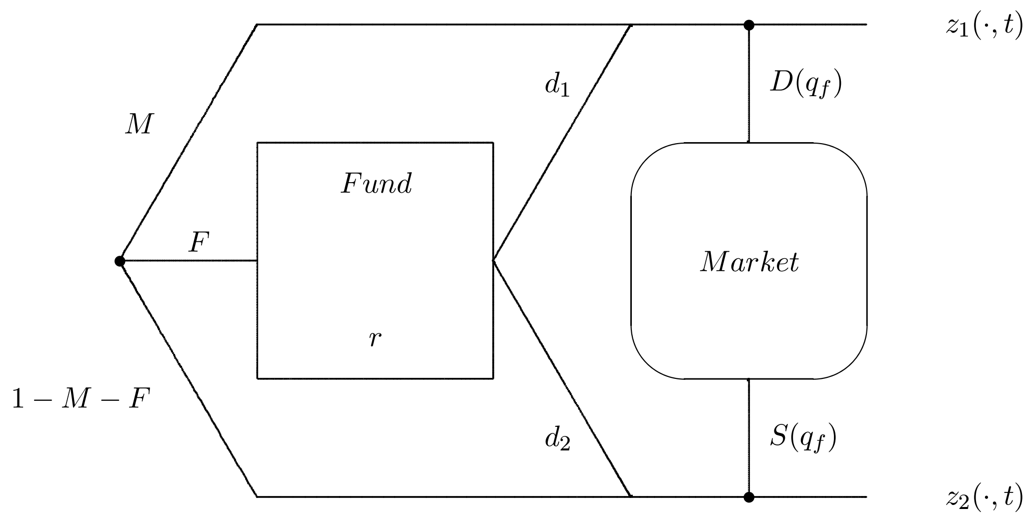

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22]As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

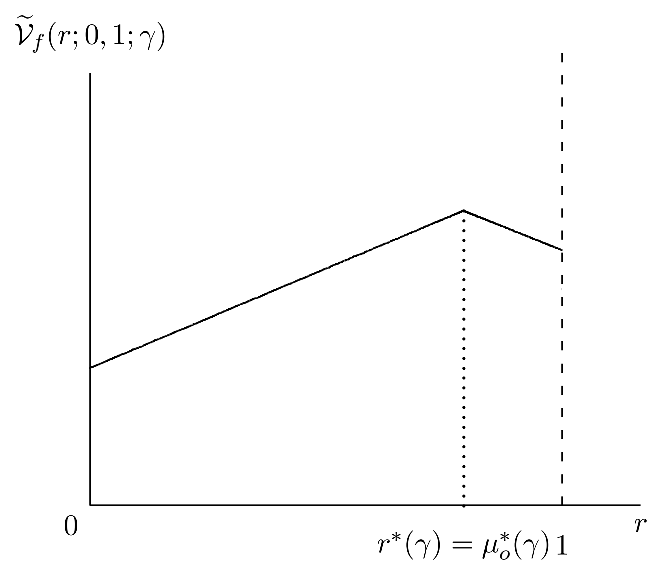

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0, 1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

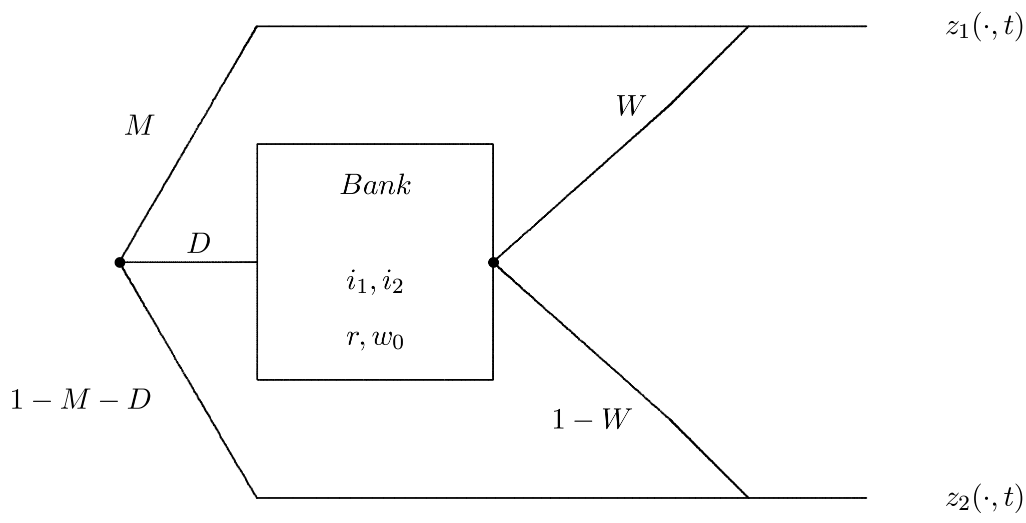

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

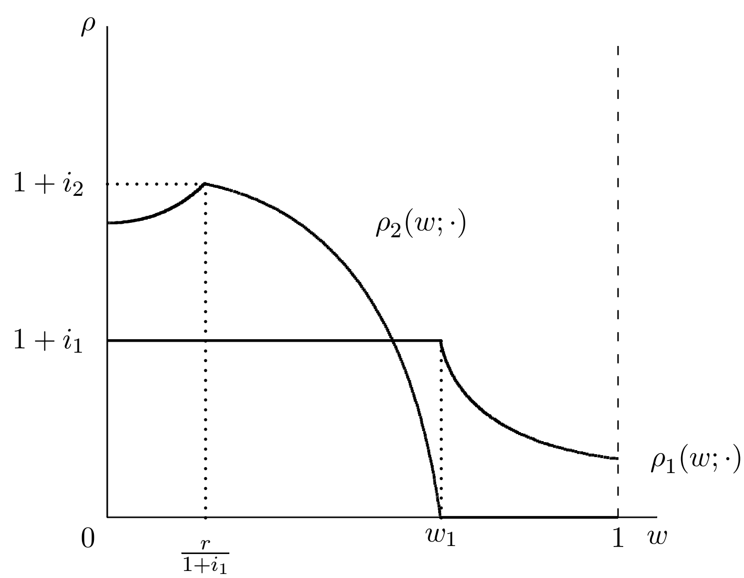

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

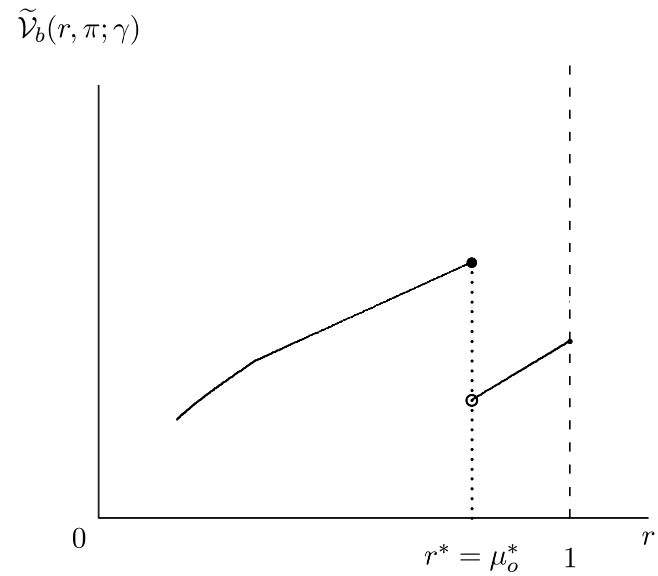

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

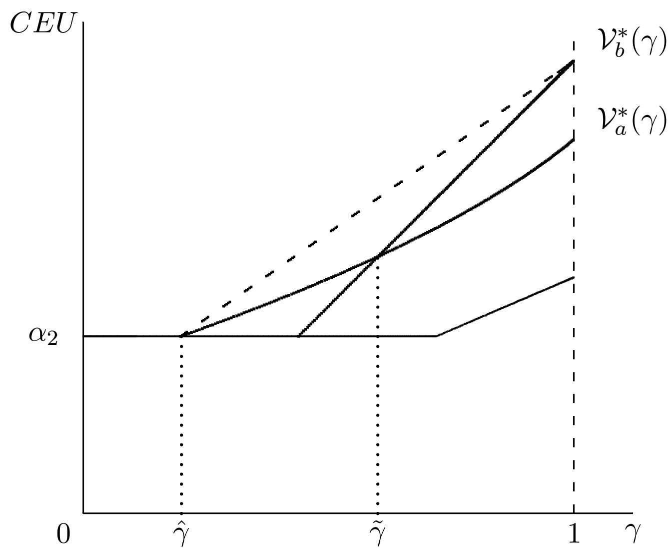

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t,\tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t,\tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S |\ f(s) \geq t\}$ and $\{s \in S |\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S |\ f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S |\ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)] + (1-\gamma)\cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_\ell] + (1-\gamma)\cdot \beta_\ell] + (1-M)\cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\begin{aligned}\underline{\tau}(\mu) &:= \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)}, \\ \overline{\tau}(\mu) &:= \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.\end{aligned}$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \text{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \text{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \text{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \text{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \text{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \text{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ*. This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$*, depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ *in the asset market exists and the price*

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) :=\ & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma)\\ =\ & \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell,\\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. *For investing in the illiquid asset it occurs for* $(t,\tau) = (\ell,\tau)$ *for any* $\tau\in[0,1]$. *From this it follows that for an asset price* q_a *the ex-ante Choquet expected utility for a consumer who holds a fraction* M *of his initial one unit of wealth as money is*

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t,\tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t,τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$
$$+ (1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ,φ). Thus, its decision problem for given (ψ,φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2 \cdot (1-r) + r - (1+i_1) \cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)} \cdot [(1-r) - \dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w=\tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \leq \beta_h \cdot \rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell \cdot \rho_1(\tau;r,w_0) \leq \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ the constraint

$$\beta_\ell \cdot \rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \leq \beta_h \cdot \rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h \cdot \rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau;r,w_0;\ell) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) < \beta_\ell \cdot \rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) \geq \beta_\ell \cdot \rho_1(\tau;r,w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D,M;r,w_0;\gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D,M;r,w_0;\gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r,w_0;\gamma) &:= \max_{\substack{(D,M)\in[0,1]\times[0,1] \\ s.t.\ D+M\leq 1}} \mathcal{V}_b(D,M;r,w_0;\gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi;r,w_0;h) + (1-\pi)\cdot v_b(\pi;r,w_0;\ell)\} \\ &\qquad + (1-\gamma)\cdot \min_{\tau\in[0,1]} v_b(\tau;r,w_0;t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r,w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r,w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$, taking into account the withdrawal behaviour $W(\tau;r,w_0;t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau=\pi$ and $\tau=1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1-\pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \; \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ =& \; \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1-r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1-r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau; 0, 1; \mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell), \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ is a best reply for $r = \mu_\ell$ and $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ *is convex*

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1 - \pi) \cdot \alpha_2 + (1 - \gamma) \cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a} \left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c} \right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a :=& \quad 2 \cdot \pi \cdot \beta_h \\ b_1 :=& \quad \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 :=& \quad \gamma \cdot (\alpha_2 - \beta_\ell) \\ c :=& \quad 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$
\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \; & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\
= \; & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\
& -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \; & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}
$$

As $c = 4 \cdot (1-\pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$
[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}
$$

and its second derivative is

$$
-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.
$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$
\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}
$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$
\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').
$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$
\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.
$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$
\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).
$$

(iii) $\mathcal{V}_b^*(\widetilde{\gamma}) = \mathcal{V}_a^*(\widetilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a, b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a, b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a, b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1-\mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned} \tau \cdot z_{1h}(\tau,\mu) + (1-\tau) \cdot z_{1\ell}(\tau,\mu) &= \mu, \\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu). \end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)} = \mu + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$.

(iii) $\tau \in [\overline{\tau}, 1]$

For these values of τ there are "too many" $h-$types in the economy. So it is optimal to choose $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$. Even for this choice, however, the proportion of $h-$types is so high, that for $z_{2h}(\tau,\mu) = 0$ the self selection constraint S_h is violated. The payout $z_{2h}(\tau,\mu)$ obtains its lowest value when S_h binds and its highest value when S_ℓ binds. The self selection constraint S_h reads

$$\beta_h \cdot z_{1h}(\tau,\mu) + z_{2h}(\tau,\mu) \geq \beta_h \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu)$$

and after substitution and rearranging we obtain

$$z_{2h}(\tau,\mu) = z_{2\ell}(\tau,\mu) - \beta_h \cdot \frac{\mu}{\tau}.$$

Using $z_{2\ell}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)]$ we find $z_{2h}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)] - \beta_h \cdot \frac{\mu}{\tau}$, from which

$$z_{2h}(\tau,\mu) = \alpha_2 \cdot (1-\mu) - \frac{(1-\tau)}{\tau} \cdot \beta_h \cdot \mu = \alpha_2 - \frac{\mu}{\tau} \cdot (\tau \cdot \alpha_2 + (1-\tau) \cdot \beta_h)$$

and

$$z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu) + \beta_h \cdot \mu$$

follow.

■

Proof of Lemma 14

The structure of this proof is as follows. First the average interim utility for the investment policy μ is determined for $\tau = \pi$. Then the interim utility of the worst case is determined and both are combined to calculate the ex-ante Choquet expected indirect utility of investment policy μ.

(i) Average interim utility

For any $\mu \in [0,1]$ let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13. For any investment policy μ and for $\tau = \pi$ the average interim utility is

$$\widetilde{U}(\pi,\mu) := \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu).$$

From Lemma 12 we have

$$\pi \cdot z_{2h}(\pi,\mu) + (1-\pi)\cdot z_{2\ell}(\pi,\mu) = \alpha_2 \cdot (1-\mu)$$

substitution of which yields

$$\widetilde{U}(\pi,\mu) = \pi\cdot\beta_h\cdot z_{1h}(\pi,\mu) + (1-\pi)\cdot\beta_\ell\cdot z_{1\ell}(\pi,\mu) + \alpha_2\cdot(1-\mu).$$

(ia) $\pi \in [0, \underline{\tau}(\mu))$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi\cdot\beta_h\cdot\left(\frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}+\mu\right) \\
&\quad +(1-\pi)\cdot\beta_\ell\cdot\left(\mu-\frac{\pi}{(1-\pi)}\cdot\frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right)+\alpha_2\cdot(1-\mu) \\
&= \left(1+(\frac{\beta_h}{\beta_\ell}-1)\cdot\pi\right)\cdot(\alpha_2+(\beta_\ell-\alpha_2)\cdot\mu)\,.
\end{aligned}$$

(ib) $\pi \in [\underline{\tau}(\mu), 1]$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi\cdot\beta_h\cdot z_{1h}(\pi,\mu)+(1-\pi)\cdot\beta_\ell\cdot z_{1\ell}(\pi,\mu)+\alpha_2\cdot(1-\mu) \\
&= \pi\cdot\beta_h\cdot\frac{\mu}{\pi}+\alpha_2\cdot(1-\mu) = \alpha_2+(\beta_h-\alpha_2)\cdot\mu,
\end{aligned}$$

which complete this part of the proof.

(ii) The worst case

The interim utility a type h consumer obtains from payouts (z_h, z_ℓ) that satisfy the self selection constrains is as least as last as the corresponding utility of a type ℓ consumer since $\beta_h > \beta_\ell$. To determine the worst case, we can therefore restrict our attention to a consumer with a low preference for liquidity. The utility of $\ell-$types from payouts that satisfy the self selection constraints for τ is increasing with τ for given μ.

Therefore, for any value of μ the utility of $\ell-$types obtains its minimum value for $\tau = 0$. Using this, we obtain

$$\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\mu,\tau;t) = \beta_\ell\cdot z_{1\ell}(0,\mu)+z_{2\ell}(0,\mu) = \alpha_2\cdot(1-\mu)+\beta_\ell\cdot\mu.$$

(iii) The ex-ante Choquet expected utility

After rearranging terms, from $\pi = \underline{\tau}(\mu)$ we obtain

$$\mu = \frac{\pi \cdot \alpha_2}{(1-\pi)\cdot \beta_\ell + \pi \cdot \alpha_2} = \mu_\ell.$$

For given investment policy μ and given level of confidence γ, substitution of the results of part *(i)* and part *(ii)* of this proof yields

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) &:= \gamma \cdot \widetilde{U}(\pi,\mu) + (1-\gamma)\cdot \widetilde{U}(0,\mu) \\ &= \begin{cases} (1+\gamma\cdot\left(\frac{\beta_h}{\beta_\ell}-1\right)\cdot\pi)\cdot(\alpha_2+(\beta_\ell-\alpha_2)\cdot\mu) & \text{if} \quad \mu > \mu_\ell \\ \alpha_2 + (\gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell - \alpha_2)\cdot\mu & \text{if} \quad \mu \le \mu_\ell, \end{cases} \end{aligned}$$

which completes the proof.

■

Proof of Proposition 16

Denote

$$\hat{v}_a(M;q;t) := M \cdot R^m(q;t) + (1-M)\cdot R^a(q;t).$$

For all $q \in [\frac{\alpha_2}{\beta h}, \frac{\alpha_2}{\beta_\ell}]$ we have $\hat{v}_a(M;q;h) > \hat{v}_a(M;q;\ell)$. Furthermore, $\hat{v}_a(M;q;\ell)$ is decreasing in q. From $q_a^E(\tau,\mu) \le \frac{\alpha_2}{\beta_\ell}$ it now follows that

$$\begin{aligned} &\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \left[\hat{v}_a(M;q_a^E(\tau,\mu);t)\right] \\ =\ &\min_{q\in[\frac{\alpha_2}{\beta_h},\frac{\alpha_2}{\beta_\ell}]} \hat{v}_a(M;q;\ell) = M\cdot\beta_\ell + (1-M)\cdot\alpha_2. \end{aligned}$$

To simplify notation, we denote $q_a := q_a^E(\pi,\mu)$. From Remark 7 it follows that in an equilibrium with $\mu \in (0,1)$, the asset price $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$ must satisfy

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

(i) $q_a = \frac{\alpha_2}{\beta_\ell}$

In this case consumers anticipate the asset price to be at its upper bound for $\tau = \pi$. By deriving a contradiction, we show this cannot occur in equilibrium. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma\cdot[\pi\cdot\beta_h + (1-\pi)\cdot\beta_\ell] + (1-\gamma)\cdot\beta_\ell = \gamma\cdot\pi\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma \cdot [\pi \cdot \frac{\alpha_2}{\beta_\ell} \cdot \beta_h + (1-\pi) \cdot \alpha_2] + (1-\gamma) \cdot \alpha_2 = \gamma \cdot \pi \cdot \frac{\alpha_2}{\beta_\ell} \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \alpha_2.$$

This implies $\widehat{\mathcal{V}}_a^m(q_a;\gamma) < \widehat{\mathcal{V}}_a^a(q_a;\gamma)$. In this case, any consumer would invest all his wealth in the illiquid asset, i.e. $\mu = 0$. This aggregate investment policy, however, leads to the asset price $q_a = \frac{\alpha_2}{\beta_h}$, which contradicts assumption *(i)*.

(ii) $q_a = \frac{\alpha_2}{\beta_h}$

In this case, the consumers anticipate the asset price to obtain its lower bound for $\tau = \pi$. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_h] + (1-\gamma) \cdot \beta_\ell = \gamma \cdot \beta_h + (1-\gamma) \cdot \beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma \cdot [\pi \cdot \alpha_2 + (1-\pi) \cdot \alpha_2] + (1-\gamma) \cdot \alpha_2 = \alpha_2.$$

This lower bound for the asset price will only be obtained in equilibrium if

$$\alpha_2 \geq \gamma \cdot \beta_h + (1-\gamma) \cdot \beta_\ell.$$

Rearranging terms, this inequality leads to $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. From Proposition 10 it follows that for these levels of confidence financial institutions are superfluous form the ex-ante point of view.

(iii) $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$

For any such q_a to be the anticipated equilibrium price in Period 1 for $\tau = \pi$, we must have

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Substitution of $R^m(q_a;h) = \beta_h$, $R^m(q_a;\ell) = \frac{\alpha_2}{q_a}$, $R^a(q_a;h) = \beta_h \cdot q_a$ and $R^a(q_a;\ell) = \alpha_2$ leads to

$$\begin{aligned} & \gamma \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \frac{1}{q_a} \cdot \alpha_2 + (1-\gamma) \cdot \beta_\ell \\ = \; & \gamma \cdot \pi \cdot \beta_h \cdot q_a + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2. \end{aligned}$$

Rearranging yields

$$(1-\gamma) \cdot [\alpha_2 - \beta_\ell] = \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot \frac{1}{q_a}\right] \cdot \gamma \cdot (1-q_a).$$

For $\gamma < 1$ we have $q_a < 1$, from which we obtain

$$\pi \cdot \beta_h - (1-\pi) \cdot \alpha_2 + \frac{1}{q_a} \cdot (1-\pi) \cdot \alpha_2 - q_a \cdot \pi \cdot \beta_h = (\frac{1}{\gamma} - 1) \cdot (\alpha_2 - \beta_\ell).$$

Denote the solution of this equation w.r.t. q_a by $\widehat{q}_a(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$.

The derivative of the left hand side of this expression with respect to q_a is

$$-\frac{(1-\pi) \cdot \alpha_2}{(q_a)^2} - \pi \cdot \beta_h < 0.$$

Therefore, the left hand side of the expression is decreasing in q_a.

The right hand side of the expression is decreasing in γ. An increase in the level of ambiguity, which corresponds to a decrease in the level of confidence, increases the right hand side of the equation. To maintain equality, the left hand side of the equation has to increase as well, which leads to a decrease in the asset price q_a, keeping all other parameters constant. The asset price is, however, bound from below by the value $\frac{\alpha_2}{\beta_h}$. After substituting this lower bound for the asset price and solving for the level of confidence γ, we find that $q_a > \frac{\alpha_2}{\beta_h}$, whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.

■

Proof of Lemma 18

If the self selection constraint for type ℓ consumers is violated for the deposit contract (r, π) and withdrawals $w = \pi$, then a bankrun in Period 1 can not be avoided. Hence, all illiquid assets will be liquidated. In this case, it is optimal for the consumers not to deposit their wealth in the bank. This leads to an ex-ante utility of $\mathcal{V}_n^*(\gamma)$.

Therefore, consider deposit contracts (r, π) such that for withdrawals $w = \pi$ the self selection constraint of type ℓ consumers is satisfied. Using $w = \pi$ this leads to the ex-ante Choquet expected utility of

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. The economy

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

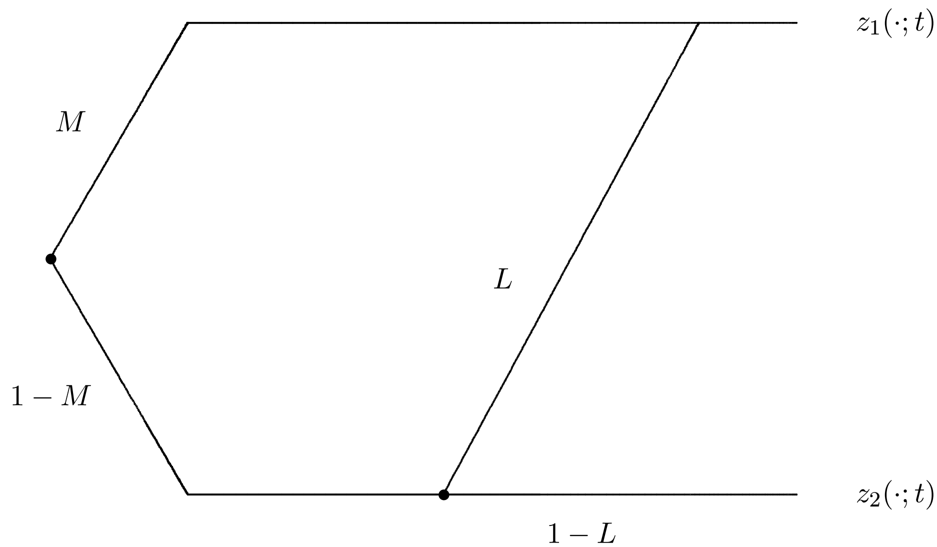

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{array}{rll} z_1(M,L) := & M + \alpha_1 \cdot L \cdot (1-M) & \text{and} \\ z_2(M,L) := & \alpha_2 \cdot (1-L) \cdot (1-M). & \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) \;:=\; u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) \;=\; \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)\\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau\\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t)\\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)]\\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) \,=\, \beta_h\cdot M + (1-M)\cdot\alpha_2 \,\geq\, \beta_\ell\cdot M + (1-M)\cdot\alpha_2 \,=\, v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

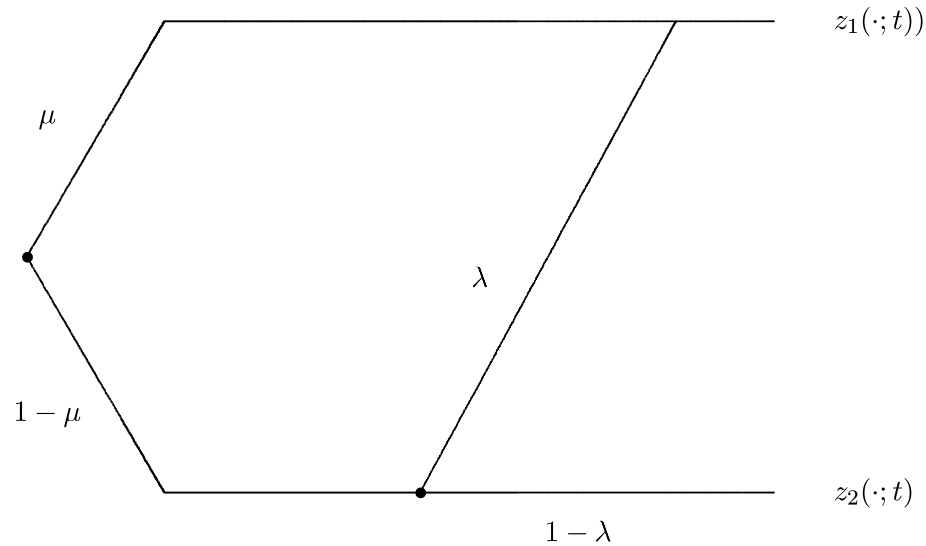

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{lllll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) :=& \quad u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ =& \quad \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1-\pi) \cdot v_o(\pi, \mu; \ell)] + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

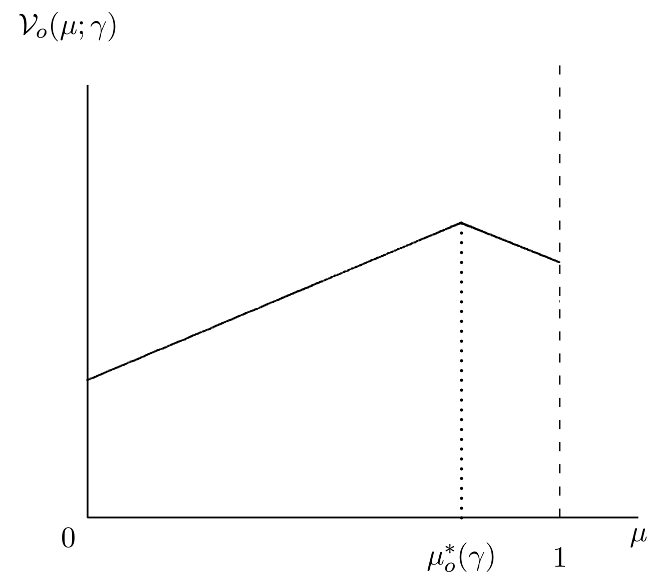

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

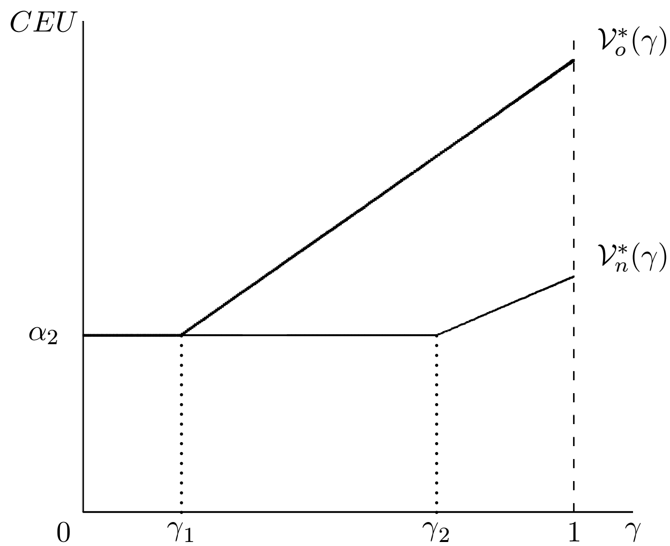

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

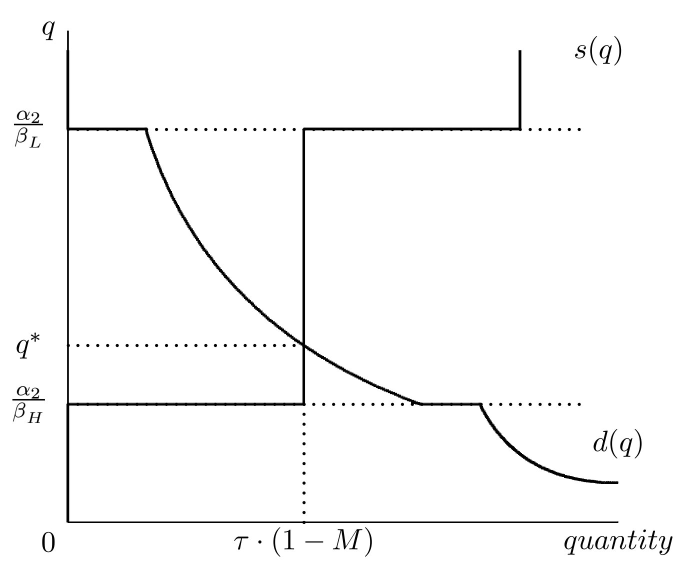

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

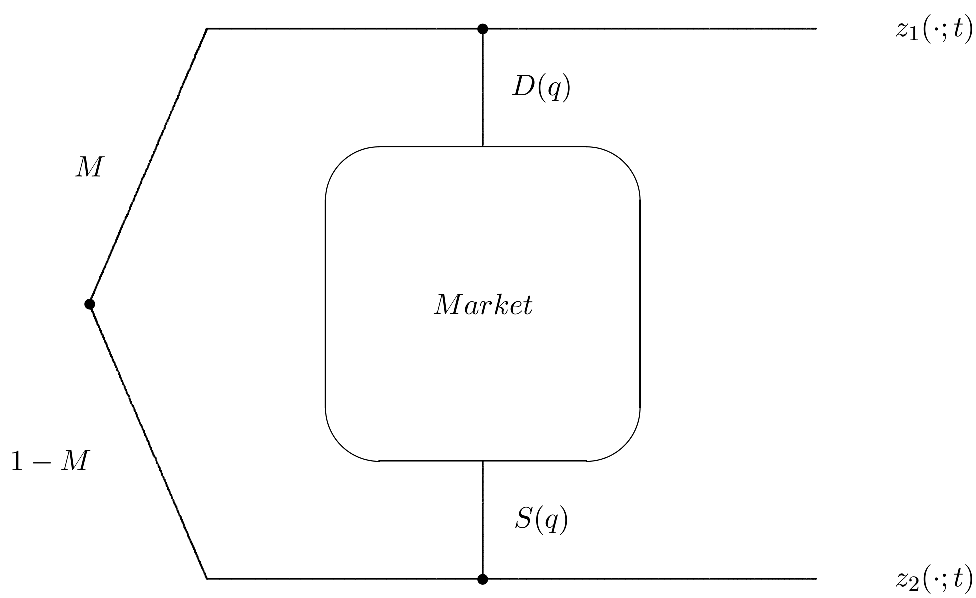

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M) \cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

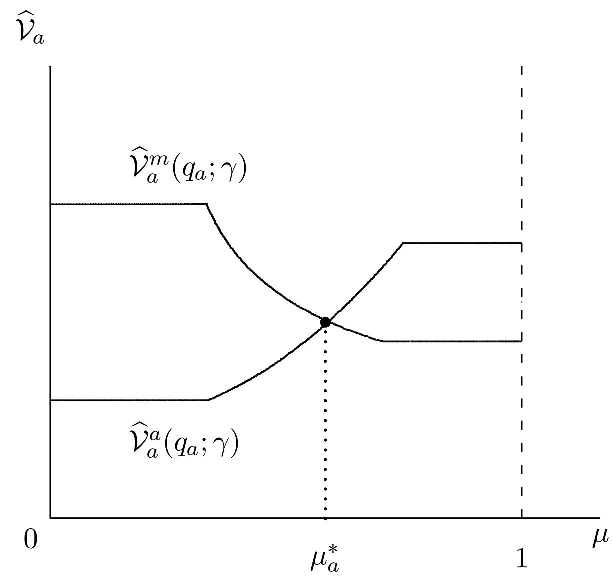

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

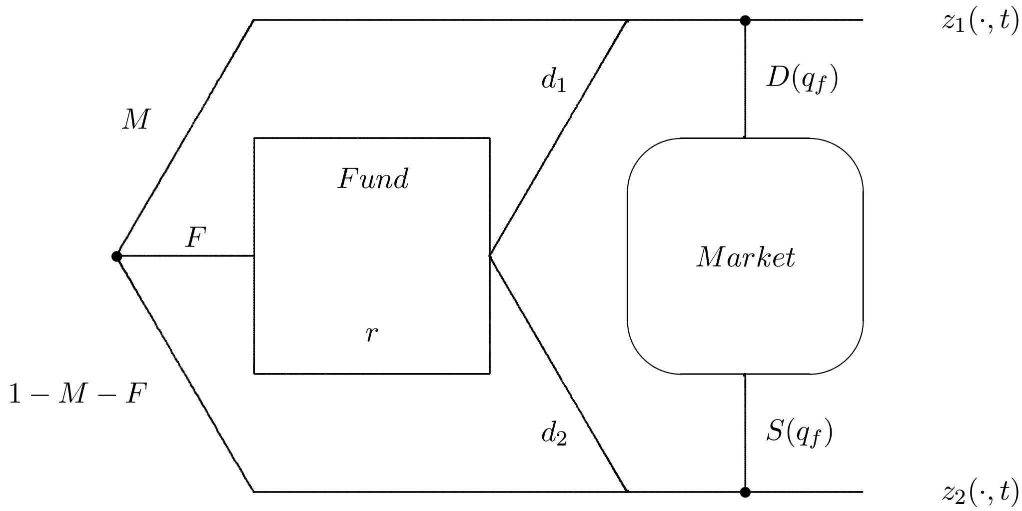

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1 - r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1 - \tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1 - \tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1 - F - M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1 - r)$ assets. Therefore, we have $q_f = (1 - r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\le 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

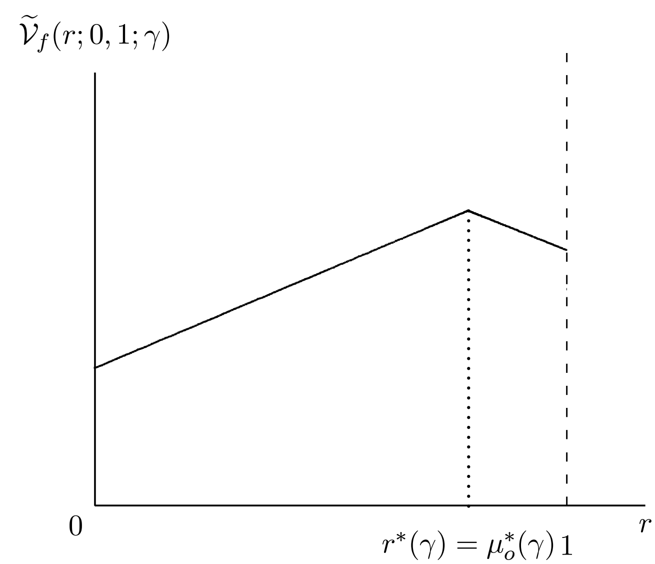

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.

[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

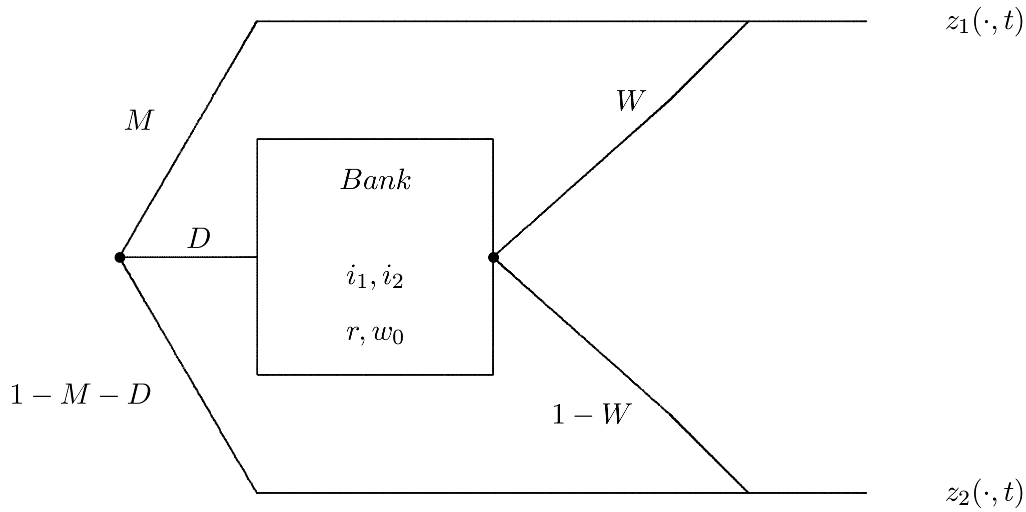

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

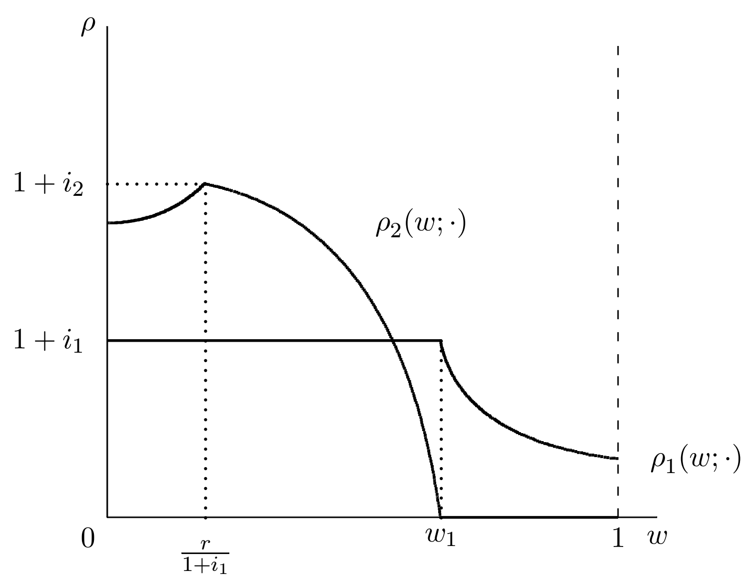

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}&\mathcal{V}_b(D, M; r, w_0; \gamma)\\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau\\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

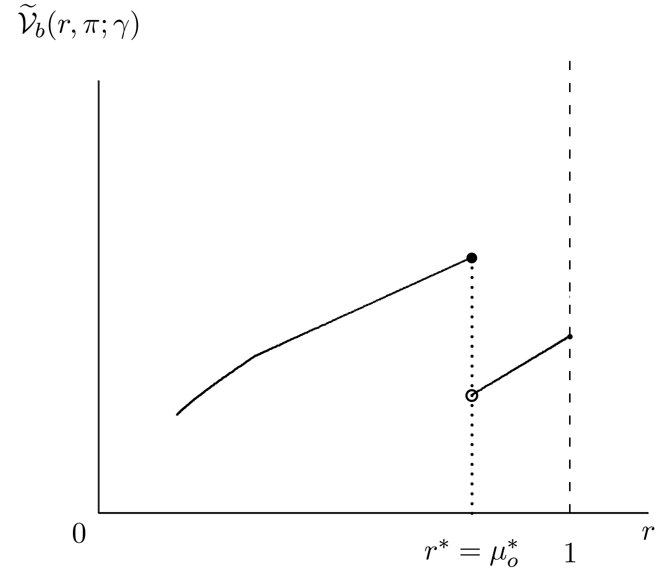

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

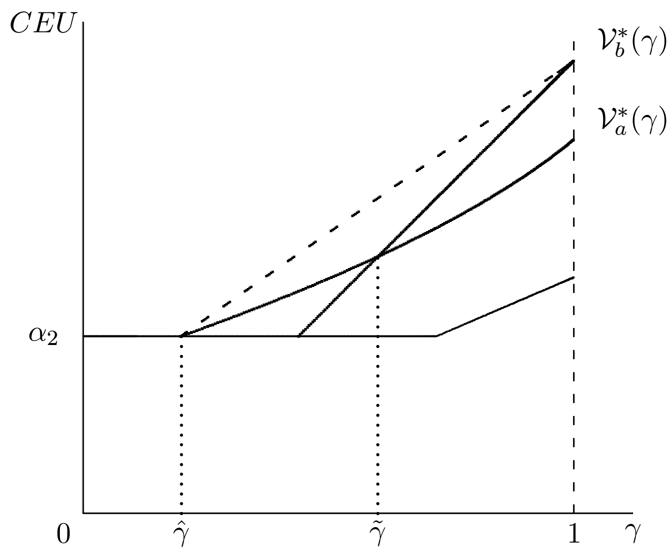

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. CONCLUDING REMARKS

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \ \text{ for all } \ E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S |\ f(s) \geq t\}$ and $\{s \in S |\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S |\ f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S |\ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)] + (1-\gamma)\cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_\ell] + (1-\gamma)\cdot \beta_\ell] + (1-M)\cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0,\underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu),1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0,\underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu),1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0,\overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu),1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0,\overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu),1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau)\cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1+\gamma\cdot(\frac{\beta_h}{\beta_\ell}-1)\cdot\tau]\cdot[\alpha_2\cdot(1-\mu)+\beta_\ell\cdot\mu] & \textit{if} \quad \pi \in [0,\underline{\tau}(\mu)) \\ \alpha_2\cdot(1-\mu) + [\gamma\cdot\pi\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell]\cdot\mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu),1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} &\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ &+ \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) := \;& \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma)\\ =\;& \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell,\\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau\in[0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t,\tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t,τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1-F-M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F,M;\tau,\psi,\varphi,r;h) := \beta_h \cdot [r\cdot F + q_f^*(\tau;\psi,\varphi,r)\cdot F + M] + \alpha_2\cdot(1-F-M),$$

if their type is h, *and*

$$v_f(F,M;\tau,\psi,\varphi,r;\ell) := \alpha_2\cdot[\frac{(M+r\cdot\varphi)}{q_f^*(\tau;\psi,\varphi,r)} + (1-r)] + \alpha_2\cdot(1-F-M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1]\\ s.t.\ F+M\le 1}} \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)]$$
$$+(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau;\psi,\varphi,r)$ *attains its upper bound at* $\frac{\alpha_2\cdot(1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2)\cdot M + (r\cdot\beta_\ell - \alpha_2)\cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) \;+\; (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2\cdot(1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ,φ). Thus, its decision problem for given (ψ,φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \textit{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \textit{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0*, one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period 1, i.e.,* $w=\tau$*, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 *to lie between the type-contingent valuations of the actual return in Period* 1*. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for* $\tau<1$ *the constraint*

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau;r,w_0;\ell) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) < \beta_\ell \cdot \rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) \geq \beta_\ell \cdot \rho_1(\tau;r,w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D,M;r,w_0;\gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau) \cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D,M;r,w_0;\gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r,w_0;\gamma) &:= \max_{\substack{(D,M)\in[0,1]\times[0,1] \\ s.t.\ D+M\leq 1}} \mathcal{V}_b(D,M;r,w_0;\gamma) \\ &= \max\{[\gamma \cdot \{\pi \cdot v_b(\pi;r,w_0;h) + (1-\pi) \cdot v_b(\pi;r,w_0;\ell)\} \\ &\qquad + (1-\gamma) \cdot \min_{\tau\in[0,1]} v_b(\tau;r,w_0;t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$, taking into account the withdrawal behaviour $W(\tau;r,w_0;t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0, 1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu\in[\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\,, \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := &\quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = &\quad \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$z_{2h}(\tau) := \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right]$$
$$= \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h.$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ is a best reply for $r = \mu_\ell$ and $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell$$
$$= \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell.$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ is convex

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi)\cdot \alpha_2 + (1-\gamma)\cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a}\left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}\right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a &:= 2 \cdot \pi \cdot \beta_h \\ b_1 &:= \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 &:= \gamma \cdot (\alpha_2 - \beta_\ell) \\ c &:= 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \;& b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\
= \;& (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\
& -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \;& [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\widetilde{\gamma}) = \mathcal{V}_a^*(\widetilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a, b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a, b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a, b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1-\mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned} \tau \cdot z_{1h}(\tau,\mu) + (1-\tau)\cdot z_{1\ell}(\tau,\mu) &= \mu, \\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu). \end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau}\cdot\frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)} = \mu + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau\cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) = \alpha_2\cdot(1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$.

(iii) $\tau \in [\overline{\tau}, 1]$

For these values of τ there are "too many" $h-$types in the economy. So it is optimal to choose $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$. Even for this choice, however, the proportion of $h-$types is so high, that for $z_{2h}(\tau,\mu) = 0$ the self selection constraint S_h is violated. The payout $z_{2h}(\tau,\mu)$ obtains its lowest value when S_h binds and its highest value when S_ℓ binds. The self selection constraint S_h reads

$$\beta_h \cdot z_{1h}(\tau,\mu) + z_{2h}(\tau,\mu) \geq \beta_h \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu)$$

and after substitution and rearranging we obtain

$$z_{2h}(\tau,\mu) = z_{2\ell}(\tau,\mu) - \beta_h \cdot \frac{\mu}{\tau}.$$

Using $z_{2\ell}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)]$ we find $z_{2h}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)] - \beta_h \cdot \frac{\mu}{\tau}$, from which

$$z_{2h}(\tau,\mu) = \alpha_2 \cdot (1-\mu) - \frac{(1-\tau)}{\tau} \cdot \beta_h \cdot \mu = \alpha_2 - \frac{\mu}{\tau} \cdot (\tau \cdot \alpha_2 + (1-\tau) \cdot \beta_h)$$

and

$$z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu) + \beta_h \cdot \mu$$

follow.

■

Proof of Lemma 14

The structure of this proof is as follows. First the average interim utility for the investment policy μ is determined for $\tau = \pi$. Then the interim utility of the worst case is determined and both are combined to calculate the ex-ante Choquet expected indirect utility of investment policy μ.

(i) Average interim utility

For any $\mu \in [0,1]$ let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13. For any investment policy μ and for $\tau = \pi$ the average interim utility is

$$\widetilde{U}(\pi,\mu) := \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu).$$

From Lemma 12 we have

$$\pi \cdot z_{2h}(\pi,\mu) + (1-\pi)\cdot z_{2\ell}(\pi,\mu) = \alpha_2 \cdot (1-\mu)$$

substitution of which yields

$$\widetilde{U}(\pi,\mu) = \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi)\cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu).$$

(ia) $\pi \in [0, \underline{\tau}(\mu))$

Using Lemma 12 we obtain

$$\begin{aligned}\widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot \left(\frac{\alpha_2\cdot(1-\mu)}{\beta_\ell} + \mu\right) \\ &\quad +(1-\pi)\cdot\beta_\ell\cdot\left(\mu - \frac{\pi}{(1-\pi)}\cdot\frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) + \alpha_2\cdot(1-\mu) \\ &= \left(1 + (\frac{\beta_h}{\beta_\ell} - 1)\cdot \pi\right)\cdot(\alpha_2 + (\beta_\ell - \alpha_2)\cdot\mu)\,.\end{aligned}$$

(ib) $\pi \in [\underline{\tau}(\mu), 1]$

Using Lemma 12 we obtain

$$\begin{aligned}\widetilde{U}(\pi,\mu) &= \pi\cdot\beta_h\cdot z_{1h}(\pi,\mu) + (1-\pi)\cdot\beta_\ell\cdot z_{1\ell}(\pi,\mu) + \alpha_2\cdot(1-\mu) \\ &= \pi\cdot\beta_h\cdot\frac{\mu}{\pi} + \alpha_2\cdot(1-\mu) = \alpha_2 + (\beta_h - \alpha_2)\cdot\mu,\end{aligned}$$

which complete this part of the proof.

(ii) The worst case

The interim utility a type h consumer obtains from payouts (z_h, z_ℓ) that satisfy the self selection constrains is as least as last as the corresponding utility of a type ℓ consumer since $\beta_h > \beta_\ell$. To determine the worst case, we can therefore restrict our attention to a consumer with a low preference for liquidity. The utility of $\ell-$types from payouts that satisfy the self selection constraints for τ is increasing with τ for given μ.

Therefore, for any value of μ the utility of $\ell-$types obtains its minimum value for $\tau = 0$. Using this, we obtain

$$\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\mu,\tau;t) = \beta_\ell \cdot z_{1\ell}(0,\mu) + z_{2\ell}(0,\mu) = \alpha_2\cdot(1-\mu) + \beta_\ell\cdot\mu.$$

(iii) The ex-ante Choquet expected utility

After rearranging terms, from $\pi = \underline{\tau}(\mu)$ we obtain

$$\mu = \frac{\pi \cdot \alpha_2}{(1-\pi)\cdot \beta_\ell + \pi \cdot \alpha_2} = \mu_\ell.$$

For given investment policy μ and given level of confidence γ, substitution of the results of part *(i)* and part *(ii)* of this proof yields

$$\mathcal{V}_o(\mu;\gamma) := \gamma \cdot \widetilde{U}(\pi,\mu) + (1-\gamma)\cdot \widetilde{U}(0,\mu)$$

$$= \begin{cases} (1+\gamma\cdot\left(\frac{\beta_h}{\beta_\ell}-1\right)\cdot\pi)\cdot(\alpha_2+(\beta_\ell-\alpha_2)\cdot\mu) & \text{if} \quad \mu > \mu_\ell \\ \alpha_2 + (\gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell - \alpha_2)\cdot\mu & \text{if} \quad \mu \le \mu_\ell, \end{cases}$$

which completes the proof.

■

Proof of Proposition 16

Denote

$$\hat{v}_a(M;q;t) := M \cdot R^m(q;t) + (1-M)\cdot R^a(q;t).$$

For all $q \in [\frac{\alpha_2}{\beta h}, \frac{\alpha_2}{\beta_\ell}]$ we have $\hat{v}_a(M;q;h) > \hat{v}_a(M;q;\ell)$. Furthermore, $\hat{v}_a(M;q;\ell)$ is decreasing in q. From $q_a^E(\tau,\mu) \le \frac{\alpha_2}{\beta_\ell}$ it now follows that

$$\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \left[\hat{v}_a(M;q_a^E(\tau,\mu);t)\right]$$

$$= \min_{q\in[\frac{\alpha_2}{\beta_h},\frac{\alpha_2}{\beta_\ell}]} \hat{v}_a(M;q;\ell) = M\cdot\beta_\ell + (1-M)\cdot\alpha_2.$$

To simplify notation, we denote $q_a := q_a^E(\pi,\mu)$. From Remark 7 it follows that in an equilibrium with $\mu \in (0,1)$, the asset price $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$ must satisfy

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

(i) $q_a = \frac{\alpha_2}{\beta_\ell}$

In this case consumers anticipate the asset price to be at its upper bound for $\tau = \pi$. By deriving a contradiction, we show this cannot occur in equilibrium. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma\cdot[\pi\cdot\beta_h + (1-\pi)\cdot\beta_\ell] + (1-\gamma)\cdot\beta_\ell = \gamma\cdot\pi\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma \cdot [\pi \cdot \frac{\alpha_2}{\beta_\ell} \cdot \beta_h + (1-\pi)\cdot \alpha_2] + (1-\gamma)\cdot\alpha_2 = \gamma\cdot\pi\cdot\frac{\alpha_2}{\beta_\ell}\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\alpha_2.$$

This implies $\widehat{\mathcal{V}}_a^m(q_a;\gamma) < \widehat{\mathcal{V}}_a^a(q_a;\gamma)$. In this case, any consumer would invest all his wealth in the illiquid asset, i.e. $\mu = 0$. This aggregate investment policy, however, leads to the asset price $q_a = \frac{\alpha_2}{\beta_h}$, which contradicts assumption *(i).*

(ii) $q_a = \frac{\alpha_2}{\beta_h}$

In this case, the consumers anticipate the asset price to obtain its lower bound for $\tau = \pi$. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma \cdot [\pi\cdot\beta_h + (1-\pi)\cdot\beta_h] + (1-\gamma)\cdot\beta_\ell = \gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma\cdot[\pi\cdot\alpha_2 + (1-\pi)\cdot\alpha_2] + (1-\gamma)\cdot\alpha_2 = \alpha_2.$$

This lower bound for the asset price will only be obtained in equilibrium if

$$\alpha_2 \geq \gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell.$$

Rearranging terms, this inequality leads to $\gamma \leq \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}$. From Proposition 10 it follows that for these levels of confidence financial institutions are superfluous form the ex-ante point of view.

(iii) $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$

For any such q_a to be the anticipated equilibrium price in Period 1 for $\tau = \pi$, we must have

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Substitution of $R^m(q_a;h) = \beta_h$, $R^m(q_a;\ell) = \frac{\alpha_2}{q_a}$, $R^a(q_a;h) = \beta_h\cdot q_a$ and $R^a(q_a;\ell) = \alpha_2$ leads to

$$\begin{aligned} & \gamma\cdot\pi\cdot\beta_h + \gamma\cdot(1-\pi)\cdot\frac{1}{q_a}\cdot\alpha_2 + (1-\gamma)\cdot\beta_\ell \\ = \; & \gamma\cdot\pi\cdot\beta_h\cdot q_a + \gamma\cdot(1-\pi)\cdot\alpha_2 + (1-\gamma)\cdot\alpha_2. \end{aligned}$$

Rearranging yields

$$(1-\gamma)\cdot[\alpha_2-\beta_\ell] = \left[\pi\cdot\beta_h + (1-\pi)\cdot\alpha_2\cdot\frac{1}{q_a}\right]\cdot\gamma\cdot(1-q_a).$$

For $\gamma < 1$ we have $q_a < 1$, from which we obtain

$$\pi \cdot \beta_h - (1-\pi) \cdot \alpha_2 + \frac{1}{q_a} \cdot (1-\pi) \cdot \alpha_2 - q_a \cdot \pi \cdot \beta_h = (\frac{1}{\gamma} - 1) \cdot (\alpha_2 - \beta_\ell).$$

Denote the solution of this equation w.r.t. q_a by $\widehat{q}_a(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$.

The derivative of the left hand side of this expression with respect to q_a is

$$-\frac{(1-\pi) \cdot \alpha_2}{(q_a)^2} - \pi \cdot \beta_h < 0.$$

Therefore, the left hand side of the expression is decreasing in q_a.

The right hand side of the expression is decreasing in γ. An increase in the level of ambiguity, which corresponds to a decrease in the level of confidence, increases the right hand side of the equation. To maintain equality, the left hand side of the equation has to increase as well, which leads to a decrease in the asset price q_a, keeping all other parameters constant. The asset price is, however, bound from below by the value $\frac{\alpha_2}{\beta_h}$. After substituting this lower bound for the asset price and solving for the level of confidence γ, we find that $q_a > \frac{\alpha_2}{\beta_h}$, whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.

■

Proof of Lemma 18

If the self selection constraint for type ℓ consumers is violated for the deposit contract (r, π) and withdrawals $w = \pi$, then a bankrun in Period 1 can not be avoided. Hence, all illiquid assets will be liquidated. In this case, it is optimal for the consumers not to deposit their wealth in the bank. This leads to an ex-ante utility of $\mathcal{V}_n^*(\gamma)$.

Therefore, consider deposit contracts (r, π) such that for withdrawals $w = \pi$ the self selection constraint of type ℓ consumers is satisfied. Using $w = \pi$ this leads to the ex-ante Choquet expected utility of

$$
\begin{aligned}
&\mathcal{V}_b(D,M;r,\pi;\gamma) := \\
&\quad \gamma\cdot[\mathbb{E}_\tau\{\tau\cdot v_b(D,M;\tau,r,\pi;h)+(1-\tau)\cdot v_b(D,M;\tau,r,\pi;\ell)\}] \\
&\qquad +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \cdot v_b(D,M;\tau,r,\pi;t) \\
&\quad =\gamma\cdot[\pi\cdot v_b(D,M;\pi,r,\pi;h)+(1-\pi)\cdot v_b(D,M;\pi,r,\pi;\ell)] \\
&\qquad +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \cdot v_b(D,M;\tau,r,\pi;t) \\
&\quad =\gamma\cdot[\pi\cdot[D\cdot\widetilde{v}_b(\pi;r,\pi;h)+(1-D)\cdot v_n(M;h)] \\
&\qquad +(1-\pi)\cdot[D\cdot\widetilde{v}_b(\pi;r,\pi;\ell)+(1-D)\cdot v_n(M;\ell)]] \\
&\quad +(1-\gamma)\cdot\min_{\tau\in[0,1]}\min_{t\in\{h,\ell\}}\{D\cdot\widetilde{v}_b(\tau;r,\pi;t)+(1-D)\cdot v_n(M;t)\} \\
&\quad =\gamma\cdot[D\cdot\mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\}+(1-D)\cdot\mathbb{E}_t\{v_n(M;t)\}] \\
&\quad +(1-\gamma)\cdot\min_{\tau\in[0,1]}[D\cdot\widetilde{v}_b(\tau;r,\pi;\ell)+(1-D)\cdot v_n(M;\ell)] \\
&\quad =D\cdot\left[\gamma\cdot\mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\}+(1-\gamma)\cdot\min_{\tau\in[0,1]}\widetilde{v}_b(\tau;r,\pi;\ell)\right] \\
&\qquad +(1-D)\cdot[\gamma\cdot\mathbb{E}_t\{v_n(M;t)\}+(1-\gamma)\cdot v_n(M;\ell)].
\end{aligned}
$$

For any $\tau\in[0,1]$ we have that $\widetilde{v}_b(\tau;r,\pi;h)>\widetilde{v}_b(\tau;r,\pi;\ell)$ and for every $M\in[0,1]$, that $v_n(M;h)\geq v_n(M;\ell)$. Thus we obtain:

$$
\begin{aligned}
\widetilde{\mathcal{V}}_b(r,\pi;\gamma) := \quad & \max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D,M;r,\pi;\gamma) \\
& \qquad\text{s.t.} \quad D+M\leq 1 \\
=\ & \max_{D\in[0,1]} D\cdot\Big[\gamma\cdot(\pi\cdot\widetilde{v}_b(\pi;r,\pi;h)+(1-\pi)\cdot\widetilde{v}_b(\pi;r,\pi;\ell)) \\
& \qquad +(1-\gamma)\cdot\min_{\tau\in[0,1]}\widetilde{v}_b(\tau;r,\pi;\ell)\Big] \\
& +\max_{M\in[0,1-D]}[\gamma\cdot(\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)\})+(1-\gamma)\cdot v_n(M;\ell)] \\
=\ & \max_{D\in[0,1]} D\cdot\left[\gamma\cdot\mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\}+(1-\gamma)\cdot\min_{\tau\in[0,1]}\widetilde{v}_b(\tau;r,\pi;\ell)\right]+(1-D)\cdot\mathcal{V}_n^*(\gamma).
\end{aligned}
$$

This expression obtains its maximum either for $D=0$ or for $D=1$ or for any value $D\in[0,1]$. From this the result of the Lemma follows immediately.

■

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution* π *is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ *with* $\overline{\gamma} := 1 - \gamma$*, where* ω *denotes the capacity of complete ignorance.*

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}^2_+ \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

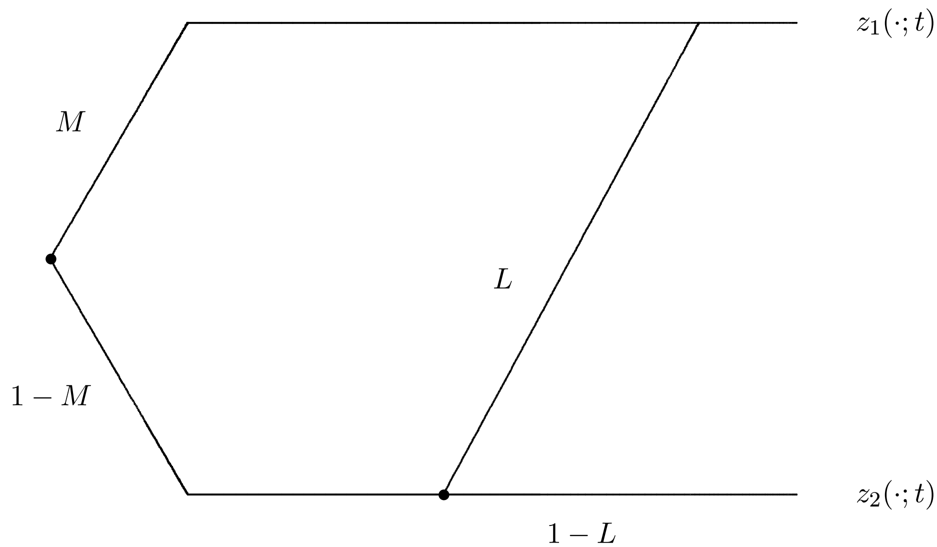

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{array}{ll} z_1(M,L) := & M + \alpha_1 \cdot L \cdot (1-M) \quad \text{and} \\ z_2(M,L) := & \alpha_2 \cdot (1-L) \cdot (1-M). \end{array}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & +(1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) \;=\; \beta_h \cdot M + (1-M)\cdot\alpha_2 \;\geq\; \beta_\ell \cdot M + (1-M)\cdot \alpha_2 \;=\; v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

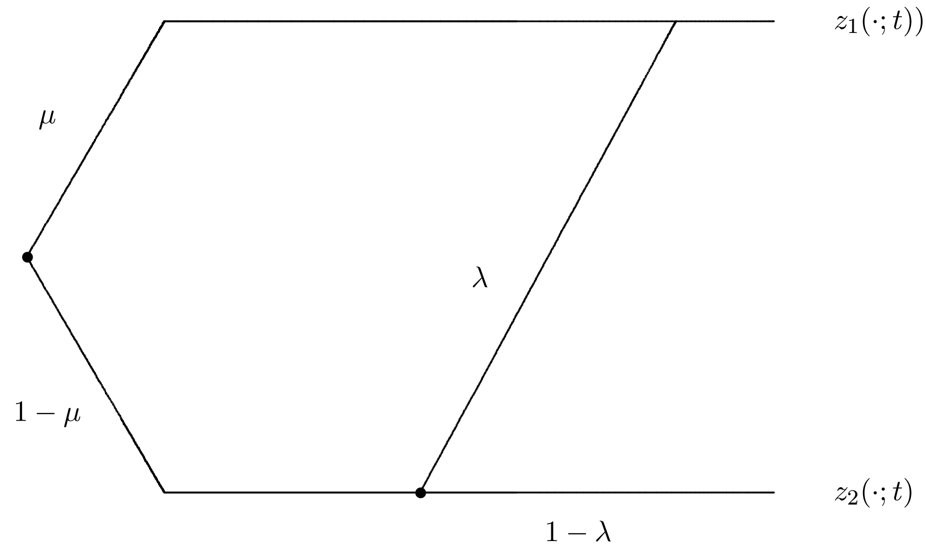

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{lllll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau, \mu; t) &:= u(z_{1t}(\tau, \mu), z_{2t}(\tau, \mu); t) \\ &= \beta_t \cdot z_{1t}(\tau, \mu) + z_{2t}(\tau, \mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu; \gamma) := \gamma \cdot [\pi \cdot v_o(\pi, \mu; h) + (1 - \pi) \cdot v_o(\pi, \mu; \ell)] + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\tau, \mu; t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu \in [0,1]} \quad \mathcal{V}_o(\mu; \gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu; \gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu; \gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

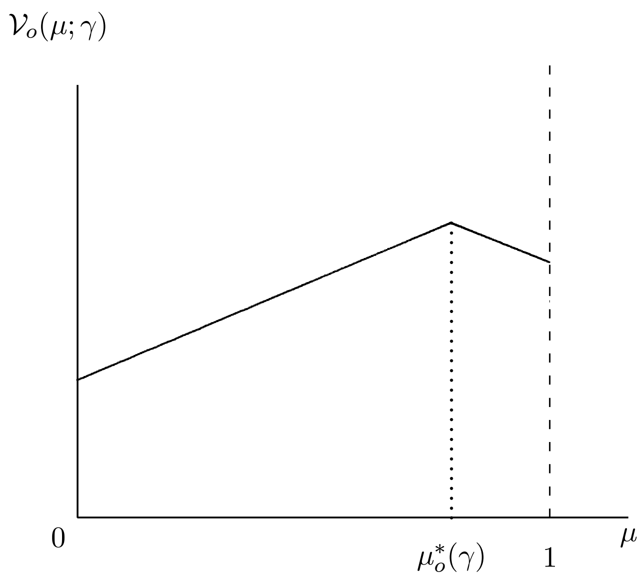

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

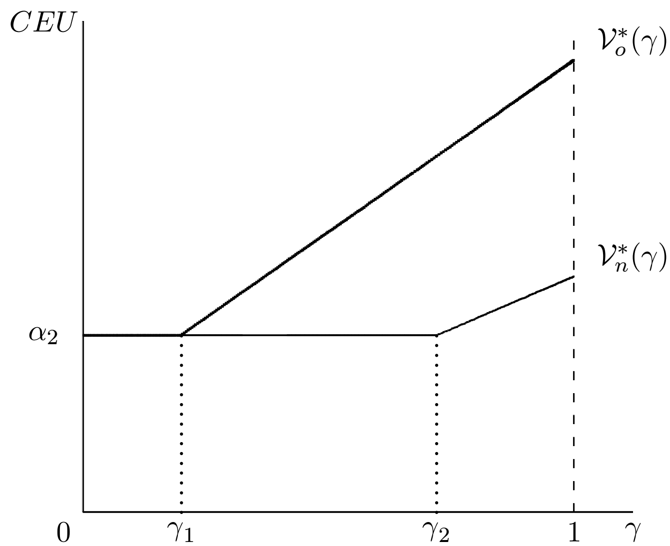

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

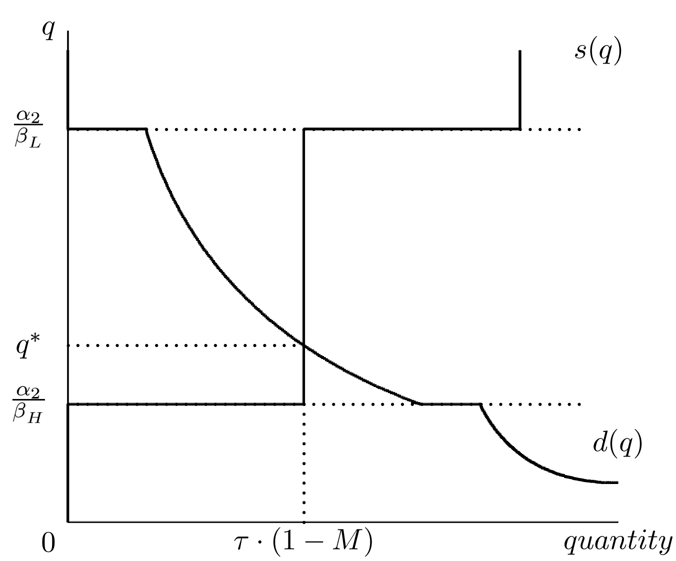

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau, \mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau, \mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

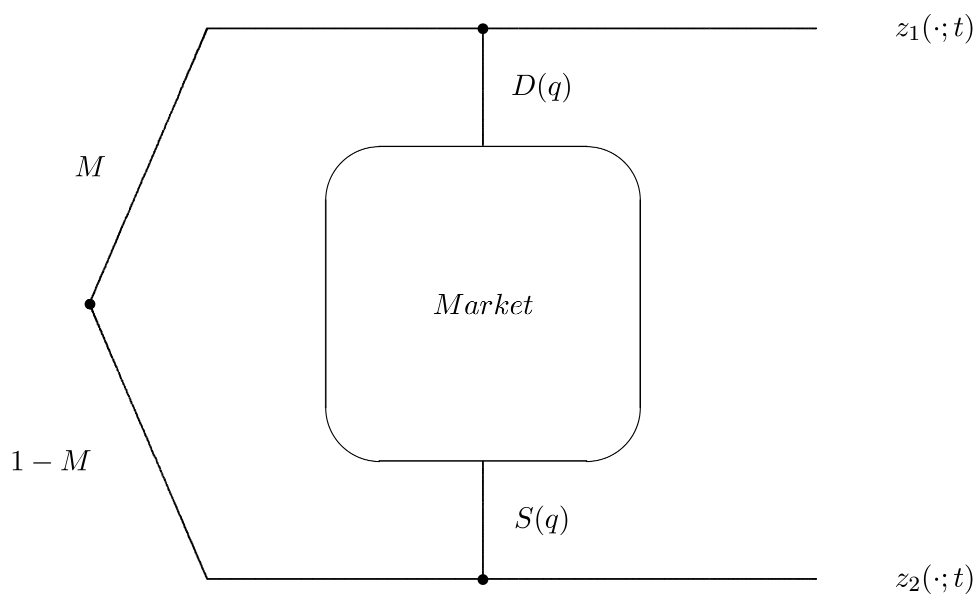

Figure 6: Asset market

Denote by $R^m(q_a; t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a; t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M; \tau, \mu; t)$, from the investment decision M is easily computed:

$$v_a(M; \tau, \mu; t) := M \cdot R^m(q_a^E(\tau, \mu); t) + (1 - M) \cdot R^a(q_a^E(\tau, \mu); t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau, \mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\mathcal{V}_a(M, \mu; \gamma) := \quad \gamma \cdot [\pi \cdot v_a(M; \pi, \mu; h) + (1-\pi) \cdot v_a(M; \pi, \mu; \ell)] \\ + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} [v_a(M; \tau, \mu; t)] \,.$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M, \mu; \gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau, \mu_a^*(\gamma)), \mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau, \mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau \in [0,1]$. We denote the equilibrium asset price for $\tau = \pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M = \mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu^*(\gamma) = \pi,$
- *if* $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}, \pi),$
- *if* $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_h}].$

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

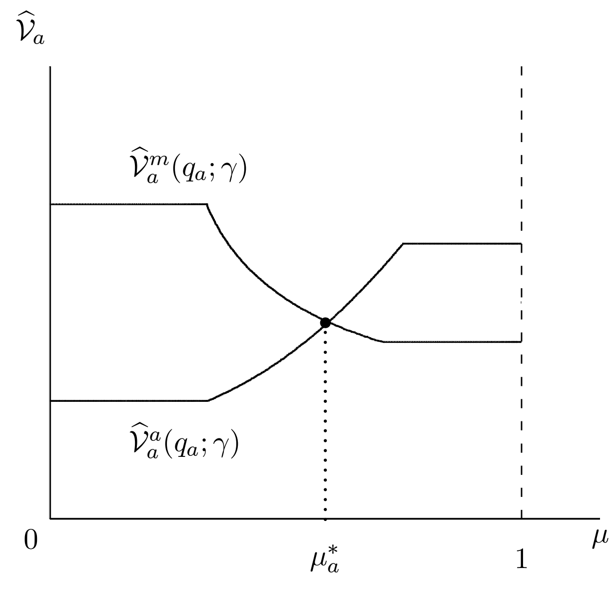

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

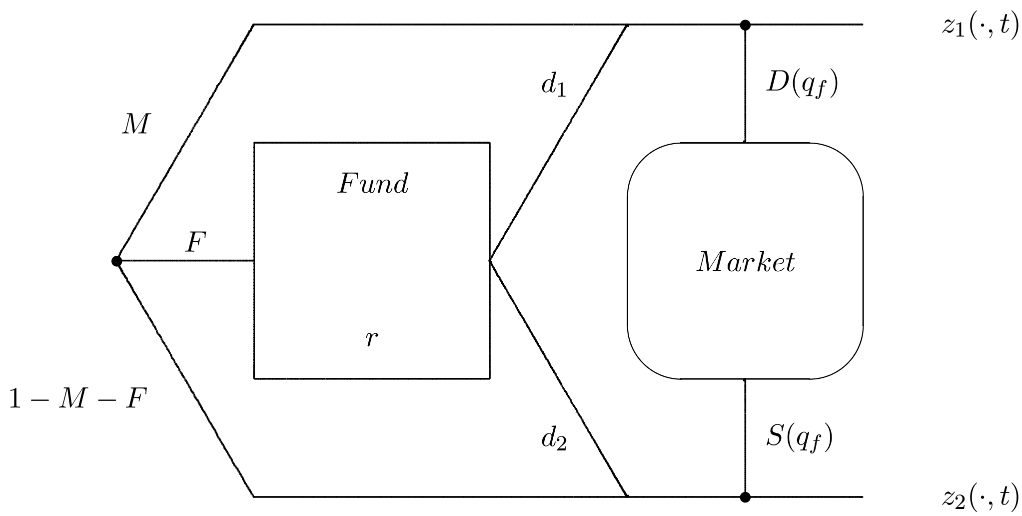

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

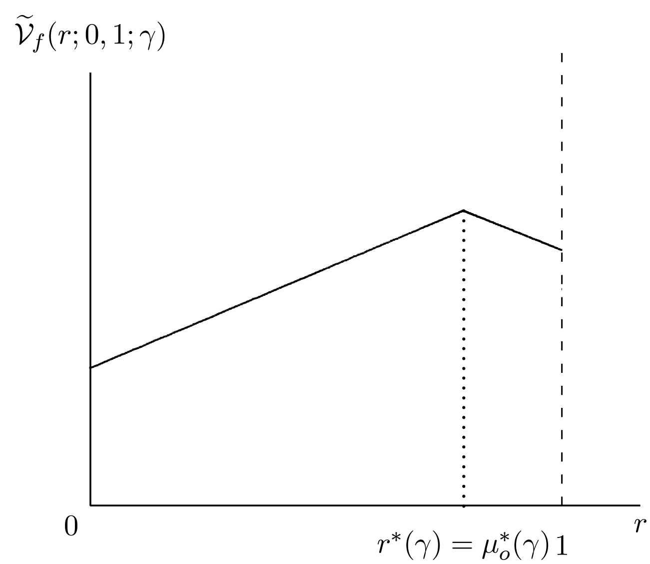

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26]*If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0, 1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27]*If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

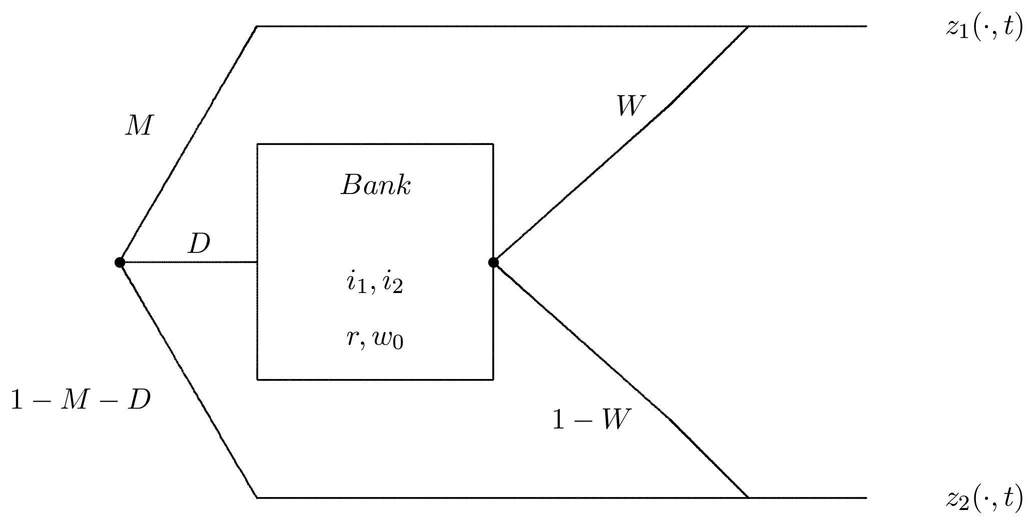

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

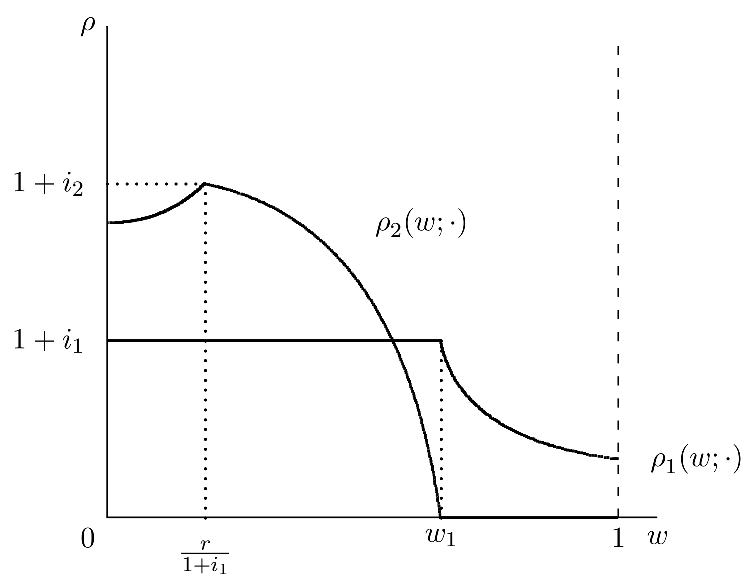

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned}\widetilde{v}_b(\tau; r, w_0; t) &:= \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ &= \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}.\end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}
&\mathcal{V}_b(D, M; r, w_0; \gamma) \\
&:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1 - \tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\
&\qquad + (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t).
\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0, 1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$
$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

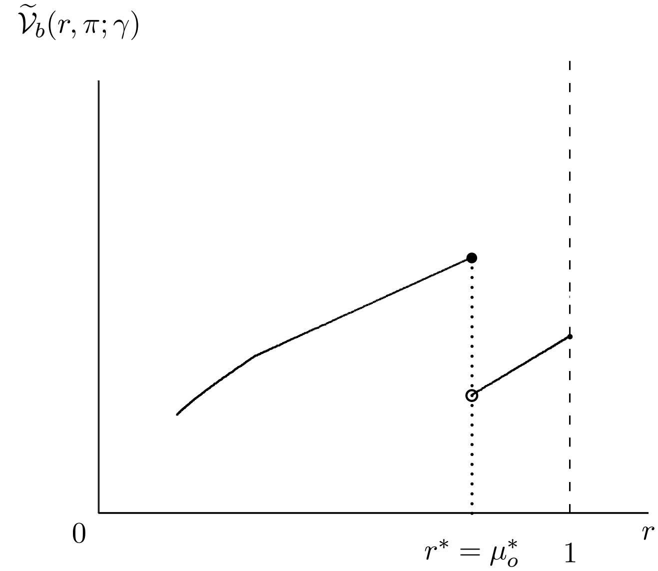

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0, 1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

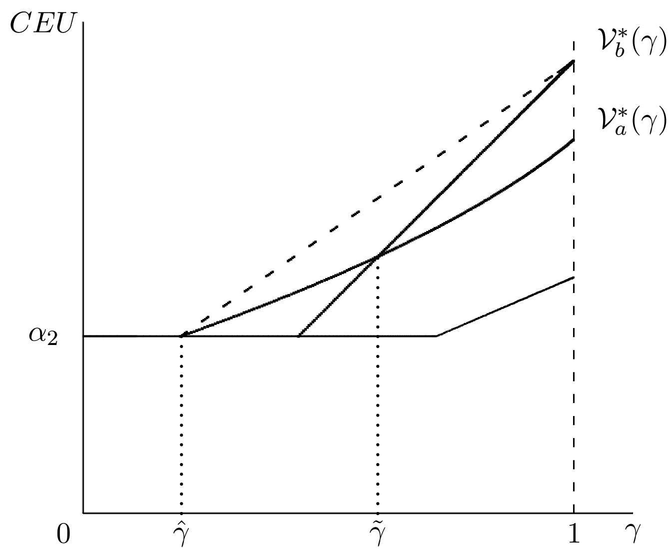

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. CONCLUDING REMARKS

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S |\ f(s) \geq t\}$ and $\{s \in S |\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S |\ f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S |\ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma\cdot[\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)]+(1-\gamma)\cdot\min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M\cdot[\gamma\cdot[\pi\cdot\beta_h+(1-\pi)\cdot\beta_\ell]+(1-\gamma)\cdot\beta_\ell]+(1-M)\cdot\alpha_2\\ &= M\cdot[\gamma\cdot\pi\cdot\beta_h+(1-\gamma\cdot\pi)\cdot\beta_\ell]+(1-M)\cdot\alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu\in[0,1]$ and $\tau\in[0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu)=0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu):=\frac{\beta_\ell\cdot\mu}{\beta_\ell\cdot\mu+\alpha_2\cdot(1-\mu)},$$
$$\overline{\tau}(\mu):=\frac{\beta_h\cdot\mu}{\beta_h\cdot\mu+\alpha_2\cdot(1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu)=\begin{cases}\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}+\mu & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ \dfrac{\mu}{\tau} & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{1\ell}(\tau,\mu)=\begin{cases}\mu-\left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2\cdot(1-\mu)}{\beta_\ell}\right) & \text{if } \tau\in[0,\underline{\tau}(\mu)]\\ 0 & \text{if } \tau\in(\underline{\tau}(\mu),1],\end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ*,* $\mathcal{V}_o(\mu;\gamma)$*, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \textit{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ*. This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$*, depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) := \quad & \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ = \quad & \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \textit{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \textit{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} & \widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ & \quad \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ & \quad + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \, \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma,\pi,\alpha_2,\beta_h,\beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}\mathcal{V}_a^*(\gamma) :=\;& \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma)\\ =\;& \max\{\gamma\cdot\left[\pi\cdot\max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot\max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell,\\ & \gamma\cdot[\pi\cdot\max\{\beta_h\cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot\max\{\beta_\ell\cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot\alpha_2\}.\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \gamma\cdot[\pi\cdot R^m(q_a;h) + (1-\pi)\cdot R^m(q_a;\ell)] + (1-\gamma)\cdot\beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) = \gamma\cdot[\pi\cdot R^a(q_a;h) + (1-\pi)\cdot R^a(q_a;\ell)] + (1-\gamma)\cdot\alpha_2.$$

The worst case for holding money occurs for $(t,\tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t,\tau) = (\ell,\tau)$ for any $\tau \in [0,1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M\cdot\widehat{\mathcal{V}}_a^m(q_a;\gamma) + (1-M)\cdot\widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t,\tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t,τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1-F-M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M) \in [0,1] \times [0,1] \\ s.t.\ F+M \leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+ (1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) \; + \; (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot [r\cdot \beta_\ell + \alpha_2 \cdot (1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ,φ). Thus, its decision problem for given (ψ,φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \textit{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \textit{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot\left[(1-r)-\dfrac{(w-r)}{\alpha_1}\right] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) *are functions of the reserve holding* r *and the predicted withdrawals* w_0*, one can write the actual return functions* $\widehat{\rho}_1(\cdot)$ *and* $\widehat{\rho}_2(\cdot)$ *also as functions of these variables only,*

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type* h *consumers withdraw their funds in Period* 1*, i.e.,* $w=\tau$*, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 *to lie between the type-contingent valuations of the actual return in Period* 1*. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for* $\tau < 1$ *the constraint*

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h *consumers is:*

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau; r, w_0; \ell) = \begin{cases} 1 & \text{if } \rho_2(\tau; r, w_0) < \beta_\ell \cdot \rho_1(\tau; r, w_0) \\ 0 & \text{if } \rho_2(\tau; r, w_0) \geq \beta_\ell \cdot \rho_1(\tau; r, w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\ d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D, M; r, w_0; \gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r, w_0; \gamma) := & \max_{\substack{(D,M) \in [0,1] \times [0,1] \\ s.t.\ D+M \leq 1}} \mathcal{V}_b(D, M; r, w_0; \gamma) \\ = & \max\{[\gamma \cdot \{\pi \cdot v_b(\pi; r, w_0; h) + (1-\pi) \cdot v_b(\pi; r, w_0; \ell)\} \\ & + (1-\gamma) \cdot \min_{\tau \in [0,1]} v_b(\tau; r, w_0; t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r, w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r, w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$, taking into account the withdrawal behaviour $W(\tau; r, w_0; t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

Appendix B: Proofs

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_h + \tau \cdot \alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_\ell + \tau \cdot \alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi+\psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell), \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := &\quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = &\quad \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ is a best reply for $r = \mu_\ell$ and $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a; \gamma) = \widehat{\mathcal{V}}_a^a(q_a; \gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi)\cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell)\cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ is convex

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma)\cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi)\cdot \alpha_2 + (1-\gamma)\cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a}\left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}\right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a &:= 2\cdot \pi \cdot \beta_h \\ b_1 &:= \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 &:= \gamma \cdot (\alpha_2 - \beta_\ell) \\ c &:= 4 \cdot \pi \cdot (1-\pi)\cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned}
& (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\
= \; & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\
= \; & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\
& - (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\
= \; & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2.
\end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\widetilde{\gamma}) = \mathcal{V}_a^*(\widetilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a, b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a, b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a, b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1 - \mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1 - \mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1 - \mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1 - \mu)}{1 - \tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned} \tau \cdot z_{1h}(\tau,\mu) + (1-\tau)\cdot z_{1\ell}(\tau,\mu) &= \mu, \\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu). \end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau}\cdot\frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)} = \mu + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau\cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) = \alpha_2\cdot(1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$.

(iii) $\tau \in [\overline{\tau}, 1]$

For these values of τ there are "too many" $h-$types in the economy. So it is optimal to choose $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$. Even for this choice, however, the proportion of $h-$types is so high, that for $z_{2h}(\tau,\mu) = 0$ the self selection constraint S_h is violated. The payout $z_{2h}(\tau,\mu)$ obtains its lowest value when S_h binds and its highest value when S_ℓ binds. The self selection constraint S_h reads

$$\beta_h \cdot z_{1h}(\tau,\mu) + z_{2h}(\tau,\mu) \geq \beta_h \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu)$$

and after substitution and rearranging we obtain

$$z_{2h}(\tau,\mu) = z_{2\ell}(\tau,\mu) - \beta_h \cdot \frac{\mu}{\tau}.$$

Using $z_{2\ell}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)]$ we find $z_{2h}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)] - \beta_h \cdot \frac{\mu}{\tau}$, from which

$$z_{2h}(\tau,\mu) = \alpha_2 \cdot (1-\mu) - \frac{(1-\tau)}{\tau} \cdot \beta_h \cdot \mu = \alpha_2 - \frac{\mu}{\tau} \cdot (\tau \cdot \alpha_2 + (1-\tau) \cdot \beta_h)$$

and

$$z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu) + \beta_h \cdot \mu$$

follow.

■

Proof of Lemma 14

The structure of this proof is as follows. First the average interim utility for the investment policy μ is determined for $\tau = \pi$. Then the interim utility of the worst case is determined and both are combined to calculate the ex-ante Choquet expected indirect utility of investment policy μ.

(i) Average interim utility

For any $\mu \in [0,1]$ let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13. For any investment policy μ and for $\tau = \pi$ the average interim utility is

$$\widetilde{U}(\pi,\mu) := \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu).$$

From Lemma 12 we have

$$\pi \cdot z_{2h}(\pi,\mu) + (1-\pi)\cdot z_{2\ell}(\pi,\mu) = \alpha_2 \cdot (1-\mu)$$

substitution of which yields

$$\widetilde{U}(\pi,\mu) = \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi)\cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu).$$

(ia) $\pi \in [0, \underline{\tau}(\mu))$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot \left(\frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu\right) \\
&\quad +(1-\pi)\cdot \beta_\ell \cdot \left(\mu - \frac{\pi}{(1-\pi)} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) + \alpha_2 \cdot (1-\mu) \\
&= \left(1 + (\frac{\beta_h}{\beta_\ell} - 1)\cdot \pi\right) \cdot (\alpha_2 + (\beta_\ell - \alpha_2)\cdot \mu)\,.
\end{aligned}$$

(ib) $\pi \in [\underline{\tau}(\mu), 1]$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi)\cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu) \\
&= \pi \cdot \beta_h \cdot \frac{\mu}{\pi} + \alpha_2 \cdot (1-\mu) = \alpha_2 + (\beta_h - \alpha_2)\cdot \mu,
\end{aligned}$$

which complete this part of the proof.

(ii) The worst case

The interim utility a type h consumer obtains from payouts (z_h, z_ℓ) that satisfy the self selection constrains is as least as last as the corresponding utility of a type ℓ consumer since $\beta_h > \beta_\ell$. To determine the worst case, we can therefore restrict our attention to a consumer with a low preference for liquidity. The utility of $\ell-$types from payouts that satisfy the self selection constraints for τ is increasing with τ for given μ.

Therefore, for any value of μ the utility of $\ell-$types obtains its minimum value for $\tau = 0$. Using this, we obtain

$$\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\mu,\tau;t) = \beta_\ell \cdot z_{1\ell}(0,\mu) + z_{2\ell}(0,\mu) = \alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu.$$

(iii) The ex-ante Choquet expected utility

After rearranging terms, from $\pi = \underline{\tau}(\mu)$ we obtain

$$\mu = \frac{\pi \cdot \alpha_2}{(1-\pi)\cdot \beta_\ell + \pi \cdot \alpha_2} = \mu_\ell.$$

For given investment policy μ and given level of confidence γ, substitution of the results of part *(i)* and part *(ii)* of this proof yields

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) &:= \gamma \cdot \widetilde{U}(\pi,\mu) + (1-\gamma)\cdot \widetilde{U}(0,\mu) \\ &= \begin{cases} (1+\gamma\cdot\left(\frac{\beta_h}{\beta_\ell}-1\right)\cdot\pi)\cdot(\alpha_2+(\beta_\ell-\alpha_2)\cdot\mu) & \text{if} \quad \mu > \mu_\ell \\ \\ \alpha_2 + (\gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell - \alpha_2)\cdot\mu & \text{if} \quad \mu \leq \mu_\ell, \end{cases} \end{aligned}$$

which completes the proof.

■

Proof of Proposition 16

Denote

$$\hat{v}_a(M;q;t) := M\cdot R^m(q;t) + (1-M)\cdot R^a(q;t).$$

For all $q \in [\frac{\alpha_2}{\beta h}, \frac{\alpha_2}{\beta_\ell}]$ we have $\hat{v}_a(M;q;h) > \hat{v}_a(M;q;\ell)$. Furthermore, $\hat{v}_a(M;q;\ell)$ is decreasing in q. From $q_a^E(\tau,\mu) \leq \frac{\alpha_2}{\beta_\ell}$ it now follows that

$$\begin{aligned} &\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \left[\hat{v}_a(M;q_a^E(\tau,\mu);t)\right] \\ = &\min_{q\in[\frac{\alpha_2}{\beta_h},\frac{\alpha_2}{\beta_\ell}]} \hat{v}_a(M;q;\ell) = M\cdot\beta_\ell + (1-M)\cdot\alpha_2. \end{aligned}$$

To simplify notation, we denote $q_a := q_a^E(\pi,\mu)$. From Remark 7 it follows that in an equilibrium with $\mu \in (0,1)$, the asset price $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$ must satisfy

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

(i) $q_a = \frac{\alpha_2}{\beta_\ell}$

In this case consumers anticipate the asset price to be at its upper bound for $\tau = \pi$. By deriving a contradiction, we show this cannot occur in equilibrium. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma\cdot[\pi\cdot\beta_h + (1-\pi)\cdot\beta_\ell] + (1-\gamma)\cdot\beta_\ell = \gamma\cdot\pi\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma \cdot [\pi \cdot \frac{\alpha_2}{\beta_\ell} \cdot \beta_h + (1-\pi)\cdot \alpha_2] + (1-\gamma)\cdot \alpha_2 = \gamma \cdot \pi \cdot \frac{\alpha_2}{\beta_\ell} \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \alpha_2.$$

This implies $\widehat{\mathcal{V}}_a^m(q_a;\gamma) < \widehat{\mathcal{V}}_a^a(q_a;\gamma)$. In this case, any consumer would invest all his wealth in the illiquid asset, i.e. $\mu = 0$. This aggregate investment policy, however, leads to the asset price $q_a = \frac{\alpha_2}{\beta_h}$, which contradicts assumption *(i)*.

(ii) $q_a = \frac{\alpha_2}{\beta_h}$

In this case, the consumers anticipate the asset price to obtain its lower bound for $\tau = \pi$. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_h] + (1-\gamma)\cdot \beta_\ell = \gamma \cdot \beta_h + (1-\gamma)\cdot \beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma \cdot [\pi \cdot \alpha_2 + (1-\pi)\cdot \alpha_2] + (1-\gamma)\cdot \alpha_2 = \alpha_2.$$

This lower bound for the asset price will only be obtained in equilibrium if

$$\alpha_2 \geq \gamma \cdot \beta_h + (1-\gamma)\cdot \beta_\ell.$$

Rearranging terms, this inequality leads to $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. From Proposition 10 it follows that for these levels of confidence financial institutions are superfluous form the ex-ante point of view.

(iii) $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$

For any such q_a to be the anticipated equilibrium price in Period 1 for $\tau = \pi$, we must have

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Substitution of $R^m(q_a;h) = \beta_h$, $R^m(q_a;\ell) = \frac{\alpha_2}{q_a}$, $R^a(q_a;h) = \beta_h \cdot q_a$ and $R^a(q_a;\ell) = \alpha_2$ leads to

$$\begin{aligned} & \gamma \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi)\cdot \frac{1}{q_a} \cdot \alpha_2 + (1-\gamma)\cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h \cdot q_a + \gamma \cdot (1-\pi)\cdot \alpha_2 + (1-\gamma)\cdot \alpha_2. \end{aligned}$$

Rearranging yields

$$(1-\gamma)\cdot[\alpha_2 - \beta_\ell] = \left[\pi \cdot \beta_h + (1-\pi)\cdot \alpha_2 \cdot \frac{1}{q_a}\right] \cdot \gamma \cdot (1-q_a).$$

For $\gamma < 1$ we have $q_a < 1$, from which we obtain

$$\pi \cdot \beta_h - (1-\pi) \cdot \alpha_2 + \frac{1}{q_a} \cdot (1-\pi) \cdot \alpha_2 - q_a \cdot \pi \cdot \beta_h = (\frac{1}{\gamma} - 1) \cdot (\alpha_2 - \beta_\ell).$$

Denote the solution of this equation w.r.t. q_a by $\widehat{q}_a(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$.

The derivative of the left hand side of this expression with respect to q_a is

$$-\frac{(1-\pi) \cdot \alpha_2}{(q_a)^2} - \pi \cdot \beta_h < 0.$$

Therefore, the left hand side of the expression is decreasing in q_a.

The right hand side of the expression is decreasing in γ. An increase in the level of ambiguity, which corresponds to a decrease in the level of confidence, increases the right hand side of the equation. To maintain equality, the left hand side of the equation has to increase as well, which leads to a decrease in the asset price q_a, keeping all other parameters constant. The asset price is, however, bound from below by the value $\frac{\alpha_2}{\beta_h}$. After substituting this lower bound for the asset price and solving for the level of confidence γ, we find that $q_a > \frac{\alpha_2}{\beta_h}$, whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.

■

Proof of Lemma 18

If the self selection constraint for type ℓ consumers is violated for the deposit contract (r, π) and withdrawals $w = \pi$, then a bankrun in Period 1 can not be avoided. Hence, all illiquid assets will be liquidated. In this case, it is optimal for the consumers not to deposit their wealth in the bank. This leads to an ex-ante utility of $\mathcal{V}_n^*(\gamma)$.

Therefore, consider deposit contracts (r, π) such that for withdrawals $w = \pi$ the self selection constraint of type ℓ consumers is satisfied. Using $w = \pi$ this leads to the ex-ante Choquet expected utility of

$$
\begin{aligned}
&\mathcal{V}_b(D,M;r,\pi;\gamma) := \\
&\quad \gamma\cdot[\mathbb{E}_\tau\{\tau\cdot v_b(D,M;\tau,r,\pi;h)+(1-\tau)\cdot v_b(D,M;\tau,r,\pi;\ell)\}] \\
&\qquad\qquad +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \cdot v_b(D,M;\tau,r,\pi;t) \\
&\quad = \gamma\cdot[\pi\cdot v_b(D,M;\pi,r,\pi;h)+(1-\pi)\cdot v_b(D,M;\pi,r,\pi;\ell)] \\
&\qquad\qquad +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \cdot v_b(D,M;\tau,r,\pi;t) \\
&\quad = \gamma\cdot[\pi\cdot[D\cdot\widetilde{v}_b(\pi;r,\pi;h)+(1-D)\cdot v_n(M;h)] \\
&\qquad\qquad +(1-\pi)\cdot[D\cdot\widetilde{v}_b(\pi;r,\pi;\ell)+(1-D)\cdot v_n(M;\ell)]] \\
&\quad +(1-\gamma)\cdot \min_{\tau\in[0,1]}\min_{t\in\{h,\ell\}}\{D\cdot\widetilde{v}_b(\tau;r,\pi;t)+(1-D)\cdot v_n(M;t)\} \\
&\quad = \gamma\cdot[D\cdot\mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\}+(1-D)\cdot\mathbb{E}_t\{v_n(M;t)\}] \\
&\quad +(1-\gamma)\cdot \min_{\tau\in[0,1]}[D\cdot\widetilde{v}_b(\tau;r,\pi;\ell)+(1-D)\cdot v_n(M;\ell)] \\
&\quad = D\cdot\left[\gamma\cdot\mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\}+(1-\gamma)\cdot\min_{\tau\in[0,1]}\widetilde{v}_b(\tau;r,\pi;\ell)\right] \\
&\qquad +(1-D)\cdot[\gamma\cdot\mathbb{E}_t\{v_n(M;t)\}+(1-\gamma)\cdot v_n(M;\ell)].
\end{aligned}
$$

For any $\tau\in[0,1]$ we have that $\widetilde{v}_b(\tau;r,\pi;h)>\widetilde{v}_b(\tau;r,\pi;\ell)$ and for every $M\in[0,1]$, that $v_n(M;h)\geq v_n(M;\ell)$. Thus we obtain:

$$
\begin{aligned}
\widetilde{\mathcal{V}}_b(r,\pi;\gamma) := &\max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D,M;r,\pi;\gamma) \\
&\qquad\qquad \text{s.t.} \quad D+M\leq 1 \\
= &\max_{D\in[0,1]} D\cdot\Big[\gamma\cdot(\pi\cdot\widetilde{v}_b(\pi;r,\pi;h)+(1-\pi)\cdot\widetilde{v}_b(\pi;r,\pi;\ell)) \\
&\qquad\qquad +(1-\gamma)\cdot\min_{\tau\in[0,1]}\widetilde{v}_b(\tau;r,\pi;\ell)\Big] \\
+ &\max_{M\in[0,1-D]}[\gamma\cdot(\pi\cdot v_n(M;h)+(1-\pi)\cdot v_n(M;\ell)\})+(1-\gamma)\cdot v_n(M;\ell)] \\
= &\max_{D\in[0,1]} D\cdot\left[\gamma\cdot\mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\}+(1-\gamma)\cdot\min_{\tau\in[0,1]}\widetilde{v}_b(\tau;r,\pi;\ell)\right]+(1-D)\cdot\mathcal{V}_n^*(\gamma).
\end{aligned}
$$

This expression obtains its maximum either for $D=0$ or for $D=1$ or for any value $D\in[0,1]$. From this the result of the Lemma follows immediately.

■

Proof of Lemma 19

This proof consists of two steps. Firstly, for any reserves r, the optimal payouts in Period 1 are determined. The second step consist of finding the optimal reserve policy, under the assumption that separating the two types of consumers is worthwhile. If the separation of types is not worth while, one either obtains the reserves as derived here and deposits $\delta = 0$, or the bank hold reserves $r = \mu_n^*(\gamma)$.

(i) $w_0 = \pi$.

For given reserves r, the representative bank is to choose the interest payments $(i_1(r, w_0)$, $(i_2(r, w_0))$. Under the zero-profit condition, this is equivalent to choosing its predicted withdrawals w_0. Therefore, its decision problem is

$$\max_{w_0 \in [\underline{\tau}(r), \overline{\tau}(r)]} \gamma \cdot [\pi \cdot \beta_h \cdot \rho_1(\pi; r, w_0) + (1-\pi) \cdot \rho_2(\pi; r, w_0)] + (1-\gamma) \cdot \beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r).$$

Rearranging, dividing by γ and disregarding the constant term to $(1-\gamma)$ yields

$$\max_{w_0 \in [\underline{\tau}(r), \overline{\tau}(r)]} \pi \cdot \beta_h \cdot \rho_1(\pi; r, w_0) + (1-\pi) \cdot \rho_2(\pi; r, w_0).$$

This problem has an interior solution, which does not depend on γ and which is obtained for $w_0 = \pi$.

(ii) Optimal reserves r

Denote

$$\mu_h := \frac{\pi \cdot \alpha_2}{(1-\pi) \cdot \beta_\ell + \pi \cdot \alpha_2}.$$

Using the result from part *(i)* of this proof, the decision problem of the bank reduces to

$$\max_{r \in [\mu_h, \mu_\ell]} \gamma \cdot [\pi \cdot \beta_h \cdot \frac{r}{\pi} + (1-\pi) \cdot \frac{\alpha_2 \cdot (1-r)}{(1-\pi)}] + (1-\gamma) \cdot \beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r),$$

where $[\mu_h, \mu_\ell]$ denotes the set of reserves that are compatible with self selection if $\tau = \pi$. After rearranging the objective function can be written as

$$[\gamma \cdot (\beta_h - \alpha_2) + (1-\gamma) \cdot \beta_\ell \cdot (1-\alpha_1)] \cdot r + \gamma \cdot \alpha_2 + (1-\gamma) \cdot \beta_\ell \cdot \alpha_1.$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.

[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ where π is an additive probability distribution on a compact set S. The Choquet integral of a continuous function f on S has the following form:

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}_+^2 \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} \textit{(i)} & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ \textit{(ii)} & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

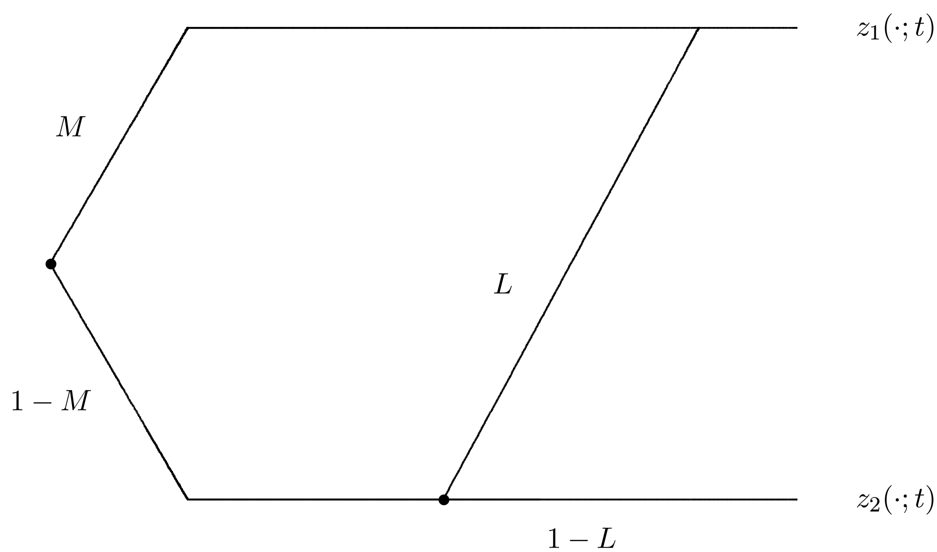

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1 - M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \textit{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1 - \tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell) \\ &= M\cdot[\tau\cdot\beta_h + (1-\tau)\cdot\beta_\ell] + (1-M)\cdot\alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma\cdot\int_0^1 [\tau\cdot v_n(M;h) + (1-\tau)\cdot v_n(M;\ell)]\cdot p(\tau)\ d\tau \\ & +(1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_n(M;t) \\ = \quad & \gamma\cdot[\mathbb{E}_p\{\tau\}\cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\})\cdot v_n(M;\ell)] \\ & +(1-\gamma)\cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \text{for} \quad \tau = \pi \\ 0 & \text{otherwise.}\end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h\cdot M + (1-M)\cdot\alpha_2 \geq \beta_\ell\cdot M + (1-M)\cdot\alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \text{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \text{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \text{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2\,,\, \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

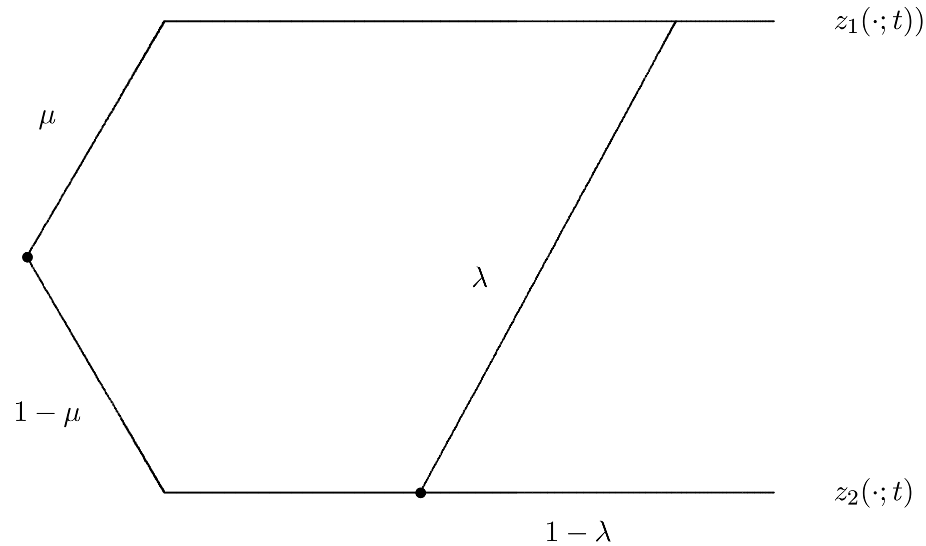

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) :=& \quad \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ :=& \quad \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \text{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \text{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \text{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \text{F}_2 \\
& \lambda \geq 0, \ z_{1h} \geq 0, \ z_{1\ell} \geq 0, \ z_{2h} \geq 0, \ z_{2\ell} \geq 0. & & & \text{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu); t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

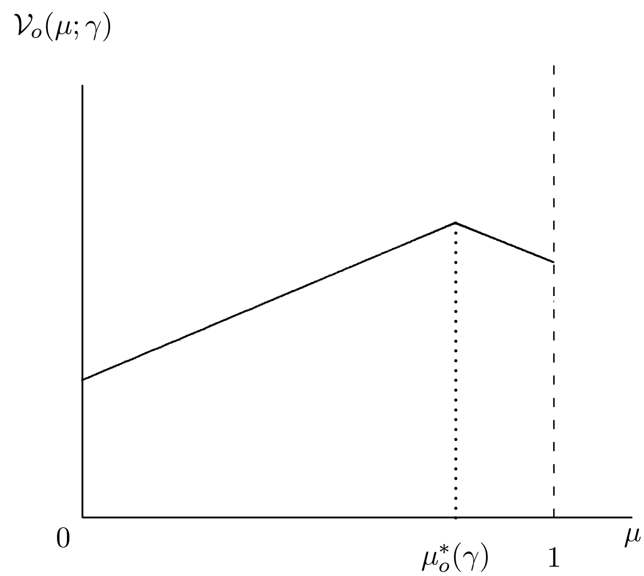

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

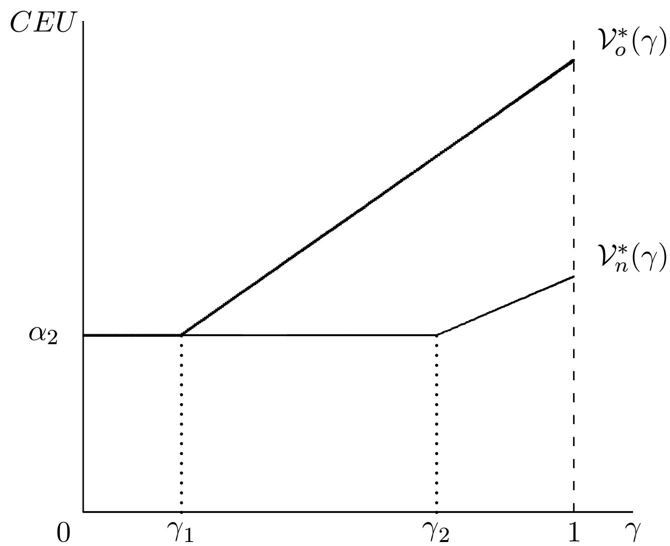

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau\cdot(1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau\cdot(1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau\cdot(1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau)\cdot\frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau)\cdot\frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau)\cdot\frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

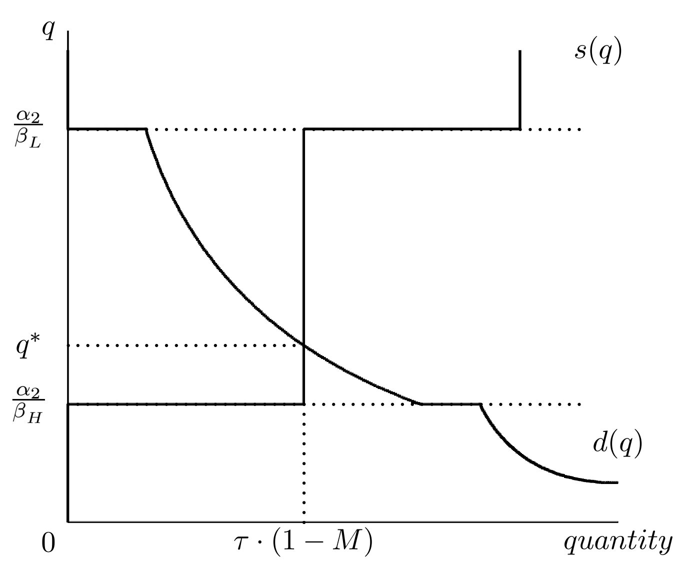

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau,\mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2} \\ \frac{1-\tau}{\tau}\cdot\frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}, \frac{\mu\cdot\beta_\ell}{\mu\cdot\beta_\ell+(1-\mu)\cdot\alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu\cdot\beta_h}{\mu\cdot\beta_h+(1-\mu)\cdot\alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau,\mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

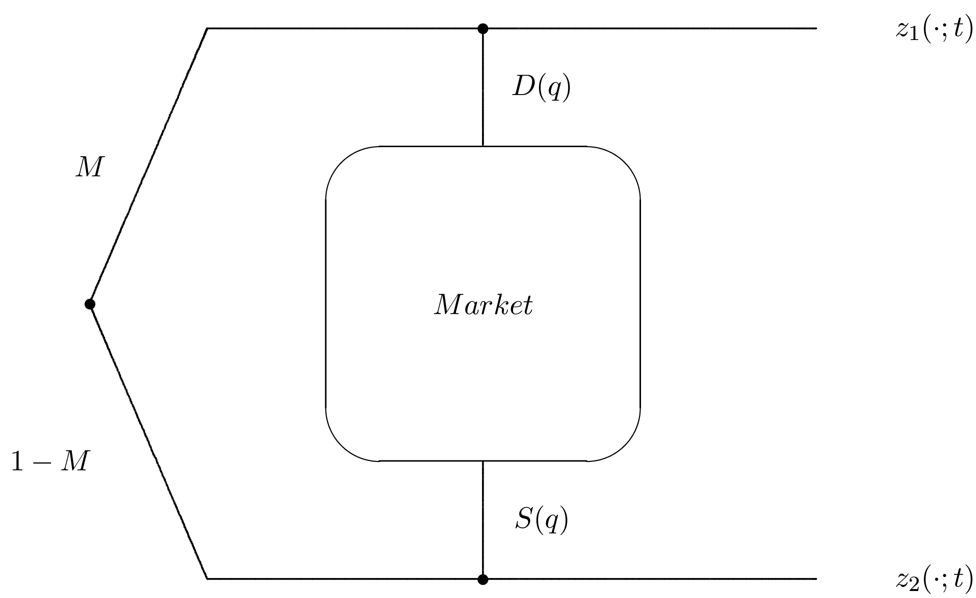

Figure 6: Asset market

Denote by $R^m(q_a;t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a;t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M;\tau,\mu;t)$, from the investment decision M is easily computed:

$$v_a(M;\tau,\mu;t) := M \cdot R^m(q_a^E(\tau,\mu);t) + (1-M)\cdot R^a(q_a^E(\tau,\mu);t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h) + (1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ *in the asset market which satisfies:*

- *if* $\gamma=1$, *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi$,
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1)$, *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi)$,
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}]$, *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}]$.

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}^a_a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

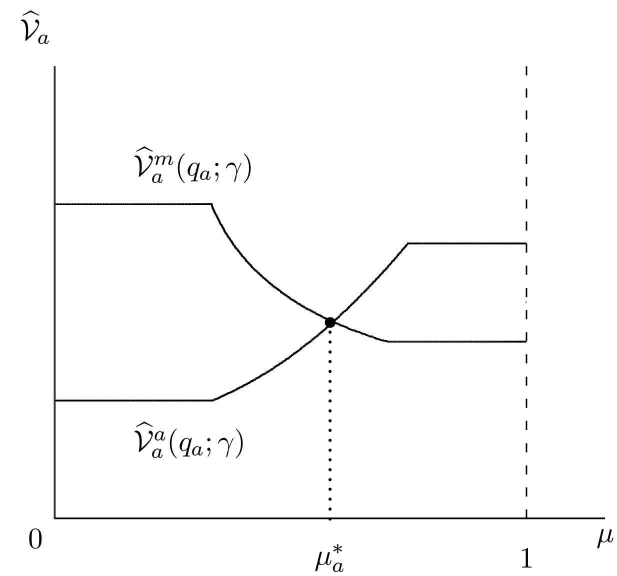

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

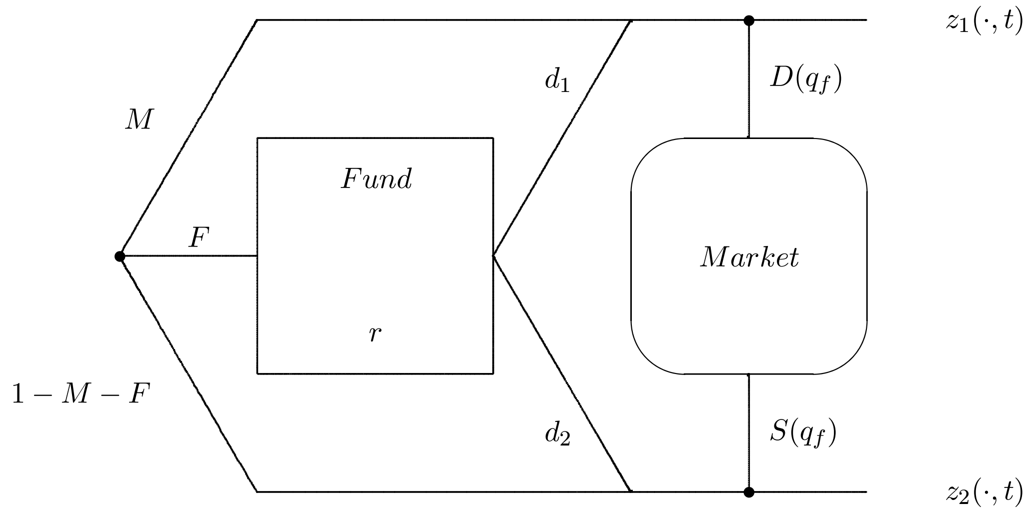

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \quad \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

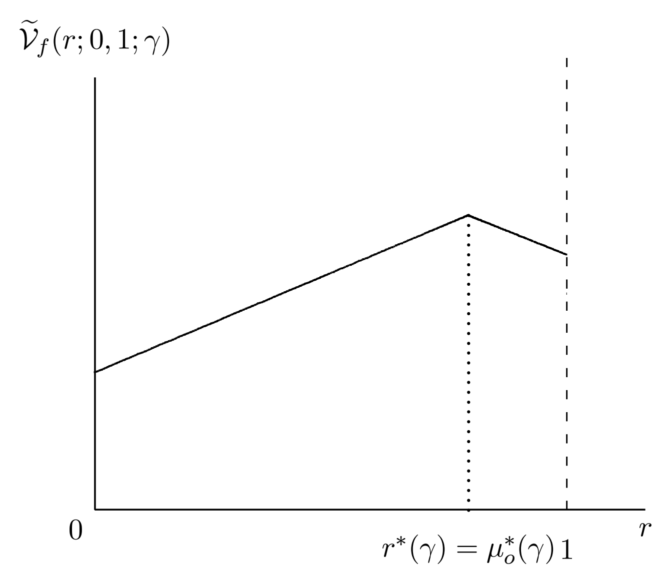

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with* $\varphi^* = 1$ *and* $r^*(\gamma) = \mu_o^*(\gamma)$.

For all $\gamma \in [0, 1]$ *the corresponding ex-ante Choquet expected utility is*

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium* (ψ^*, φ^*, r^*) *we have* $\varphi^* + \psi^* = \mu_a^*(\gamma)$.

The ex-ante expected utility is in this case, for all $\gamma \in [0, 1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.
[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

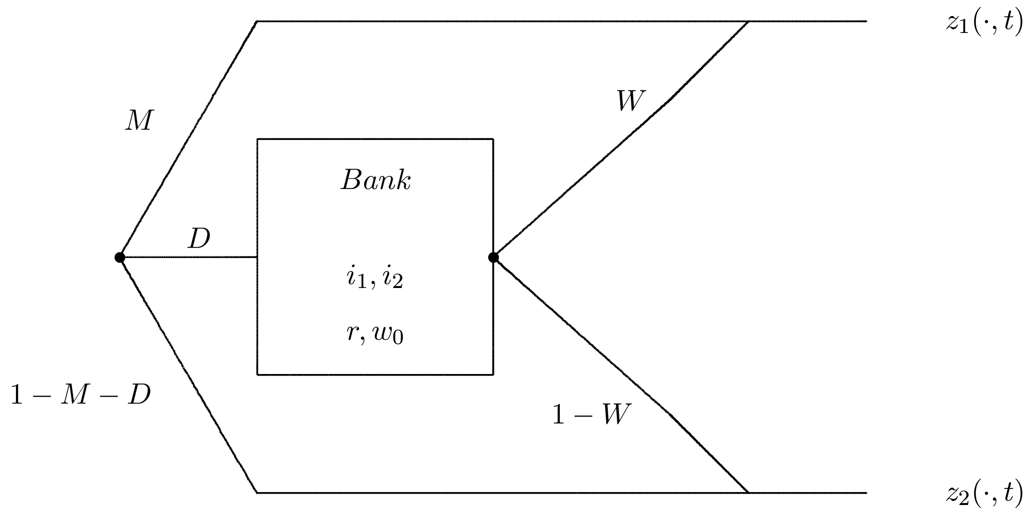

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

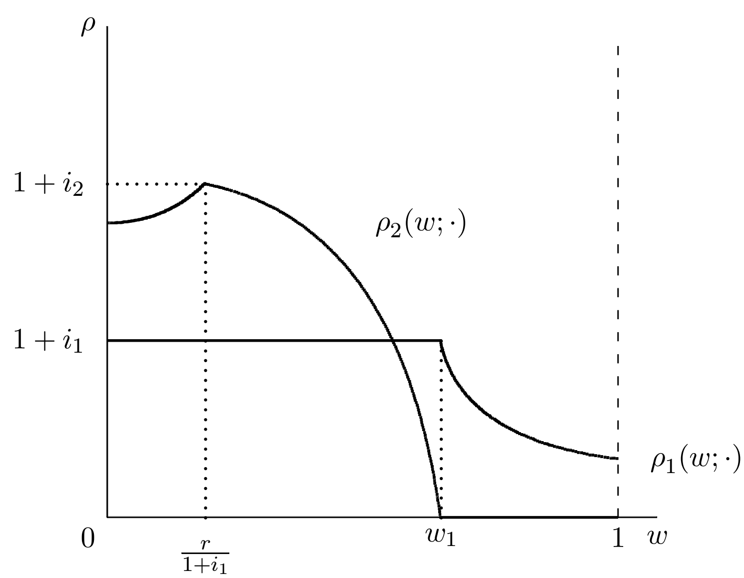

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned}&\mathcal{V}_b(D, M; r, w_0; \gamma)\\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D, M; \tau, r, w_0; t).\end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M)\in[0,1]\times[0,1]} \mathcal{V}_b(D, M; r, w_0; \gamma) \quad \text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

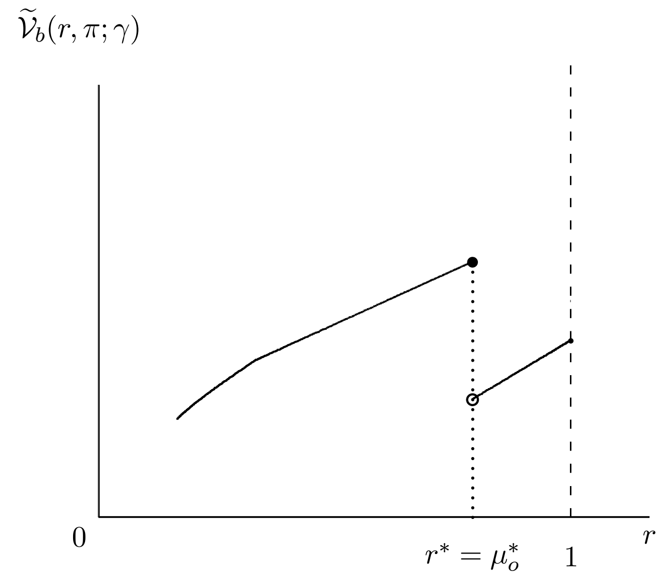

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \textit{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \textit{if} \quad \gamma = \overline{\gamma} \\ 0 & \textit{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := & \quad \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = & \quad \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

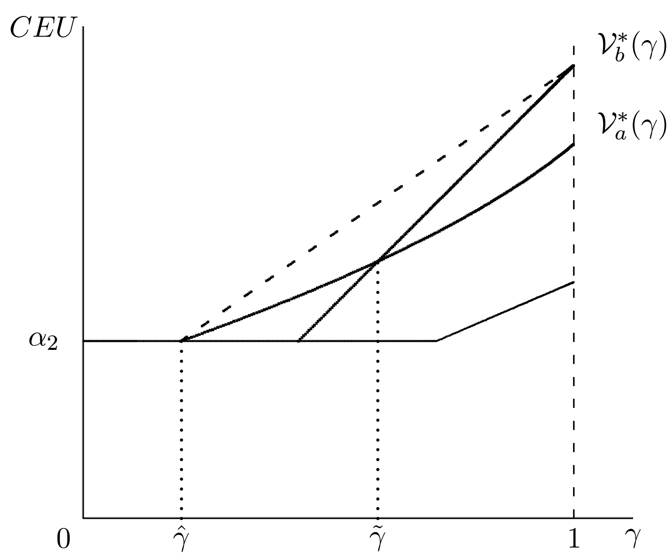

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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APPENDIX A: RESULTS AND REMARKS

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A* ***capacity*** *is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A* ***probability distribution*** *which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of* ***complete ignorance*** *ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S |\ f(s) \geq t\}$ and $\{s \in S |\ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the* ***Choquet integral*** *with respect to the capacity ν is defined as*

$$\int^{C} f \, d\nu = \int_0^{\infty} \nu(\{s \in S |\ f(s) \geq t\}) \, dt + \int_{-\infty}^{0} [\nu(\{s \in S |\ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)] + (1-\gamma)\cdot \min\{v_n(M;h),\ v_n(M;\ell)\} \\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_\ell] + (1-\gamma)\cdot \beta_\ell] + (1-M)\cdot \alpha_2 \\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right)\cdot\left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\dfrac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment μ, $\mathcal{V}_o(\mu;\gamma)$, has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any $\mu \in [0,1]$ we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \textit{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy μ is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ denotes the Choquet expected value of the indirect utility from the investment policy μ. This problem is complicated because the critical population shares of type h consumers, $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$, depend on the investment policy.

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned}\mathcal{V}_o^*(\gamma) :=&\quad \mathcal{V}_o(\mu^*(\gamma);\gamma) \\ =&\quad \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell)\cdot\pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \le \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}\end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma;\pi,\alpha_2,\beta_h,\beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned}&\widehat{q}_a^E(\gamma;\pi,\alpha_2,\beta_h,\beta_\ell) := \frac{1}{2\cdot\pi\cdot\beta_h}\cdot \\ &\Big[-\Big(\alpha_2 - \pi\cdot(\alpha_2+\beta_h) + \tfrac{1-\gamma}{\gamma}\cdot(\alpha_2-\beta_\ell)\Big) \\ &+\sqrt{\Big(\alpha_2 - \pi\cdot(\alpha_2+\beta_h) + \tfrac{1-\gamma}{\gamma}\cdot(\alpha_2-\beta_\ell)\Big)^2 + 4\cdot\pi\cdot(1-\pi)\cdot\alpha_2\cdot\beta_h}\,\Big].\end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\, \mu_a^*(\gamma))$ in the asset market exists and the price

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma, \pi, \alpha_2, \beta_h, \beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi)\cdot \beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi)\cdot \beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned} \mathcal{V}_a^*(\gamma) := \quad & \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\ = \quad & \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot \beta_\ell, \\ & \gamma \cdot [\pi \cdot \max\{\beta_h \cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot \max\{\beta_\ell \cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot \alpha_2\}. \end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a; \gamma) = \gamma \cdot [\pi \cdot R^m(q_a; h) + (1-\pi)\cdot R^m(q_a; \ell)] + (1-\gamma)\cdot \beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a; \gamma) = \gamma \cdot [\pi \cdot R^a(q_a; h) + (1-\pi)\cdot R^a(q_a; \ell)] + (1-\gamma)\cdot \alpha_2.$$

The worst case for holding money occurs for $(t, \tau) = (\ell, 0)$. For investing in the illiquid asset it occurs for $(t, \tau) = (\ell, \tau)$ for any $\tau \in [0, 1]$. From this it follows that for an asset price q_a the ex-ante Choquet expected utility for a consumer who holds a fraction M of his initial one unit of wealth as money is

$$M \cdot \widehat{\mathcal{V}}_a^m(q_a; \gamma) + (1-M)\cdot \widehat{\mathcal{V}}_a^a(q_a; \gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t, \tau) = (\ell, 0)$. When this is not the case, investment policy M may affect the state (t, τ) for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of $\widehat{\mathcal{V}}_a^m$ and $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and invest the remaining fraction $(1 - F - M)$ in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, and

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of $h-$types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M) \in [0,1] \times [0,1] \\ s.t.\ F+M \leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ consumers, the worst case occurs when $q_f^*(\tau; \psi, \varphi, r)$ attains its upper bound at $\frac{\alpha_2 \cdot (1-r)}{\beta_\ell}$. In this case a consumer's utility equals

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M and in F. Consumers of type h can always obtain a utility level at least as high as the consumers of type ℓ by copying the latter's behaviour.

Lemma 17. *For any* $\psi, \varphi \in [0,1]$ *we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) \; + \; (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$

$$+(1-\gamma)\cdot [r\cdot \beta_\ell + \alpha_2 \cdot (1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings* r *so that it maximizes the ex-ante utility of its investors for given* (ψ,φ)*. Thus, its decision problem for given* (ψ,φ) *is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either* $r^* = 0$ *or* $r^* = 1$*.*
- *when short-selling is allowed, any reserve holding* r^* *of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that* $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$*.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates* (i_1, i_2) *for reserve holdings* r *and an actual fraction of withdrawals* w *are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w=\tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ the constraint

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) > \beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau;r,w_0;\ell)=\begin{cases}1 & \text{if } \rho_2(\tau;r,w_0)<\beta_\ell\cdot\rho_1(\tau;r,w_0)\\ 0 & \text{if } \rho_2(\tau;r,w_0)\geq\beta_\ell\cdot\rho_1(\tau;r,w_0).\end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r,w_0),i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned}&\mathcal{V}_b(D,M;r,w_0;\gamma)\\ &:=\ \gamma\cdot\int[\tau\cdot v_b(D,M;\tau,r,w_0;h)\ +\ (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)]\cdot p(\tau)\ d\tau\\ &\qquad\qquad +(1-\gamma)\cdot\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t).\end{aligned}$$

If the consumer chooses $D,M\in[0,1]$ such that $\widetilde{\mathcal{V}}_b(D,M;r,w_0;\gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned}\widetilde{\mathcal{V}}_b(r,w_0;\gamma) &:= \max_{\substack{(D,M)\in[0,1]\times[0,1]\\ s.t.\ D+M\leq 1}} \mathcal{V}_b(D,M;r,w_0;\gamma)\\ &= \max\{[\gamma\cdot\{\pi\cdot v_b(\pi;r,w_0;h)+(1-\pi)\cdot v_b(\pi;r,w_0;\ell)\}\\ &\qquad +(1-\gamma)\cdot\min_{\tau\in[0,1]} v_b(\tau;r,w_0;t)],\mathcal{V}_n^*(\gamma)\},\end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r,w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r,w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$, taking into account the withdrawal behaviour $W(\tau;r,w_0;t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau=\pi$ and $\tau=1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1-\pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) :=& \ \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ =& \ \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1-r^*)] + (1-\gamma) \cdot [r^* + \alpha_1 \cdot (1-r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

APPENDIX B: PROOFS

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_h + \tau\cdot\alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot\beta_\ell + \tau\cdot\alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau;0,1;\mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell)\,,\, \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) :=&\quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ =&\quad \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ is a best reply for $r = \mu_\ell$ and $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1 \right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ *is convex*

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a}\left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}\right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a :=& \quad 2 \cdot \pi \cdot \beta_h \\ b_1 :=& \quad \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 :=& \quad \gamma \cdot (\alpha_2 - \beta_\ell) \\ c :=& \quad 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned} & (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\ = \ & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + {b_2}^2 + c \cdot \gamma^2 \\ = \ & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + {b_2}^2 \\ & -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\ = \ & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2. \end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\widetilde{\gamma}) = \mathcal{V}_a^*(\widetilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a, b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a, b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a, b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1 - \mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1 - \mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1 - \mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1 - \mu)}{1 - \tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned} \tau \cdot z_{1h}(\tau,\mu) + (1-\tau)\cdot z_{1\ell}(\tau,\mu) &= \mu, \\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu). \end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau}\cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau}\cdot\frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell\cdot(1-\tau)} = \mu + \frac{\alpha_2\cdot(1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau\cdot z_{2h}(\tau,\mu) + (1-\tau)\cdot z_{2\ell}(\tau,\mu) = \alpha_2\cdot(1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2\cdot(1-\mu)}{1-\tau}$.

(iii) $\tau \in [\overline{\tau}, 1]$

For these values of τ there are "too many" $h-$types in the economy. So it is optimal to choose $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$. Even for this choice, however, the proportion of $h-$types is so high, that for $z_{2h}(\tau,\mu) = 0$ the self selection constraint S_h is violated. The payout $z_{2h}(\tau,\mu)$ obtains its lowest value when S_h binds and its highest value when S_ℓ binds. The self selection constraint S_h reads

$$\beta_h \cdot z_{1h}(\tau,\mu) + z_{2h}(\tau,\mu) \geq \beta_h \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu)$$

and after substitution and rearranging we obtain

$$z_{2h}(\tau,\mu) = z_{2\ell}(\tau,\mu) - \beta_h \cdot \frac{\mu}{\tau}.$$

Using $z_{2\ell}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)]$ we find $z_{2h}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)] - \beta_h \cdot \frac{\mu}{\tau}$, from which

$$z_{2h}(\tau,\mu) = \alpha_2 \cdot (1-\mu) - \frac{(1-\tau)}{\tau} \cdot \beta_h \cdot \mu = \alpha_2 - \frac{\mu}{\tau} \cdot (\tau \cdot \alpha_2 + (1-\tau) \cdot \beta_h)$$

and

$$z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu) + \beta_h \cdot \mu$$

follow.

■

Proof of Lemma 14

The structure of this proof is as follows. First the average interim utility for the investment policy μ is determined for $\tau = \pi$. Then the interim utility of the worst case is determined and both are combined to calculate the ex-ante Choquet expected indirect utility of investment policy μ.

(i) Average interim utility

For any $\mu \in [0,1]$ let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13. For any investment policy μ and for $\tau = \pi$ the average interim utility is

$$\widetilde{U}(\pi,\mu) := \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu).$$

From Lemma 12 we have

$$\pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu) = \alpha_2 \cdot (1-\mu)$$

substitution of which yields

$$\widetilde{U}(\pi,\mu) = \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu).$$

(ia) $\pi \in [0, \underline{\tau}(\mu))$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot \left(\frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu \right) \\
&\quad + (1-\pi) \cdot \beta_\ell \cdot \left(\mu - \frac{\pi}{(1-\pi)} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} \right) + \alpha_2 \cdot (1-\mu) \\
&= \left(1 + (\frac{\beta_h}{\beta_\ell} - 1) \cdot \pi \right) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu) .
\end{aligned}$$

(ib) $\pi \in [\underline{\tau}(\mu), 1]$

Using Lemma 12 we obtain

$$\begin{aligned}
\widetilde{U}(\pi,\mu) &= \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \alpha_2 \cdot (1-\mu) \\
&= \pi \cdot \beta_h \cdot \frac{\mu}{\pi} + \alpha_2 \cdot (1-\mu) = \alpha_2 + (\beta_h - \alpha_2) \cdot \mu,
\end{aligned}$$

which complete this part of the proof.

(ii) The worst case

The interim utility a type h consumer obtains from payouts (z_h, z_ℓ) that satisfy the self selection constrains is as least as last as the corresponding utility of a type ℓ consumer since $\beta_h > \beta_\ell$. To determine the worst case, we can therefore restrict our attention to a consumer with a low preference for liquidity. The utility of $\ell-$types from payouts that satisfy the self selection constraints for τ is increasing with τ for given μ.

Therefore, for any value of μ the utility of $\ell-$types obtains its minimum value for $\tau = 0$. Using this, we obtain

$$\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\mu,\tau;t) = \beta_\ell \cdot z_{1\ell}(0,\mu) + z_{2\ell}(0,\mu) = \alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu.$$

(iii) The ex-ante Choquet expected utility

After rearranging terms, from $\pi = \underline{\tau}(\mu)$ we obtain

$$\mu = \frac{\pi \cdot \alpha_2}{(1-\pi) \cdot \beta_\ell + \pi \cdot \alpha_2} = \mu_\ell.$$

For given investment policy μ and given level of confidence γ, substitution of the results of part *(i)* and part *(ii)* of this proof yields

$$\begin{aligned} \mathcal{V}_o(\mu; \gamma) &:= \gamma \cdot \widetilde{U}(\pi, \mu) + (1-\gamma) \cdot \widetilde{U}(0, \mu) \\ &= \begin{cases} (1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu) & \text{if} \quad \mu > \mu_\ell \\ \\ \alpha_2 + (\gamma \cdot \beta_h + (1-\gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu & \text{if} \quad \mu \leq \mu_\ell, \end{cases} \end{aligned}$$

which completes the proof.

■

Proof of Proposition 16

Denote

$$\hat{v}_a(M; q; t) := M \cdot R^m(q; t) + (1-M) \cdot R^a(q; t).$$

For all $q \in [\frac{\alpha_2}{\beta h}, \frac{\alpha_2}{\beta_\ell}]$ we have $\hat{v}_a(M; q; h) > \hat{v}_a(M; q; \ell)$. Furthermore, $\hat{v}_a(M; q; \ell)$ is decreasing in q. From $q_a^E(\tau, \mu) \leq \frac{\alpha_2}{\beta_\ell}$ it now follows that

$$\begin{aligned} &\min_{(t,\tau) \in \{h,\ell\} \times [0,1]} \left[\hat{v}_a(M; q_a^E(\tau, \mu); t)\right] \\ = &\min_{q \in [\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell}]} \hat{v}_a(M; q; \ell) = M \cdot \beta_\ell + (1-M) \cdot \alpha_2. \end{aligned}$$

To simplify notation, we denote $q_a := q_a^E(\pi, \mu)$. From Remark 7 it follows that in an equilibrium with $\mu \in (0,1)$, the asset price $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$ must satisfy

$$\widehat{\mathcal{V}}_a^m(q_a; \gamma) = \widehat{\mathcal{V}}_a^a(q_a; \gamma).$$

(i) $q_a = \frac{\alpha_2}{\beta_\ell}$

In this case consumers anticipate the asset price to be at its upper bound for $\tau = \pi$. By deriving a contradiction, we show this cannot occur in equilibrium. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a; \gamma) := \gamma \cdot [\pi \cdot \beta_h + (1-\pi) \cdot \beta_\ell] + (1-\gamma) \cdot \beta_\ell = \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma\cdot[\pi\cdot\frac{\alpha_2}{\beta_\ell}\cdot\beta_h + (1-\pi)\cdot\alpha_2] + (1-\gamma)\cdot\alpha_2 = \gamma\cdot\pi\cdot\frac{\alpha_2}{\beta_\ell}\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\alpha_2.$$

This implies $\widehat{\mathcal{V}}_a^m(q_a;\gamma) < \widehat{\mathcal{V}}_a^a(q_a;\gamma)$. In this case, any consumer would invest all his wealth in the illiquid asset, i.e. $\mu = 0$. This aggregate investment policy, however, leads to the asset price $q_a = \frac{\alpha_2}{\beta_h}$, which contradicts assumption *(i)*.

(ii) $q_a = \frac{\alpha_2}{\beta_h}$

In this case, the consumers anticipate the asset price to obtain its lower bound for $\tau = \pi$. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma\cdot[\pi\cdot\beta_h + (1-\pi)\cdot\beta_h] + (1-\gamma)\cdot\beta_\ell = \gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma\cdot[\pi\cdot\alpha_2 + (1-\pi)\cdot\alpha_2] + (1-\gamma)\cdot\alpha_2 = \alpha_2.$$

This lower bound for the asset price will only be obtained in equilibrium if

$$\alpha_2 \geq \gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell.$$

Rearranging terms, this inequality leads to $\gamma \leq \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}$. From Proposition 10 it follows that for these levels of confidence financial institutions are superfluous form the ex-ante point of view.

(iii) $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$

For any such q_a to be the anticipated equilibrium price in Period 1 for $\tau = \pi$, we must have

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Substitution of $R^m(q_a;h) = \beta_h$, $R^m(q_a;\ell) = \frac{\alpha_2}{q_a}$, $R^a(q_a;h) = \beta_h\cdot q_a$ and $R^a(q_a;\ell) = \alpha_2$ leads to

$$\begin{aligned} & \gamma\cdot\pi\cdot\beta_h + \gamma\cdot(1-\pi)\cdot\frac{1}{q_a}\cdot\alpha_2 + (1-\gamma)\cdot\beta_\ell \\ = \; & \gamma\cdot\pi\cdot\beta_h\cdot q_a + \gamma\cdot(1-\pi)\cdot\alpha_2 + (1-\gamma)\cdot\alpha_2. \end{aligned}$$

Rearranging yields

$$(1-\gamma)\cdot[\alpha_2-\beta_\ell] = \left[\pi\cdot\beta_h + (1-\pi)\cdot\alpha_2\cdot\frac{1}{q_a}\right]\cdot\gamma\cdot(1-q_a).$$

For $\gamma < 1$ we have $q_a < 1$, from which we obtain

$$\pi \cdot \beta_h - (1-\pi)\cdot \alpha_2 + \frac{1}{q_a}\cdot(1-\pi)\cdot\alpha_2 - q_a\cdot\pi\cdot\beta_h = (\frac{1}{\gamma}-1)\cdot(\alpha_2-\beta_\ell).$$

Denote the solution of this equation w.r.t. q_a by $\widehat{q}_a(\gamma;\pi,\alpha_2,\beta_h,\beta_\ell)$.

The derivative of the left hand side of this expression with respect to q_a is

$$-\frac{(1-\pi)\cdot\alpha_2}{(q_a)^2} - \pi\cdot\beta_h < 0.$$

Therefore, the left hand side of the expression is decreasing in q_a.

The right hand side of the expression is decreasing in γ. An increase in the level of ambiguity, which corresponds to a decrease in the level of confidence, increases the right hand side of the equation. To maintain equality, the left hand side of the equation has to increase as well, which leads to a decrease in the asset price q_a, keeping all other parameters constant. The asset price is, however, bound from below by the value $\frac{\alpha_2}{\beta_h}$. After substituting this lower bound for the asset price and solving for the level of confidence γ, we find that $q_a > \frac{\alpha_2}{\beta_h}$, whenever $\gamma > \frac{(\alpha_2-\beta_\ell)}{(\beta_h-\beta_\ell)}$.

■

Proof of Lemma 18

If the self selection constraint for type ℓ consumers is violated for the deposit contract (r,π) and withdrawals $w=\pi$, then a bankrun in Period 1 can not be avoided. Hence, all illiquid assets will be liquidated. In this case, it is optimal for the consumers not to deposit their wealth in the bank. This leads to an ex-ante utility of $\mathcal{V}_n^*(\gamma)$.

Therefore, consider deposit contracts (r,π) such that for withdrawals $w=\pi$ the self selection constraint of type ℓ consumers is satisfied. Using $w=\pi$ this leads to the ex-ante Choquet expected utility of

$$
\begin{aligned}
&\mathcal{V}_b(D,M;r,\pi;\gamma) := \\
&\quad \gamma \cdot [\mathbb{E}_\tau\{\tau \cdot v_b(D,M;\tau,r,\pi;h) + (1-\tau)\cdot v_b(D,M;\tau,r,\pi;\ell)\}] \\
&\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \cdot v_b(D,M;\tau,r,\pi;t) \\
&\quad = \gamma \cdot [\pi \cdot v_b(D,M;\pi,r,\pi;h) + (1-\pi)\cdot v_b(D,M;\pi,r,\pi;\ell)] \\
&\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \cdot v_b(D,M;\tau,r,\pi;t) \\
&\quad = \gamma \cdot [\pi \cdot [D\cdot \widetilde{v}_b(\pi;r,\pi;h) + (1-D)\cdot v_n(M;h)] \\
&\qquad + (1-\pi)\cdot [D\cdot \widetilde{v}_b(\pi;r,\pi;\ell) + (1-D)\cdot v_n(M;\ell)]] \\
&\quad + (1-\gamma)\cdot \min_{\tau\in[0,1]} \min_{t\in\{h,\ell\}} \{D\cdot \widetilde{v}_b(\tau;r,\pi;t) + (1-D)\cdot v_n(M;t)\} \\
&\quad = \gamma\cdot [D\cdot \mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\} + (1-D)\cdot \mathbb{E}_t\{v_n(M;t)\}] \\
&\quad + (1-\gamma)\cdot \min_{\tau\in[0,1]} [D\cdot \widetilde{v}_b(\tau;r,\pi;\ell) + (1-D)\cdot v_n(M;\ell)] \\
&\quad = D\cdot \left[\gamma\cdot \mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\} + (1-\gamma)\cdot \min_{\tau\in[0,1]} \widetilde{v}_b(\tau;r,\pi;\ell)\right] \\
&\qquad + (1-D)\cdot [\gamma\cdot \mathbb{E}_t\{v_n(M;t)\} + (1-\gamma)\cdot v_n(M;\ell)].
\end{aligned}
$$

For any $\tau \in [0,1]$ we have that $\widetilde{v}_b(\tau;r,\pi;h) > \widetilde{v}_b(\tau;r,\pi;\ell)$ and for every $M \in [0,1]$, that $v_n(M;h) \geq v_n(M;\ell)$. Thus we obtain:

$$
\begin{aligned}
\widetilde{\mathcal{V}}_b(r,\pi;\gamma) := \quad & \max_{(D,M)\in[0,1]\times[0,1]} \quad \mathcal{V}_b(D,M;r,\pi;\gamma) \\
& \qquad \text{s.t.} \quad D+M \leq 1 \\
= & \max_{D\in[0,1]} D\cdot [\gamma\cdot(\pi\cdot \widetilde{v}_b(\pi;r,\pi;h) + (1-\pi)\cdot \widetilde{v}_b(\pi;r,\pi;\ell)) \\
& \qquad + (1-\gamma)\cdot \min_{\tau\in[0,1]} \widetilde{v}_b(\tau;r,\pi;\ell)\Big] \\
+ & \max_{M\in[0,1-D]} [\gamma\cdot(\pi\cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)\}) + (1-\gamma)\cdot v_n(M;\ell)] \\
= & \max_{D\in[0,1]} D\cdot \left[\gamma\cdot \mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\} + (1-\gamma)\cdot \min_{\tau\in[0,1]} \widetilde{v}_b(\tau;r,\pi;\ell)\right] + (1-D)\cdot \mathcal{V}_n^*(\gamma).
\end{aligned}
$$

This expression obtains its maximum either for $D = 0$ or for $D = 1$ or for any value $D \in [0,1]$. From this the result of the Lemma follows immediately.

■

Proof of Lemma 19

This proof consists of two steps. Firstly, for any reserves r, the optimal payouts in Period 1 are determined. The second step consist of finding the optimal reserve policy, under the assumption that separating the two types of consumers is worthwhile. If the separation of types is not worth while, one either obtains the reserves as derived here and deposits $\delta = 0$, or the bank hold reserves $r = \mu_n^*(\gamma)$.

(i) $w_0 = \pi$.

For given reserves r, the representative bank is to choose the interest payments $(i_1(r, w_0)$, $(i_2(r, w_0))$. Under the zero-profit condition, this is equivalent to choosing its predicted withdrawals w_0. Therefore, its decision problem is

$$\max_{w_0 \in [\underline{\tau}(r), \overline{\tau}(r)]} \gamma \cdot [\pi \cdot \beta_h \cdot \rho_1(\pi; r, w_0) + (1-\pi) \cdot \rho_2(\pi; r, w_0)] + (1-\gamma) \cdot \beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r).$$

Rearranging, dividing by γ and disregarding the constant term to $(1-\gamma)$ yields

$$\max_{w_0 \in [\underline{\tau}(r), \overline{\tau}(r)]} \pi \cdot \beta_h \cdot \rho_1(\pi; r, w_0) + (1-\pi) \cdot \rho_2(\pi; r, w_0).$$

This problem has an interior solution, which does not depend on γ and which is obtained for $w_0 = \pi$.

(ii) Optimal reserves r

Denote

$$\mu_h := \frac{\pi \cdot \alpha_2}{(1-\pi) \cdot \beta_\ell + \pi \cdot \alpha_2}.$$

Using the result from part *(i)* of this proof, the decision problem of the bank reduces to

$$\max_{r \in [\mu_h, \mu_\ell]} \gamma \cdot [\pi \cdot \beta_h \cdot \frac{r}{\pi} + (1-\pi) \cdot \frac{\alpha_2 \cdot (1-r)}{(1-\pi)}] + (1-\gamma) \cdot \beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r),$$

where $[\mu_h, \mu_\ell]$ denotes the set of reserves that are compatible with self selection if $\tau = \pi$. After rearranging the objective function can be written as

$$[\gamma \cdot (\beta_h - \alpha_2) + (1-\gamma) \cdot \beta_\ell \cdot (1-\alpha_1)] \cdot r + \gamma \cdot \alpha_2 + (1-\gamma) \cdot \beta_\ell \cdot \alpha_1.$$

This expression is maximised when r is at its upper bound, whenever

$$\gamma \cdot (\beta_h - \alpha_2) + (1-\gamma) \cdot \beta_\ell \cdot (1-\alpha_1) > 0.$$

From Assumption 2 we know that $\beta_h > \alpha_2$ and $\alpha_1 < 1$. Therefore the inequality holds and we have

$$r = \mu_\ell := \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1-\pi) \cdot \beta_\ell}.$$

■

Proof of Proposition 20

The ex-ante Choquet expected indirect utility of the consumers from the deposit contract (r, π) is obtained as

$$\max_{D \in [0,1]} \quad D \cdot \gamma \cdot [\beta_h \cdot r + \alpha_2 \cdot (1-r)] + (1-\gamma) \cdot D \cdot [\beta_\ell \cdot (\alpha_1 \cdot (1-r) + r)]$$
$$+ (1-D) \cdot \mathcal{V}_n^*(\gamma).$$

After rearranging the objective function, it follows directly that a consumer deposits all his wealth in the bank, i.e. $D^* = 1$, if

$$\gamma \cdot [(\beta_h - \alpha_2) \cdot r + \alpha_2] + (1-\gamma) \cdot [\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r)] - \mathcal{V}_n^*(\gamma) > 0.$$

To prove the Proposition, two cases are considered.

(i) $\alpha_2 \geq \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell$

For these parameter values we have $\mathcal{V}_n^*(\gamma) = \alpha_2$ and substitution yields

$$\gamma \cdot [(\beta_h - \alpha_2) \cdot \mu + \alpha_2] + (1-\gamma) \cdot [\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot \mu)] - \alpha_2 > 0.$$

For the left hand side of this expression we obtain

$$\gamma \cdot [(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell]$$
$$-(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi - (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell.$$

This equals zero for the level of confidence

$$\widehat{\gamma}' = \frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell} > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}.$$

Liquidity and Ambiguity: Banks or Asset Markets? [1]

by

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Abstract. We study the impact of ambiguity on financial intermediation in an economy where agents have random liquidity needs. The ambiguity the agents face is modeled by the degree of confidence in their additive beliefs. We compare an optimal liquidity allocation with the allocation achieved by trade in an asset market, by a mutual fund, and by a competitive banking sector.

For low levels of confidence, intermediation is superfluous. For intermediate levels the asset market, and for high levels of confidence banks are the preferred intermediary arrangements.

The desirability of mutual funds depends crucially on the possibility of short sales. When short selling of fund shares is feasible, the asset market outcome results. If short sales are impossible, then a mutual fund can implement the optimal outcome for any degree of confidence.

Keywords: Financial institutions, Liquidity, Ambiguity, Choquet Expected Utility.
JEL Classification Codes: D8, G1, G2.

1. Introduction

In modern economies, financial intermediaries have three main functions. They provide and monitor credit and insurance contracts cost efficiently, they play a prominent role in organizing financial markets, and they act as providers of liquidity. Focusing on the third role of intermediaries, we ask in this paper which type of financial intermediary is optimal in an economy in which consumers face uncertainty both about their individual future liquidity needs and about the economy-wide need for liquidity. We study this question in a simple model in which consumers have the opportunity of investing in an illiquid asset which is costly to liquidate before maturity.

The uncertainty of consumers with respect to liquidity needs is modelled by ambiguity.

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Uncertainty has long been recognized as an important factor determining economic activities. The distinction between *risk*, i.e., situations where the probabilities of events are known, and *ambiguity*, i.e., situations where this is not the case, has served Knight (1921, [16]) as a foremost explanation of economic phenomena such as profit and entrepreneurial activity. [4] [5]

For several decades, however, the behaviourist theory of subjective expected utility by Savage (1954, [19]) appeared to have rendered this distinction obsolete. If individuals faced with uncertainty behave as if they held a subjective probability distribution over events, then, from an analytical point of view, behaviour under risk and under uncertainty can be treated in the same way. Yet early evidence by Ellsberg (1961, [10]) suggested that the hypothesis of a well-defined subjective probability function could not be maintained empirically. Systematic laboratory experiments have confirmed Ellsberg's conjecture, see Camerer and Weber (1992, [2]). Some aspects of uncertainty cannot be captured by the assumption of a subjective probability distribution. Partial information about events, in particular, cannot be properly incorporated.

Despite these obvious inconsistencies, expected utility theory proved to be a very successful modelling tool over the same period. Important economic insights were obtained from the distinction between risk preferences and beliefs which can be made in this approach. The economics of insurance and information were developed in this context. Lacking an alternative theory of decision making under ambiguity, it was impossible to pursue with the same rigor those aspects of uncertainty which subjective probabilities could not capture properly.

1.1. Modelling ambiguity. In recent years, major progress has been made in modelling decision-making under uncertainty without subjective probabilities. Schmeidler (1989, [20]) and Gilboa (1987, [11]) proposed a theory where the decision maker's beliefs are represented by *non-additive probabilities* (or *capacities*). *Choquet expected utility (CEU)* theory, a generalization of subjective expected utility, can accommodate many different weighting schemes for

[4] The subdividing uncertainty in risk and ambiguity is in accordence with modern terminology. Knight (1921, [16]) used "uncertainty" where we use "ambiguity".

[5] See Kelsey and Spanjers (2004, [15]) for a formalization of Knight's intuition using the Choquet Expected Utility framework.

events while maintaining the separation of beliefs and outcome evaluation, which is important in economic applications in order to identify risk preferences.

Generalizing additive beliefs to non-additive beliefs allows one to accommodate some empirically observed anomalies like, e.g., the Ellsberg-paradox as described in Ellsberg (1961, [10]). Without imposing additional restrictions on capacities, however, the predictions about economic behaviour are typically less precise. Eichberger and Kelsey (1999, [8]) propose several desiderata for applications in economics and game theory and identify the class of capacities satisfying them.

In this paper, we will restrict attention to beliefs that can be represented by *simple capacities*. Simple capacities are convex combinations of an additive probability distribution and the capacity of complete ignorance. They reduce the large number of free parameters, which are typical for general capacities, and serve as a controlled deviation from additive beliefs. Moreover, with simple capacities, one can give *ambiguity* and *confidence* a parametric interpretation. This enables us to conduct a comparative static analysis of endogenously determined economic outcomes with respect to the players' ambiguity.

In Eichberger and Kelsey (1999, [9]) it is demonstrated that Choquet expected utility with simple capacities is as easy to apply to economic problems as traditional expected utility theory. It is intuitive and can explain some puzzles of the traditional theory. More importantly however, it enables us to study the impact of ambiguity and confidence on economic outcomes, an analysis which is impossible with expected utility theory. The ambiguity and confidence of a player may reflect his familiarity with a particular situation.

1.2. Liquidity and financial intermediation. Diamond and Dybvig (1983, [5]), Jacklin (1987, [13]) and Jacklin and Bhattacharya (1988, [14]) show that deposit contracts and mutual funds can implement an optimal allocation of liquidity when there is perfect foresight about the aggregate need for liquidity in the economy as a whole. Eichberger (1992, [7]) shows that even if the aggregate need for liquidity is known in advance, asset markets do not implement an optimal allocation of liquidity. In order to enable a comparison with the above results

and to isolate the effect of ambiguous beliefs, we assume perfect foresight on the hand of the consumers.

Spanjers (1999, [21]) presents a simple framework for the comparison between banks and asset markets in the presence of ambiguity. Apart from ambiguity about the demand for liquidity, which is the focus of the paper at hand, he also analyses the effects of ambiguity about asset returns.

Using this framework, we obtain to the following results. In the presence of ambiguity, neither the deposit contracts by competing banks nor the asset market implement the ex-ante optimal allocation of liquidity. Which institutional framework is preferred ex-ante depends on the degree of ambiguity the consumers face.

In the case of a high level of ambiguity, neither institutional framework can improve upon the allocation of liquidity without an intermediary. For an intermediate level of ambiguity, the asset market is the preferred institutional arrangement. If the level of ambiguity is low, then (uninsured) deposit contracts offer the ex-ante preferred method of liquidity provision.

Finally, we analyse mutual funds. We find that if there is only one mutual fund in the economy that acts in the sole interest of its share holders, and short selling the shares of the fund is not possible, then this mutual fund implements the optimal outcome. If, however, it is possible to short sell the shares of the fund, than the asset market outcome results.

2. Non-additive beliefs and the Choquet integral

In this section we present the concepts of a *capacity* and of the *Choquet integral,* an expected value of a function with respect to a capacity. Moreover, *simple capacities* are introduced as a special case, and a representation of the Choquet integral for simple capacities is derived. [6]

Simple capacities form a special class of capacities. A simple capacity is a convex combination of an additive probability distribution and the capacity of complete ignorance. [7] Simple capacities maintain many properties of additive probability distributions and have a natural interpretation in terms of beliefs. The degree of ambiguity reflects the deviation from additivity

[6] For a definition of capacities see Appendix A, Definition 2.
[7] For a definition of the capacity of complete ignorance see Appendix A, Remark 1.

of the prediction made in the probabilistic part.

In economic and game-theoretic applications, one can determine the additive part of beliefs endogenously as the rational prediction of the players, derived from the knowledge of the game structure and the assumed rationality of the other players. Players may however distrust these "rational" predictions to some degree. If they do not trust their rational prediction at all, they are in a state of complete ignorance.

Definition 1. *A* ***simple capacity*** *based on an additive probability distribution π is defined as*

$$\nu(E) = \gamma \cdot \pi(E) + \overline{\gamma} \cdot \omega(E)$$

for all $E \subseteq S$ with $\overline{\gamma} := 1 - \gamma$, where ω denotes the capacity of complete ignorance.

Simple capacities can be interpreted as an additive, i.e. completely consistent, probability **assessment**, which is held with some degree of doubt. The **degree of confidence** $\gamma \in [0, 1]$ measures the confidence of the decision maker in the additive probability π. In contrast, the weight given to the capacity of complete ignorance, $\overline{\gamma}$, denotes the **degree of ambiguity**. In this interpretation, ambiguity is simply the counterpart of confidence in a probabilistic assessment.

Various measures of ambiguity have been suggested in the literature. [8] Ambiguity is usually measured by the "deviation" of a capacity from an additive probability distribution. Since there are different notions of "deviation" from additivity which are not compatible, comparative static results with regard to players' confidence or ambiguity are difficult to compare. A special, and for the economic analysis convenient, feature of simple capacities is the fact that these notions all coincide for simple capacities.

For applications to economic problems, it is particularly important that the expected value of a function with respect to a capacity, the Choquet integral, takes the form of a convex combination with weight γ of the expected utility with respect to the assessment π and the worst

[8] Dow and Werlang (1992, [6]), Marinacci (2000, [17]), Ghirardato and Marinacci (2002, [12]) suggest alternative measures of ambiguity and ambiguity aversion.

outcome. [9] [10] The following result is proved in Eichberger and Kelsey (1999, [9], Proposition 2.1).

Proposition 1. Choquet integral of a simple capacity

Consider a simple capacity $\nu := \gamma \cdot \pi + \overline{\gamma} \cdot \omega$ *where* π *is an additive probability distribution on a compact set* S*. The Choquet integral of a continuous function* f *on* S *has the following form:*

$$\int^C f \, d\nu := \gamma \cdot \int_S f(s) \cdot \pi(s) \, ds + \overline{\gamma} \cdot \min_{s \in S} f(s).$$

Proposition 1 gives a very simple and parsimonious description of ambiguity. In particular, for the case of full confidence, $\gamma = 1$, one has the familiar expected utility form, whereas under complete ignorance, $\gamma = 0$, the maximin decision rule arises.

Clearly, the Choquet integral of a simple capacity models ambiguity in a very special way. We feel, however, that this pay-off description captures at least some aspects of Knight's ideas. In particular, it allows us to relate intermediary institutions in an economy to the intrinsic ambiguity the consumers face.

3. THE ECONOMY

We consider an economy with many consumer. As usual, this is modelled by assuming the set of consumers is the interval $[0, 1]$. [11] The economy extends over three periods. In Period 0, each agent is endowed with one unit of wealth (money) and faces the following investment opportunities:

Payoff in	Period 0	Period 1	Period 2
Asset liquidated	-1	α_1	0
Asset matured	-1	0	α_2
Money 0 to 1	-1	1	0
Money 1 to 2	0	-1	1

[9] For a general definition of the Choquet Integral see Appendix A, Definition 3.
[10] Eichberger and Kelsey (1999, [8]) provide an axiomatization for this representation.
[11] To aggregate the behaviour of the individual agents, the usual Lebesgue integral is applied.

We assume

$$\alpha_2 > 1 > \alpha_1.$$

This payoff structure of the asset justifies referring to it as an **illiquid asset**. There is no uncertainty about the pay-offs of the assets. [12]

There are two types of ex-ante identical consumers. In Period 1, consumers privately learn their type $t \in \{h, \ell\}$. The type of a consumer determines his preference for liquidity in Period 1. The type-dependent preferences are represented by a risk-neutral von Neumann-Morgenstern utility index $u : \mathbb{R}_+^2 \times \{h, \ell\} \to \mathbb{R}$:

$$u(z_1, z_2; t) := \beta_t \cdot z_1 + z_2$$

where z_1 and z_2 denote consumption in Period 1 and Period 2, respectively. The proportion of consumers of type h in the economy is π. This proportion is unknown in Period 0, but becomes common knowledge in Period 1. As in Diamond and Dybvig (1983, [5]), it is assumed that the probability of a consumer being assigned a particular type equals the proportion of this type in the economy. Hence, π denotes also the true probability of becoming type h. In Period 0, however, consumers are uncertain both about the proportion of $h-$types and about their own probability of having a high liquidity preference.

Throughout the paper, the following assumptions about the parameter values of our model is maintained.

Assumption 2. Liquidity preference

$$\begin{array}{ll} (i) & \beta_h > \alpha_2 > \beta_\ell \geq 1, \\ (ii) & \alpha_2 > \beta_h \cdot \alpha_1. \end{array}$$

According to Assumption 2 *(i)*, consumers of type h strictly prefer to hold money, while consumers of type ℓ prefer an investment in the illiquid asset. Condition *(ii)* is not strictly necessary for the analysis, but reduces the number of different cases to be considered substan-

[12] Such uncertainty can, however, be introduced in the form of risk as, e.g., in Eichberger (1992, [7]) or Spanjers (1999, [21], Chapter 3). The latter formalization allows one to incorporate ambiguity about the asset returns in a natural way, as in Spanjers (1999, [21], Chapter 5).

tially. It assumes that even consumers with a high preference for liquidity will not consider liquidating the asset prematurely.

3.1. Investment without intermediation. In Period 0, each consumer decides on the fraction M of his wealth to be held as money and, consequently, on the fraction $1 - M$ to be invested in the illiquid asset. In Period 1, knowing their types, consumers determine the fraction L of their illiquid investment which they want to liquidate.

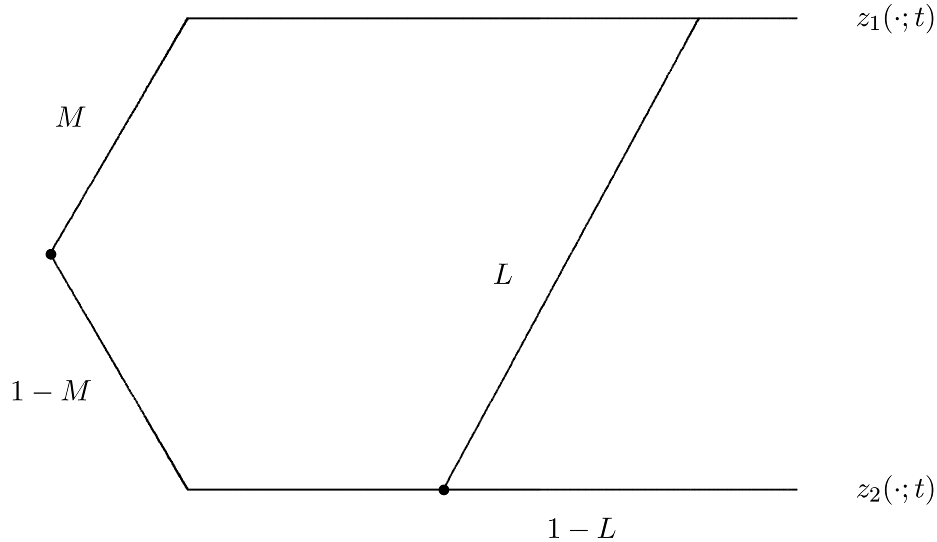

Figure 1: No intermediation

By Assumption 2, we need not consider the case that a consumer would want to hold money over to Period 2.

Given an investment decision in Period 0, $M \in [0,1]$, and a liquidation decision in Period 1, $L \in [0,1]$, the following consumption levels result:

$$\begin{aligned} z_1(M,L) &:= \quad M + \alpha_1 \cdot L \cdot (1-M) \qquad \text{and} \\ z_2(M,L) &:= \quad \alpha_2 \cdot (1-L) \cdot (1-M). \end{aligned}$$

The optimal liquidation policy of a consumer in Period 1 depends only on his type $t \in \{h, \ell\}$,

$$L^*(t) = \begin{cases} 1 & \text{for} \quad \beta_t \cdot \alpha_1 > \alpha_2 \\ \in [0,1] & \text{for} \quad \beta_t \cdot \alpha_1 = \alpha_2 \\ 0 & \text{for} \quad \beta_t \cdot \alpha_1 < \alpha_2. \end{cases}$$

Since, by Assumption 2, $\alpha_2 > \beta_h \cdot \alpha_1 (> \beta_\ell \cdot \alpha_1)$,

$$L^*(h) = L^*(\ell) = 0$$

follows. The indirect interim utility function v for $t \in \{h, \ell\}$ is:

$$v_n(M;t) := u(z_1(M, L^*(t)), z_2(M, L^*(t)); t) = \beta_t \cdot M + (1-M) \cdot \alpha_2,$$

where the subscript n denotes that the indirect utility is obtained when no financial institutions are present.

In Period 0, when consumers have to decide on an investment strategy M, they do not know what type they will be in Period 1, nor do they know the proportion of types. Therefore, decisions have to be based on beliefs about these parameters.

The beliefs of consumers are usually modelled by a subjective probability distribution over the unknown parameters. As discussed in the previous section, ambiguity is modelled here as lack of confidence in an additive probability distribution. For each consumer, the unknown parameters are his type, t, and the proportion τ of $h-$types in the population. We assume that these parameters are drawn from an additive joint probability distribution $P(t, \tau)$ with marginal distribution $p(\tau)$ for the population parameter τ.

With ex-ante identical consumers, it appears reasonable to assume further that the likelihood of a consumer being either type equals the proportion of the type in the population. Therefore the following assumption is justified.

Assumption 3. Population shares and probabilities of types

The conditional probability distribution over types equals the proportions of types in the economy,

$$P(h|\tau) = \tau \qquad \text{and} \qquad P(\ell|\tau) = 1 - \tau.$$

Assumption 3 restricts the conditional probability distributions over types and implies $P(h, \tau) = \tau \cdot p(\tau)$ and $P(\ell, \tau) = (1-\tau) \cdot p(\tau)$.

Assume for a moment that the fraction of type h consumers τ were common knowledge. Then, the ex-ante expected utility of a consumer, with respect to the conditional probability distribution over types, $\mathbb{E}_t\{v(M;t)|\tau\}$, can be computed in a straightforward way,

$$\begin{aligned}\mathbb{E}_t\{v_n(M;t)|\tau\} &= \tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell) \\ &= M \cdot [\tau \cdot \beta_h + (1-\tau) \cdot \beta_\ell] + (1-M) \cdot \alpha_2.\end{aligned}$$

Suppose now that consumers have doubts as to the correct probability distribution $P(t,\tau)$, which are modelled by a degree of confidence γ less than 1. Then the ex-ante utility of the consumer is obtained by applying the Choquet integral with respect to the simple capacity with assessment P and degree of confidence γ. The resulting Choquet expected indirect utility for money holdings M is [13]

$$\begin{aligned}\mathcal{V}_n(M;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_n(M;h) + (1-\tau) \cdot v_n(M;\ell)] \cdot p(\tau)\ d\tau \\ & + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_n(M;t) \\ = \quad & \gamma \cdot [\mathbb{E}_p\{\tau\} \cdot v_n(M;h) + (1-\mathbb{E}_p\{\tau\}) \cdot v_n(M;\ell)] \\ & + (1-\gamma) \cdot v_n(M;\ell).\end{aligned}$$

A final assumption is needed in order to link the beliefs of consumers to the actual proportion π of $h-$types in the population.

Assumption 4. Correct assessment

Consumers' assessments of the population share of $h-$types are concentrated on the true proportion π,

$$p(\tau) = \begin{cases} 1 & \textit{for} \quad \tau = \pi \\ 0 & \textit{otherwise.} \end{cases}$$

Assumption 4 guarantees that beliefs about the proportion of $h-$types within the population remain related to its true population share π. Hence, in case of full confidence in this

[13] Note that, by Assumption 2,

$$v_n(M;h) = \beta_h \cdot M + (1-M) \cdot \alpha_2 \geq \beta_\ell \cdot M + (1-M) \cdot \alpha_2 = v_n(M;\ell).$$

assessment, $\gamma = 1$, consumers have rational expectations. The only distortion introduced by ambiguity is restricted confidence in the assessment.

Maximizing the ex-ante indirect utility function $\mathcal{V}_n(M;\gamma)$ over $M \in [0,1]$ yields the following solution. [14]

Proposition 5. *In an economy without intermediation the following investment policy is optimal:*

$$M_n^*(\gamma) := \begin{cases} 0 & \textit{for} \quad \gamma \cdot \pi < \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0,1] & \textit{for} \quad \gamma \cdot \pi = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 1 & \textit{for} \quad \gamma \cdot \pi > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}, \end{cases}$$

yielding an ex-ante Choquet expected utility of

$$\mathcal{V}_n^*(\gamma) := \mathcal{V}_n(M_n^*(\gamma);\gamma) = \max\{\alpha_2 , \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell\}.$$

Consumers of type h prefer to hold money in Period 1. Hence, money holding is optimal if the assessed probability of becoming a type h consumer, π, is high and there is a high degree of confidence γ in this belief. Otherwise, all money is invested in the illiquid asset.

3.2. Optimal contract. The allocation which consumers can generate individually in this economy is suboptimal. Even taking the informational constraints in Period 0 as given, there is scope for Pareto improvements by pooling the resources in Period 0 and investing them jointly. With a reasonable forecast of the liquidity demand in Period 1, i.e., of the proportions of the types, it is feasible for consumers to mutually insure each other. [15] Since the type of a consumer is private information, such a asset pooling scheme needs to be studied carefully. In this section, we investigate the optimal allocation in this economy subject to the informational constraints.

For an optimal allocation, the resources of the consumers are pooled in Period 0 and optimally invested in liquid and illiquid assets. In periods 1 and 2, type contingent payouts,

[14] For a precise expression for $\mathcal{V}_n(M;\gamma)$ see Appendix A, Remark 2.

[15] Consumers are interested in insurance, eventhough they are risk neutral. The reason for this is that the consequence for each state of nature is an allocation of wealth over two periods. The insurance scheme redistributes wealth between the two periods for a given state of nature.

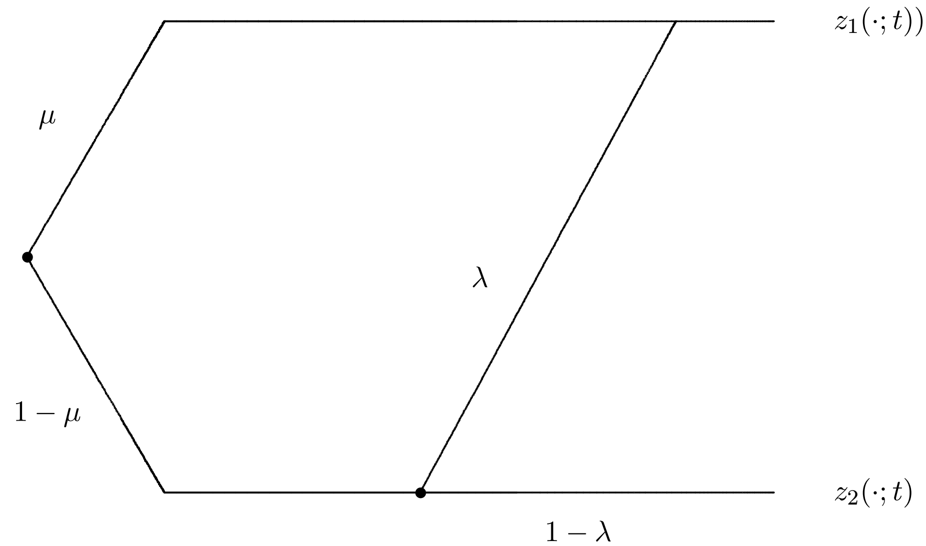

Figure 2: Optimal contract

$z_h := (z_{1h}, z_{2h})$ and $z_\ell := (z_{1\ell}, z_{2\ell})$, occur subject to self selection constraints. We call such a payout scheme an **optimal allocation** if it maximizes the ex-ante Choquet expected utility of a representative consumer.

In Period 0, before types $t \in \{h, \ell\}$ are *privately known* and before the proportion of types τ is *common knowledge*, all consumers are identical. In Period 1, however, the situation is different. Since consumers' types are not publicly known, an optimal allocation which would maximize individual utility would assume away the informational asymmetry. The type contingent payout scheme, (z_h, z_ℓ), must therefore be based only on the proportion of $h-$types τ. We assume that this type contingent payout scheme is selected such that the *average interim utility* of consumers is maximized. The **average interim utility** for a population fraction τ of $h-$types, is

$$\begin{aligned} U(z_h, z_\ell; \tau) &:= \tau \cdot u(z_h; h) + (1-\tau) \cdot u(z_\ell; \ell) \\ &:= \tau \cdot [\beta_h \cdot z_{1h} + z_{2h}] + (1-\tau) \cdot [\beta_\ell \cdot z_{1\ell} + z_{2\ell}]. \end{aligned}$$

The choice problem has two stages. In Period 0, the fraction μ of aggregate wealth held as money is determined and the remainder is invested in the illiquid asset. The second stage occurs in Period 1, after the investment decision μ has been made and the proportion of $h-$types

$\tau \in [0,1]$ has become known. Here it is decided on the fraction of the illiquid investment that is liquidated, λ, and on the consumers' type-contingent payouts in periods 1 and 2, (z_h, z_ℓ). We analyse these two stages in turn beginning with the determination of the type contingent payout scheme.

The optimal payout scheme. For an investment policy μ chosen in Period 0 and a realized proportion of $h-$type consumers $\tau \in [0,1]$,

- the pay-offs $(z_{1h}(\tau,\mu), z_{2h}(\tau,\mu))$ for $h-$types,
- the pay-offs $(z_{1\ell}(\tau,\mu), z_{2\ell}(\tau,\mu))$ for $\ell-$types, and
- the fraction $\lambda(\tau,\mu)$ of illiquid assets to be liquidated in Period 1

are determined.

The optimal payout scheme (z_h, z_ℓ) and the optimal liquidation policy λ maximize the average interim utility, subject to self selection and feasibility constraints:

$$\begin{array}{llcll}
\max\limits_{\lambda, z_h, z_\ell} & U(z_h, z_\ell; \tau) & & & \\
\text{s.t.} & \beta_h \cdot z_{1h} + z_{2h} & \geq & \beta_h \cdot z_{1\ell} + z_{2\ell} & \mathrm{S}_h \\
& \beta_\ell \cdot z_{1\ell} + z_{2\ell} & \geq & \beta_\ell \cdot z_{1h} + z_{2h} & \mathrm{S}_\ell \\
& \tau \cdot z_{1h} + (1-\tau) \cdot z_{1\ell} & = & \mu + \alpha_1 \cdot (1-\mu) \cdot \lambda & \mathrm{F}_1 \\
& \tau \cdot z_{2h} + (1-\tau) \cdot z_{2\ell} & = & \alpha_2 \cdot (1-\mu) \cdot (1-\lambda) & \mathrm{F}_2 \\
& \lambda \geq 0,\ z_{1h} \geq 0,\ z_{1\ell} \geq 0,\ z_{2h} \geq 0,\ z_{2\ell} \geq 0. & & & \mathrm{NN}
\end{array}$$

Feasibility, F_1 and F_2, requires that aggregate payouts in periods 1 and 2 can be financed given the investment policy μ. Furthermore, in order to guarantee that consumers select the type-contingent payout designed for them, the two self selection constraints S_h and S_ℓ must be satisfied.

Remember that it is never optimal to liquidate existing assets in Period 1 as a consequence of Assumption 2. [16] From the linearity of the average interim utility function $U(z_h, z_\ell; \tau)$ it is immediately clear that the optimal allocation fails to be unique at an optimum where the self selection constraints are not binding. In this case, the Period 2 payout of one of the types may

[16] See Appendix A, Lemma 12.

be decreased and the proceeds can be used to increase the payout in Period 2 for the other type without violating the self selection constraints.

The payout scheme proposed in Proposition 13 in Appendix A yields as the indirect interim utility of a type t consumer for an investment policy μ and a proportion τ of $h-$types:

$$\begin{aligned} v_o(\tau,\mu;t) := & \quad u(z_{1t}(\tau,\mu), z_{2t}(\tau,\mu);t) \\ = & \quad \beta_t \cdot z_{1t}(\tau,\mu) + z_{2t}(\tau,\mu), \end{aligned}$$

where the subscript o denotes that the indirect interim utility is obtained by use of the optimal contract. It remains to determine the optimal investment policy μ.

The optimal investment policy. In Period 0, aggregate money holdings μ are chosen to maximize the ex-ante utility of the representative consumer. Recall that the ex-ante utility of a consumer depends on the perceived ambiguity which is represented by the degree of confidence γ. For investment policy μ the ex-ante Choquet expected indirect utility function is: [17]

$$\mathcal{V}_o(\mu;\gamma) := \quad \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi)\cdot v_o(\pi,\mu;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t).$$

The optimal investment policy μ is determined as the solution to the following optimization problem:

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma).$$

Using the precise expression for $\mathcal{V}_o(\mu;\gamma)$, this optimization problem is depicted in Figure 3

As the solution of the problem we obtain: [18]

Proposition 6. *The optimal fraction of wealth held as money is*

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi} & \text{if} \quad \gamma > \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ \in [0, \dfrac{\alpha_2\cdot\pi}{\beta_\ell + (\alpha_2-\beta_\ell)\cdot\pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}. \end{cases}$$

[17] For the precise expression for $\mathcal{V}_o(\mu;\gamma)$ see Appendix A, Lemma 14.
[18] For the exact expression for $\mathcal{V}_o^*(\gamma)$ see Appendix A, Lemma 15.

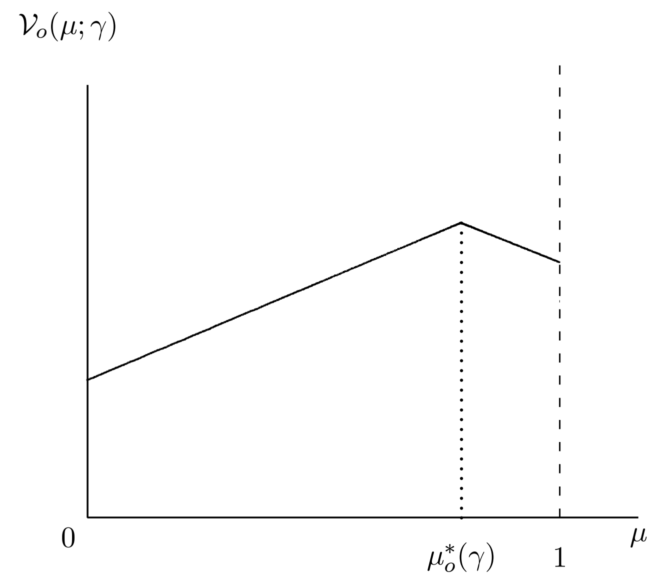

Figure 3: Optimal reserves

The corresponding ex-ante expected utility of the consumers is

$$\mathcal{V}_o^*(\gamma) := \quad \mathcal{V}_o(\mu_o^*(\gamma);\gamma)$$

The ex-ante obtainable Choquet expected utility of a representative consumer in the absence of intermediary institutions was derived in the previous section. It yields a lower bound for what can be achieved through intermediation. [19] The optimal contract discussed in this section provides an upper bound for what an intermediary institution can achieve. The following diagram shows the obtainable ex-ante Choquet expected utility levels for the case of no intermediation and for the optimal contract.

In the following sections, we investigate different institutional settings and evaluate them in their capability to achieve an optimal allocation. We deal successively with an asset market for the illiquid asset, with a mutual fund arrangement, and with deposit contracts offered by competing banks.

[19] Assuming the consumers can always decline to use intermediation.

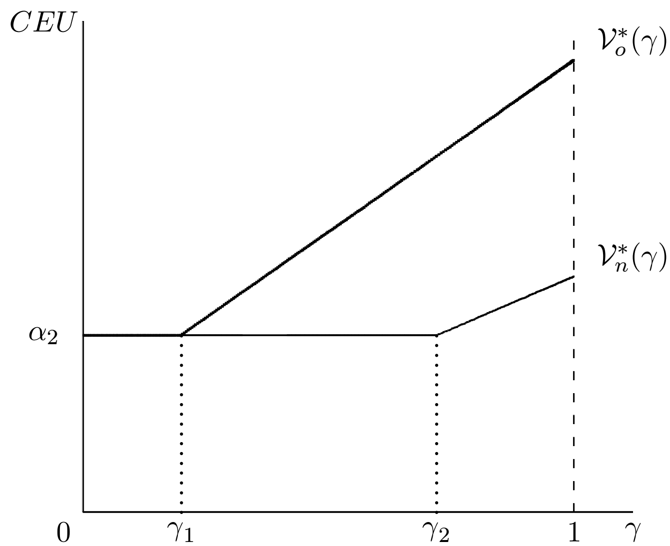

Figure 4: Optimal contract vs. no intermediation

4. Asset market and mutual fund

In this section we consider two types of markets:

- a market for claims on the illiquid asset and
- a market for claims on the ex-dividend portfolio of a mutual fund.

We refer to both types of claims as securities. In both cases, there is a market for securities in Period 1 in which the claims are traded.

4.1. Market for securities. As a first institutional environment, suppose that in Period 1 consumers can directly or indirectly sell claims to their investment in the illiquid asset. The possibility to trade claims on the asset makes this investment more liquid and provides an extra incentive to invest in it. As before, consumers decide in Period 0 how much of their wealth to invest in the asset and how much to keep as liquid money holdings.

Let us consider first the security market in Period 1. At this stage, the aggregate investment policy μ is given, the types are private knowledge and the actual proportion τ of h-types is common knowledge. Since all consumers are identical in Period 0, individual investment

policies can be assumed to equal their aggregates. Denote by q the price of a security that is a claim on one unit of the illiquid asset in terms of money. If this price is high enough, consumers of type h, who hold some securities, will try to sell them in order to benefit from their high preference for liquidity. The aggregate supply of the securities is:

$$s(q) := \begin{cases} 1-\mu & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [\tau \cdot (1-\mu), 1-\mu] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ \tau \cdot (1-\mu) & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [0, \tau \cdot (1-\mu)] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ 0 & \text{for} \quad \frac{\alpha_2}{\beta_h} > q. \end{cases}$$

For very high prices, $q > \frac{\alpha_2}{\beta_\ell}$, both types of consumers would like to sell their securities. For very low prices, $\frac{\alpha_2}{\beta_h} > q$, no one wants to sell them.

Similarly, consumers of type ℓ will want to buy securities if the price q is not too high. Hence, one obtains the following aggregate demand for securities:

$$d(q) := \begin{cases} 0 & \text{for} \quad q > \frac{\alpha_2}{\beta_\ell} \\ [0, (1-\tau) \cdot \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_\ell} \\ (1-\tau) \cdot \frac{\mu}{q} & \text{for} \quad \frac{\alpha_2}{\beta_\ell} > q > \frac{\alpha_2}{\beta_h} \\ [(1-\tau) \cdot \frac{\mu}{q}, \frac{\mu}{q}] & \text{for} \quad q = \frac{\alpha_2}{\beta_h} \\ \frac{\mu}{q} & \text{for} \quad q < \frac{\alpha_2}{\beta_h}. \end{cases}$$

The demand and supply for the security are depicted in Figure 5.

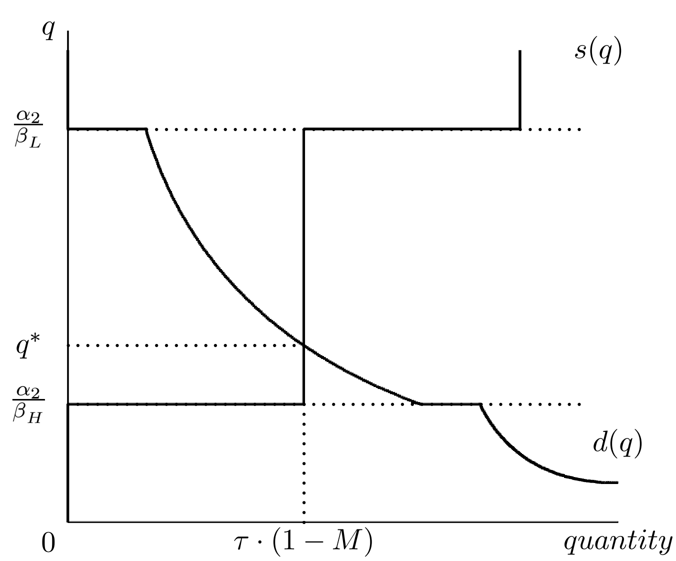

Figure 5: Market for securities

In equilibrium, the market for securities clears, which requires a price in the range $\frac{\alpha_2}{\beta_\ell} \geq q \geq \frac{\alpha_2}{\beta_h}$. The equilibrium price depends on the proportion of $h-$types τ and the aggregate investment policy μ:

$$q^E(\tau, \mu) := \begin{cases} \frac{\alpha_2}{\beta_\ell} & \text{if} \quad \tau < \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2} \\ \frac{1-\tau}{\tau} \cdot \frac{\mu}{1-\mu} & \text{if} \quad \tau \in [\frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}, \frac{\mu \cdot \beta_\ell}{\mu \cdot \beta_\ell + (1-\mu) \cdot \alpha_2}] \\ \frac{\alpha_2}{\beta_h} & \text{if} \quad \tau > \frac{\mu \cdot \beta_h}{\mu \cdot \beta_h + (1-\mu) \cdot \alpha_2}. \end{cases}$$

For equilibrium prices $q^E(\tau, \mu) \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$, all consumers of type h sell their securities and all consumers of type ℓ use their money holdings to buy securities.

4.2. Asset Market: Claims to the illiquid asset. We now turn to the investment decision in Period 0. Since there is a continuum of consumers, a single consumer's share in the aggregate investment is negligible.

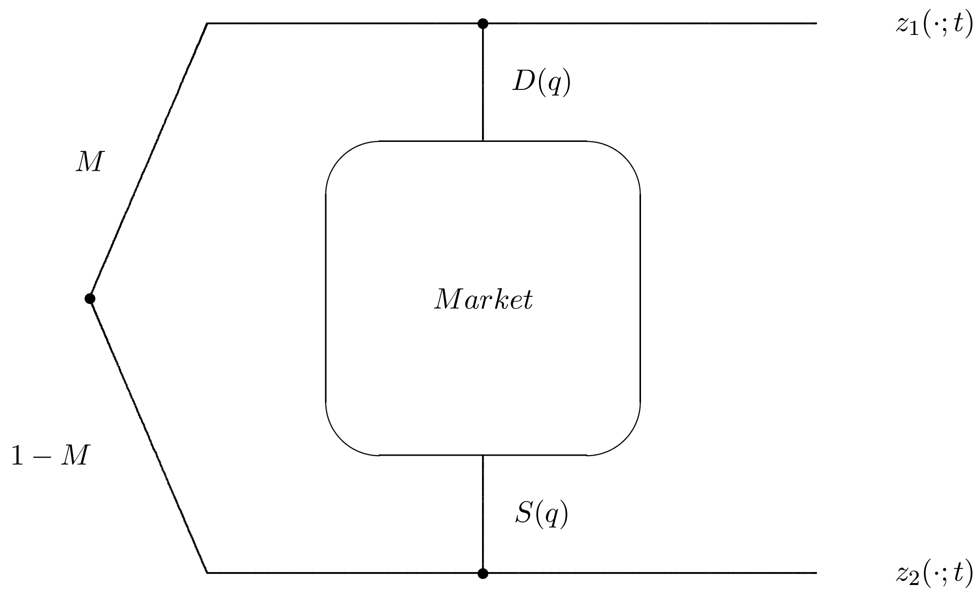

Figure 6: Asset market

Denote by $R^m(q_a; t) := \max\{\beta_t, \frac{\alpha_2}{q_a}\}$ and $R^a(q_a; t) =: \max\{\beta_t \cdot q_a, \alpha_2\}$ the implicit returns on holding money and the illiquid asset, respectively, given a trading opportunity in the asset market at a price q_a. The indirect interim utility of a type t consumer, $v_a(M; \tau, \mu; t)$, from the investment decision M is easily computed:

$$v_a(M; \tau, \mu; t) := M \cdot R^m(q_a^E(\tau, \mu); t) + (1 - M) \cdot R^a(q_a^E(\tau, \mu); t).$$

The subscript a of the indirect utility function denotes that the indirect utility is obtained in the institutional framework of an asset market. A consumer's prediction of the equilibrium asset price $q_a^E(\tau,\mu)$ is subject to the uncertainty the consumer faces regarding the proportion of h-types τ. [20] As before, this uncertainty is modelled by the degree of confidence γ which consumers hold in the point expectation π. The ex-ante Choquet expected indirect utility now easily follows:

$$\begin{aligned}\mathcal{V}_a(M,\mu;\gamma) := \quad & \gamma\cdot[\pi\cdot v_a(M;\pi,\mu;h)+(1-\pi)\cdot v_a(M;\pi,\mu;\ell)] \\ & + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} [v_a(M;\tau,\mu;t)]\,.\end{aligned}$$

Consumers choose their investment M to maximize their ex-ante utility $\mathcal{V}_a(M,\mu;\gamma)$ for given aggregate money holdings μ. In an asset market equilibrium, $(q_a^E(\tau,\mu_a^*(\gamma)),\mu_a^*(\gamma))$, consumers choose investment policies consistent with the aggregate money holdings $\mu_a^*(\gamma)$, and the asset price $q_a^E((\tau,\mu_a^*(\gamma))$ clears the market in Period 1 for each $\tau\in[0,1]$. We denote the equilibrium asset price for $\tau=\pi$ by $q_a^*(\gamma)$.

The individual money holdings as fractions of wealth are assumed to equal the aggregate proportion of wealth held as money, i.e. $M=\mu$. As all consumers are ex-ante identical, this assumption is without loss of generality.

Proposition 16 in Appendix A characterizes the equilibrium in the asset market and the ex-ante Choquet expected utility of a representative consumer. The following Corollary restates its main result.

Corollary 7. Equilibrium in the asset market

There exists a unique equilibrium $(q_a^*(\gamma),\,\mu_a^*(\gamma))$ in the asset market which satisfies:

- *if* $\gamma=1,$ *then* $q_a^*(\gamma)=1$ *and* $\mu^*(\gamma)=\pi,$
- *if* $\gamma\in(\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell},1),$ *then* $q_a^*(\gamma)\in(\frac{\alpha_2}{\beta_h},1)$ *and* $\mu^*(\gamma)\in(\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h},\pi),$
- *if* $\gamma\in[0,\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma)=\frac{\alpha_2}{\beta_h}$ *and* $\mu^*(\gamma)\in[0,\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The corresponding ex-ante expected utility of a consumer is denoted as

$$\mathcal{V}_a^*(\gamma) := \quad \mathcal{V}_a(\mu_a^*(\gamma),\mu_a^*(\gamma);\gamma).$$

[20] For a precise expression of the equilibrium price q_a^E see Appendix A, Remark 6.

Intuitively, equilibrium is obtained when the ex-ante Choquet expected return from investing in assets, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$, equals that of money holdings, $\widehat{\mathcal{V}}_a^a(q_a;\gamma)$. [21] Thus, the equilibrium investment policy is obtained as in Figure 7.

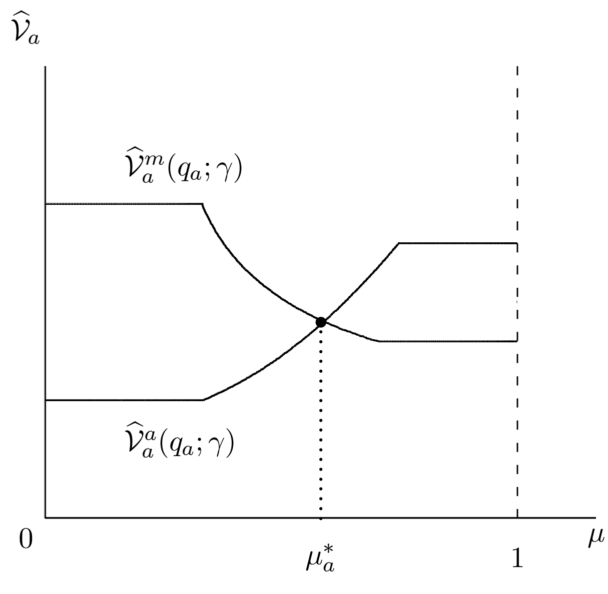

Figure 7: Equilibrium investment policy

In an asset market equilibrium, the asset price q must fulfil a dual role:

- it must clear the market in Period 1 for given holdings of money and the illiquid asset, and
- it must yield equal Choquet expected returns from holding money and from investing in the illiquid asset in Period 0.

If the latter condition were not satisfied, consumers would either hold only money or only the illiquid asset, and no trade would occur in Period 1. As we demonstrate below, this overload impairs the asset market's potential to achieve the optimal allocation in all but the trivial case of an equilibrium without trade. As an institutional arrangement, however, the asset market may dominate the other intermediary institutions.

[21] See Appendix A, Remark 7.

4.3. Mutual fund: claims to a portfolio. A mutual fund is an intermediary in which consumers invest money in Period 0, in return for shares in it. These shares entitle the holder to dividend payments (d_1, d_2) in Period 1 and in Period 2. In Period 1 there is a market where the ex-dividend shares in the mutual fund are traded. In order to obtain the most favourable case of a mutual fund for consumers, its investment policy is determined such that consumers' ex-ante welfare is maximized. Moreover, the fund charges no fees and makes no profits.

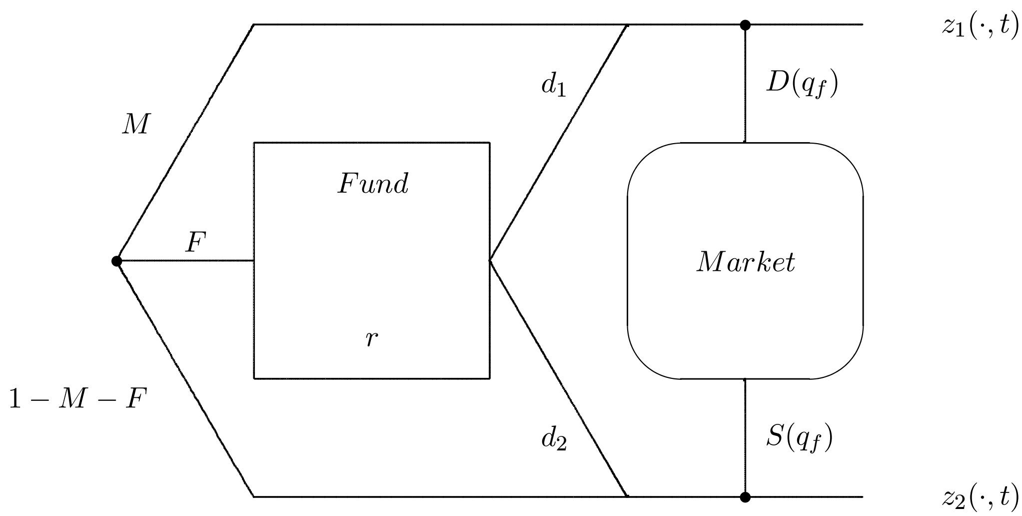

Figure 8: Mutual fund

Denote by r the fraction of its resources which the mutual fund holds as money. The mutual fund's choice of investment policy r determines its dividends (d_1, d_2),

$$\begin{aligned} d_1 &:= \quad r \\ d_2 &:= \quad \alpha_2 \cdot (1 - r). \end{aligned}$$

Since dividends (d_1, d_2) are independent of the fraction τ of h-types, the mutual fund can fulfil its obligations for every realization $\tau \in [0, 1]$.

Let μ be the aggregate money holdings in the economy, denote by φ the fraction total wealth invested in the fund in Period 0, and by ψ the fraction aggregate private money holdings. We assume the individual investment policies equal the aggregate investment policy. [22] Let q_f

[22] As before, this assumption is without loss of generality, since the consumers are identical ex-ante.

denote the price of ex-dividend shares on the fund in Period 1.[23] If the price q_f satisfies the condition

$$\beta_h \cdot q_f > \alpha_2 \cdot (1-r) > \beta_\ell \cdot q_f,$$

then type h consumers sell their shares, which represent an initially invested wealth of $\tau \cdot \varphi$, while type ℓ consumers buy as many shares as they can given their money holdings $(1-\tau) \cdot \mu$, which include their Period 1 dividends. Notice that the aggregate money holdings μ consist of the aggregate private money holdings, ψ, and the reserves $\varphi \cdot r$ of the mutual fund. A market equilibrium obtains for an ex-dividend price q_f^E which satisfies

$$(\psi + \varphi \cdot r) \cdot (1-\tau) = q_f^E \cdot \tau \cdot \varphi.$$

The equilibrium price, $q_f^E(\tau; \psi, \varphi, r)$, depends on the aggregate private money holdings ψ, the total investment in the mutual fund φ, the fund's reserve policy r, and the proportion of type h consumers, τ.

It remains to determine the aggregate investment decisions of the consumers in Period 0, (ψ, φ), and the reserve holdings of the mutual fund r. The outcome depends on whether or not asset-backed short selling of shares in the mutual fund is allowed. We first study the case with a short-selling constraint, and then proceed to analyse the case where short selling of shares of the mutual fund is allowed.

Suppose that short selling of shares is not permitted. Consumers who invest the fraction F of their wealth in the mutual fund, hold the fraction M as money, and buy $(1-F-M)$ units of the illiquid asset realize an interim indirect utility denoted by [24]

$$v_f(F, M; \tau, \psi, \varphi, r; t).$$

Allowing for the consumers' ambiguity about their own type and about the true distribution of types, the ex-ante Choquet expected indirect utility of the representative consumer can be

[23] An ex-dividend share in the mutual fund represents a claim on $(1-r)$ assets. Therefore, we have $q_f = (1-r) \cdot q$, where q is the price of one unit of securities as in Section 4.1.

[24] See Appendix A, Remark 8 for an explicit formulation of the interim indirect utility function.

written as:

$$\mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma) := \gamma\cdot[\pi\cdot v_f(F,M;\pi,\psi,\varphi,r;h) + (1-\pi)\cdot v_f(F,M;\pi,\psi,\varphi,r;\ell)] + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F,M;\tau,\psi,\varphi,r;t).$$

Taking into account the optimising behaviour of the consumer, given the aggregate investment decisions (ψ,φ) of the other agents and the investment policy r of the mutual fund, we obtain:

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) := \max_{(F,M)\in[0,1]\times[0,1]} \mathcal{V}_f(F,M;\tau,\psi,\varphi,r;\gamma)$$

$$\text{s.t. } F+M\leq 1.$$

The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of the consumers. Thus, its optimization problem for given (ψ,φ) is [25]

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

This optimization problem is illustrated in the Figure 9.

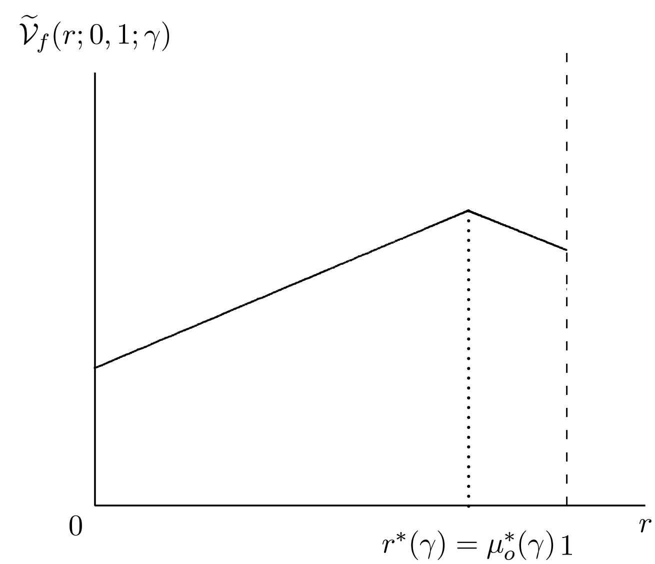

Figure 9: Mutual fund reserve holdings

Note that, as no assets are ever liquidated, this problem is identical to that of finding the reserve holdings for the optimal contract.

[25] See Appendix A, Lemma 17.

Proposition 8. [26] *If asset-backed short selling of shares of the mutual fund is not permitted, then a mutual-fund equilibrium exists with $\varphi^* = 1$ and $r^*(\gamma) = \mu_o^*(\gamma)$.*

For all $\gamma \in [0,1]$ the corresponding ex-ante Choquet expected utility is

$$\mathcal{V}_f^*(\gamma) := \widetilde{\mathcal{V}}_f(r^*(\gamma); 0, 1; \gamma) = \mathcal{V}_o^*(\gamma).$$

Proposition 8 is a remarkable result. It shows that a mutual fund can implement the optimal allocation for any level of confidence, provided short sales are not permitted. The intuition for this result is straightforward. A mutual fund is similar to the asset market discussed above with one difference. The investment policy allows the fund to manipulate the dividends and, in consequence, the share price. Proposition 8 shows that a mutual fund should invest exactly as the optimal allocation requires it, $r^* = \mu_o^*$. In this way a dividend stream is produced which leads to a share price that induces the optimal allocation.

Note that, for any positive reserve holding r, investing in the fund dominates investing in the illiquid asset. For this, the no short-sale constraint in the mutual fund share market is essential. If short selling of the mutual fund's shares is permitted in Period 1, then investment in the mutual fund must not dominate investment in the illiquid asset, as otherwise arbitrage is possible. Hence, the equilibrium in the economy with a mutual fund equals the asset market equilibrium. Short selling opens the opportunity for consumers to "free ride" on the coordination efforts of the mutual fund.

Proposition 9. [27] *If short selling of asset-backed shares of the fund is permitted, then for any mutual-fund equilibrium (ψ^*, φ^*, r^*) we have $\varphi^* + \psi^* = \mu_a^*(\gamma)$.*

The ex-ante expected utility is in this case, for all $\gamma \in [0,1]$,

$$\mathcal{V}_f^*(\gamma) = \mathcal{V}_a^*(\gamma).$$

[26] See Appendix A, Remark 10.

[27] See Appendix A, Remark 10.

One may wonder how competition amongst several mutual funds would affect these equilibria. Suppose the market for mutual funds is competitive and characterized by free entry. Then competition for consumers drives the market equilibrium to the allocation obtained with the asset market. Otherwise some mutual fund could make arbitrage profits by investing all its resources in illiquid assets. Hence, in order to obtain ex-ante efficiency, competition between mutual funds must be ruled out.

5. Banks

In this section, liquidity is provided by a competing banks. Banks offer deposit contracts to their customers. Deposit contracts specify the repayments for withdrawals in Period 1 or in Period 2 according to the following rules:

1. Withdrawals in Period 1 are treated as senior to withdrawals in Period 2. If withdrawals in Period 1 exceed a bank's reserves, then consumers withdrawing their deposit in Period 1 are served at the expense of consumers who will withdraw in Period 2.

2. Within each period, consumers have the same priority. If the withdrawals exceed the resources of the bank, then consumers calling back their deposits obtain a repayment proportional to their initial deposit.

In Period 0, consumers deposit their wealth with the banks, and banks decide how to invest these funds. Based on a prediction about the withdrawals in Period 1, banks hold part of the deposited wealth as reserves in the form of money and invest the remainder in the illiquid asset.

Competition between banks is assumed to be Bertrand competition in deposit contracts. This ensures zero profits from deposit contracts, as free entry would. It guarantees deposit contracts which maximize the representative consumers' ex-ante utility. [28] These assumptions allow us to portray the competing banks by a representative bank.

[28] Competition among banks is in the spirit of Allen and Gale (1998, [1]). For a more extensive discussion in a similar context we refer to Spanjers (1999, [21], Chapter 3).

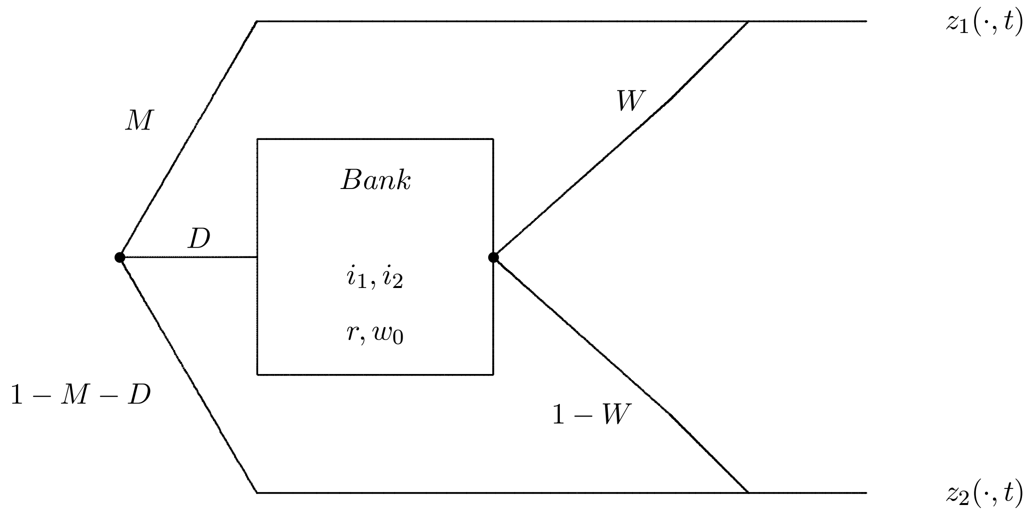

Figure 10: Banks

5.1. The deposit contract. The deposit contract of a bank is characterized by the interest rates (i_1, i_2) paid in periods 1 and 2, respectively. Since liquidation of funds invested in the illiquid asset is costly, $\alpha_1 < 1$, the bank holds reserves r equal to the withdrawals it predicts for Period 1, w_0. If a fraction w_0 of its depositors withdraws their funds in Period 1, the bank has to pay out $(1 + i_1) \cdot w_0$. Hence, reserve holdings will be equal to

$$r = (1 + i_1) \cdot w_0.$$

Since free entry to banking enforces zero profits, the bank must pay out all its returns from investment in Period 2, $\alpha_2 \cdot (1 - r)$, to those depositors who did not withdraw in Period 1. [29] With an initial amount of deposits equal to 1, the zero-profit condition determines the interest rate i_2 :

$$(1 + i_2) \cdot (1 - w_0) = \alpha_2 \cdot (1 - r).$$

Given the reserve policy r and the zero-profit assumption, the interest rates (i_1, i_2) are determined as functions of the reserves r and the predicted withdrawals w_0:

$$i_1(r, w_0) = \frac{r}{w_0} - 1, \qquad i_2(r, w_0) = \frac{\alpha_2 \cdot (1 - r)}{1 - w_0} - 1.$$

[29] If the zero profit condition holds w.r.t. $r = (1 + i_1) \cdot w_0$ and withdrawals w_0, the bank fails to make a profit for any other fraction of withdrawals w as well.

Notice that the zero-profit assumption directly relates the contracted interest rates (i_1, i_2) to the reserve holdings r and the predicted withdrawals w_0.

The feasibility of the contracted interest payments depends on the actual fraction of withdrawals w in Period 1. By nature of their high discount factor, consumers of type h will want to withdraw in Period 1. Hence, actual withdrawals in Period 1 depend on the distribution of types in the economy. Not knowing the true distribution of types when it offers the deposit contracts in Period 0, a bank is bound to make errors which force it to modify the contracted interest payments (i_1, i_2) according to the priority rules set out before:

- Period 1 claims are settled before Period 2 claims and
- consumers within a period are equally treated.

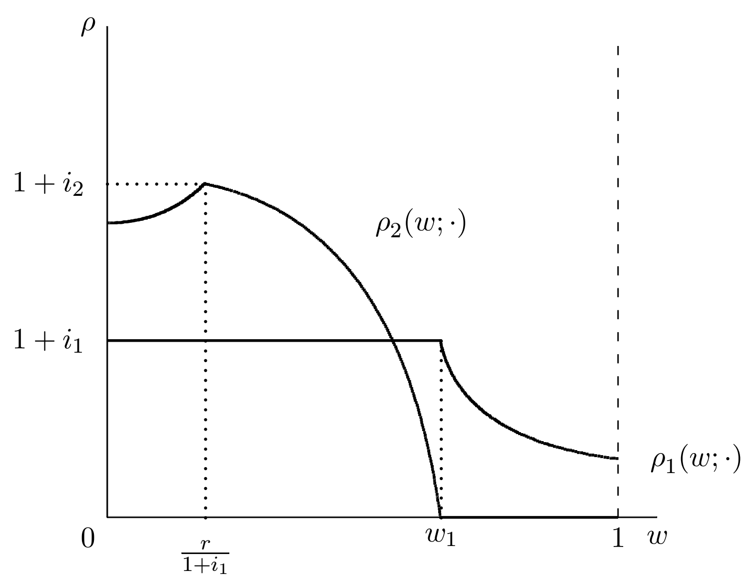

Figure 11: Actual returns of deposits

Since the contracted interest rates (i_1, i_2) are functions of the predicted withdrawals w_0 and the reserve holdings r, one can write the actual returns also as functions of these variables only, $\rho_1(w; r, w_0)$ and $\rho_2(w; r, w_0)$. [30] The actual returns of deposits as a function of the withdrawals in Period 1 are depicted in Figure 11.

[30] See also Appendix A, Remark 11.

5.2. The depositor's problem. In Period 0, consumers decide what fraction D of their initial wealth to deposit with the bank. This decision depends on their beliefs about their prospective type $t \in \{h, \ell\}$ and about the proportion τ of h-types in Period 1. Since in the absence of a bankrun only consumers of type h will withdraw their funds in Period 1, predicting the actual payout from a deposit contract corresponds to predicting the proportion of these types, $w = \tau$.

In Period 1, once the proportion of types τ is common knowledge, the actual payouts of the deposit contract are determined. At this stage, consumers, knowing their type t, will decide what fraction of their deposits, $W(\tau; t)$, to withdraw early, i.e., in Period 1, and what fraction, $1 - W(\tau; t)$, to leave with the bank until Period 2. Choosing the fraction $W(\tau; t)$ optimally, a type t consumer obtains an indirect utility from depositing with the bank equal to [31]

$$\begin{aligned} \widetilde{v}_b(\tau; r, w_0; t) := & \max_{W \in [0,1]} [W \cdot \beta_t \cdot \rho_1(\tau; r, w_0) + (1 - W) \cdot \rho_2(\tau; r, w_0)] \\ = & \max\{\beta_t \cdot \rho_1(\tau; r, w_0), \rho_2(\tau; r, w_0)\}. \end{aligned}$$

We recall from Section 3 that $v_n(M; t)$ is the indirect utility a type t consumer obtains from holding M units of money and investing $1 - M$ units of funds in the illiquid asset. In Period 0, consumers can choose what fraction D of their wealth they wish to deposit with the bank and what fraction M to hold as money. The remaining fraction $1 - D - M$ is invested in the illiquid asset. A choice of (D, M) yields a type t consumer the indirect interim utility

$$v_b(D, M; \tau, r, w_0; t) := D \cdot \widetilde{v}_b(\tau; r, w_0; t) + (1 - D) \cdot v_n(M; t),$$

if a fraction τ of consumers is of type h, and if the bank holds reserves r and predicts withdrawals w_0.

In Period 0, when deciding how much to deposit with the bank, consumers are still uncertain about their type t and the actual proportion of h-types in the population τ. As before, a consumer's beliefs about the proportion of $h-$types are assumed to be

- rational and concentrated on the true proportion of $h-$types π, but

[31] See Appendix A, Remark 12.

- held with a degree of confidence equal to γ which may be less than 1.

Given a deposit contract $(i_1(r, w_0), i_2(r, w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of wealth with a bank and holding a fraction M as money is

$$\begin{aligned} &\mathcal{V}_b(D, M; r, w_0; \gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D, M; \tau, r, w_0; h) + (1-\tau) \cdot v_b(D, M; \tau, r, w_0; \ell)] \cdot p(\tau)\, d\tau \\ &\qquad + (1-\gamma) \cdot \min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_b(D, M; \tau, r, w_0; t). \end{aligned}$$

The consumer's decision problem is to choose $D, M \in [0,1]$ such that $\mathcal{V}_b(D, M; r, w_0; \gamma)$ is maximized, i.e.

$$\max_{(D,M) \in [0,1] \times [0,1]} \quad \mathcal{V}_b(D, M; r, w_0; \gamma)$$

$$\text{s.t.} \quad D + M \leq 1,$$

the optimal value of which is denoted by $\widetilde{\mathcal{V}}_b(r, w_0; \gamma)$.

It remains to determine the deposit contract parameters (r, w_0). From Appendix A, Lemma 18 it follows that only two values for τ play a role in determining the deposit contract, i.e. $\tau = \pi$ and $\tau = 1$. The first value occurs when the additive probability assessment applies, whereas for the second value a bankrun is inevitable for any fraction of reserves $r < 1$. As a consequence, banks offer deposit contracts on the basis of $w_0 = \pi$. [32] Competition forces banks to choose the most favourable contract for the consumers. Thus, the decision problem of the bank reduces to

$$\max_{r \in [0,1]} \widetilde{\mathcal{V}}_b(r, \pi; \gamma).$$

This problem is depicted in Figure 12.

The outcome of the competitive banking sector is summarized in the following proposition: [33]

[32] Assuming this and disregarding $w_0 = 0$ and $w_0 = 1$ is without loss of generality. Both cases are implicitly covered when agents choose not to deposit their money in the bank.

[33] See Appendix A, Lemma 19 and Proposition 20.

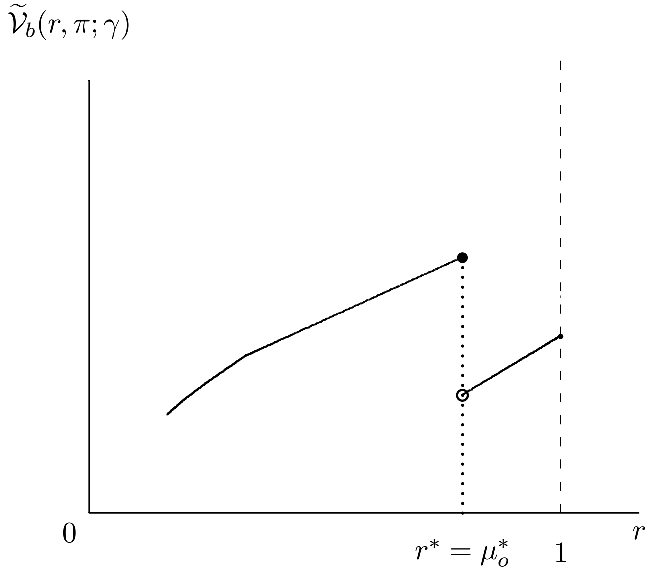

Figure 12: Bank reserves

Proposition 10. *There exist an equilibrium deposit contract* (r^*, w_0^*), *with* $w_0^* = \pi$ *and*

$$r^* = \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}$$

and some $\overline{\gamma} < 1$ *such that the equilibrium the aggregate level of deposits is*

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0,1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding ex-ante Choquet expected utility of the consumers is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, w_0^*; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

The reserves r^*, which the bank holds in order to accommodate withdrawals in Period 1, equal the money holdings of the optimal contract for levels of confidence $\gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. Comparing the payouts for $\tau = \pi$ shows that the equilibrium deposit contract is optimal if there is full confidence in the probability assessment, $\gamma = 1$. The "worst-case" pay-offs,

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \alpha_2$ in state $(t, \tau) = (\ell, 0)$ for the optimal contract, and

- $r^* \cdot \beta_\ell + (1 - r^*) \cdot \beta_\ell \cdot \alpha_1$ in state $(t, \tau) = (\ell, 1)$ for the equilibrium deposit contract,

however, differ as the worst case for the deposit contract involves the inefficient liquidation of assets. Thus, the equilibrium deposit contract in Proposition 10 is optimal if and only if $\gamma = 1$.

6. Banks, mutual funds, or asset markets?

Three types of intermediation were studied in order to assess their potential to deal with the liquidity allocation problem of the economy, namely

- direct trade of claims to the illiquid asset, the *asset market,*
- claims on a portfolio of money and the illiquid asset, the *mutual fund,* and
- deposit contracts issued by intermediary institutions, *banks.*

We consider a framework in which the allocation of funds among consumers according to their preference is severely impaired. One reason is that the individual preference for liquidity is private knowledge. Another, less familiar, cause is the ambiguity the consumers face both about their individual future preference for liquidity and about the aggregate liquidity demand. For low levels of confidence, even the best possible anonymous incentive-compatible allocation, the optimal contract, cannot improve upon the allocation which a consumer can obtain in isolation. Hence, in this case intermediation is superfluous. Let $\widehat{\gamma}$ be the critical level of confidence below which no intermediation is wanted. For values of γ above $\widehat{\gamma}$ the optimal contract yields an allocation which is preferred to the allocation without intermediation. This opens some scope for improvements by intermediary institutions.

A first approach to the problem suggests a single mutual fund without short-selling of its claims as the best alternative. It can implement the optimal liquidity allocation for any level of confidence γ. For the appropriate portfolio choice by the fund manager, a mutual fund can offer a payoff scheme which dominates the allocation under all other intermediary institutions. By choosing the right dividend scheme, the fund can influence its share price in Period 1 in a way that leaves the consumers better off in Period 0.

This advantage of the well-managed mutual fund is also its Achilles heel. By holding only the illiquid asset, a consumer can sell the same claims as the fund without any risk and still make a profit. This arbitrage must be prohibited in order to make a mutual fund with positive reserve holdings viable. A similar argument can be made when there is competition between mutual funds.

The need to enforce trading constraints in order to make a mutual fund practicable has been noted by Jacklin (1987, [13]) in a situation without ambiguity. [34] If short-selling constraints cannot be enforced, trading shares in the mutual fund is like trading claims to the illiquid asset.

When short-selling constraints cannot be enforced for a mutual fund, a comparison between competing banks and the asset market becomes important. The following result provides a full comparison of these two institutions.

Theorem 11. *Let* $\widehat{\gamma} = \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$. *There exists some level of confidence* $\widetilde{\gamma} \in (\widehat{\gamma}, 1)$ *such that,*

(i)	*for* $\gamma \leq \widehat{\gamma}$,	*consumers need no intermediation at all,*
(ii)	*for* $\widehat{\gamma} < \gamma < \widetilde{\gamma}$,	*consumers strictly prefer the asset market,*
(iii)	*for* $\gamma > \widetilde{\gamma}$,	*consumers strictly prefer competing banks.*

The result of Theorem 11 is remarkable because the evaluation of these two institutions depends only on the degree of uncertainty. Neither the individual discount factors nor the payoff parameters of the illiquid asset have any influence on this basic comparison, although they influence the critical levels of confidence $\widehat{\gamma}$ and $\widetilde{\gamma}$. Even the additive assessment of actual proportion of h-types in the economy, π, does not matter for this result.

Roughly speaking, three cases can arise:

1. For high degrees of ambiguity, i.e. low levels of confidence, neither banks nor the asset market can improve upon the investment opportunities without intermediation.

[34] Jacklin and Bhattacharya (1988, [14]) demonstrate how sensitive the dominance of mutual funds is to the predictability of aggregate liquidity needs.

2. For intermediate degrees of ambiguity, the asset market is preferred to the deposit contract offered by competing banks.

3. For low degrees of ambiguity, consumers prefer the outcome of banks over the asset market allocation.

The following diagram illustrates this result.

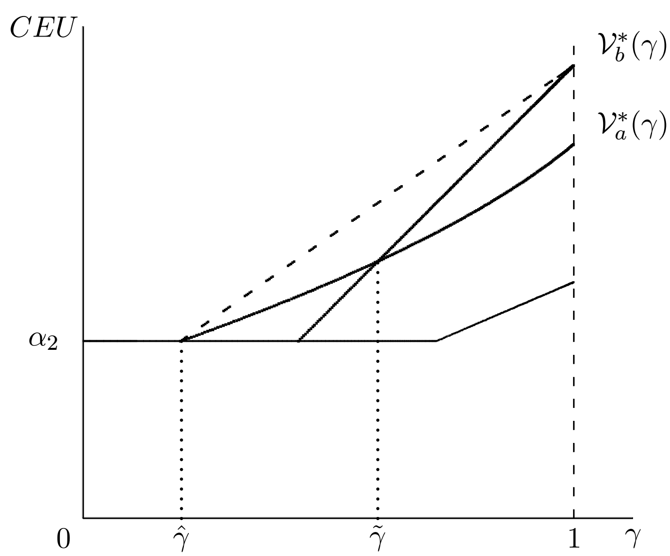

Figure 13: Evaluation of intermediary institutions

To obtain some insight into the reason for this ranking, consider the case of no ambiguity, $\gamma = 1$. The second column of the table below gives the ex-ante expected utility of the institutions. The fourth column indicates the state in which the worst expected utility would be encountered, and the third column lists the levels of utility in these worst states. The bank deposit contract depends on the optimal aggregate investment policy

$$r^* = \mu_o^* = \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1-\pi) \cdot \beta_\ell}.$$

$\gamma = 1$	Expected utility	Worst utility	Worst state
Asset market	$\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2$	$\pi \cdot \beta_\ell + (1-\pi) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$
Banks	$r^* \cdot \beta_h + (1-r^*) \cdot \alpha_2$	$\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r^*)$	$(t, \tau) = (\ell, 1)$
Optimal contract	$\mu_o^* \cdot \beta_h + (1-\mu_o^*) \cdot \alpha_2$	$\mu_o^* \cdot \beta_\ell + (1-\mu_o^*) \cdot \alpha_2$	$(t, \tau) = (\ell, 0)$

In case of no intermediation, a consumer's ex-ante utility does not depend on the fraction of type h consumers. The ranking of the other institutions according to expected utility, however, is reversed for the worst utility.

Ranking of intermediary institutions	
Expected utility:	asset market $\prec$ optimal allocation $\sim$ banks
Worst utility:	banks $\prec$ optimal allocation $\prec$ asset market.

With full confidence, $\gamma = 1$, bank deposit contracts yield the same expected utility as the optimal liquidity allocation and a higher expected utility than the asset market can achieve. Nevertheless, the worst possible utility for a consumer may occur with the bank deposit contract, if this consumer is of type ℓ and all other consumers are of type h and withdraw their funds. On the other hand, an asset market yields an even better utility than the optimal allocation in their worst state, where a consumer is of type ℓ and all other consumers are of type ℓ as well.

As the degree of confidence runs from 1 to 0 the utility in the worst case becomes more important than the expected utility. Though this comparison is more complicated, because the optimal aggregate investment policy $\mu_a^*(\gamma)$ for the asset market changes with the degree of confidence γ, the intuition remains correct. Hence, there is a switch over point $\widetilde{\gamma}$ below which the expected utility ranking of banks and asset markets is reversed.

Notice that, for the case of $\gamma = 1$, our analysis confirms the Diamond and Dybvig (1983, [5]) result that a bank deposit contract can implement the optimal allocation, and the Jacklin (1987, [13]) result that mutual funds without short sales will also achieve the optimal outcome. Moreover, as also noted by Jacklin, a mutual fund with short sales will not achieve a second-best efficient liquidity allocation, because it coincides with the asset market outcome.

7. Concluding remarks

In this paper, we have presented a model of an economy where consumers face uncertain liquidity needs and investment opportunities are illiquid. First-best efficient contracts are not feasible because consumers have private information about their true liquidity needs. This scenario is similar to a version of the Diamond and Dybvig (1983, [5]) model proposed by

Chari and Jagannathan (1988, [3]).

When consumers face no ambiguity, we obtain the well-known results that a bank deposit contract and a single mutual fund in the absence of short selling opportunities can implement a second best efficient liquidity allocation. Moreover, an asset market equilibrium where claims to the illiquid asset are traded directly, is shown to be equivalent to a mutual fund if short sales are permitted. The allocation of such an asset market (or mutual fund with short sale opportunities) is inferior to the allocation achieved by a bank deposit contract and by a mutual fund with short selling constraints.

A main contribution of this paper is the introduction of ambiguity, modelled here by the degree of confidence of consumers regarding their additive beliefs about the states of the world. If consumers are concerned about the worst outcome that may arise, then different intermediary institutions can no longer be judged according to the, maybe rationally, expected utility alone. The worst possible case needs to be given more emphasis than indicated by its probability. Introducing ambiguity into our analysis shows that the, from a purely expected utility point of view, dominated asset market allocation becomes attractive for consumers if ambiguity is sufficiently high. The reason is that the worst consequences of such an institutional framework are less dramatic than in the case of a bank deposit contract.

There is a more positive analytical aspect to our analysis. If one takes as a starting point an economy where the degree of confidence differs across the population, ranging from consumers facing extremely low degrees of ambiguity to those with very high degrees of ambiguity, then one may expect to see bank deposit contracts coexist with asset markets. Consumers with low degrees of confidence may prefer the asset market while consumers with high degrees of confidence may favour the bank deposit contract. In a different institutional setting, one may find consumers with high degrees of confidence providing equity to banks that offer deposit contracts, effectively insuring the consumers with low degrees of confidence. Such result would be in line with the finding of Diamond (1997, [4]) for risk averse consumers in the absence of ambiguity. Both settings appear to be promising avenues for future research.

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Appendix A: Results and Remarks

To Section 2 Choquet Expected Utility

Consider a set of states $S \subseteq \mathbb{R}$ and an associated algebra of subsets $\mathfrak{S}$. A capacity is a set-valued function representing the beliefs about events in $\mathfrak{S}$.

Definition 2. *Let $S \subseteq \mathbb{R}$ and $\mathfrak{S}$ be an algebra of subsets of S. A **capacity** is a function $\nu : \mathfrak{S} \to \mathbb{R}$ such that*

(i) $A, B \in \mathfrak{S}$, $A \subseteq B$ *implies* $\nu(A) \leq \nu(B)$, [monotonicity]

(ii) $\nu(\emptyset) = 0$ *and* $\nu(S) = 1$. [normalization]

Remark 1. *As special cases of capacities we consider:*

1. *A **probability distribution** which is a capacity such that, for all $A, B \in \mathfrak{S}$, $A \cap B = \emptyset$*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

2. *The capacity of **complete ignorance** ω defined by*

$$\omega(E) = 0 \text{ for all } E \subsetneqq S.$$

A real-valued function $f : S \to \mathbb{R}$, is **measurable** if the sets $\{s \in S | \ f(s) \geq t\}$ and $\{s \in S | \ f(s) > t\}$ are elements of $\mathfrak{S}$ for all $t \in \mathbb{R}$. Let F be the set of bounded and measurable real-valued functions on S. A weighted average of a function $f \in F$ with respect to a capacity ν is defined by the Choquet integral. [35]

Definition 3. *For any $f \in F$, the **Choquet integral** with respect to the capacity ν is defined as*

$$\overset{C}{\int} f \, d\nu = \int_0^\infty \nu(\{s \in S | \ f(s) \geq t\}) \, dt + \int_{-\infty}^0 [\nu(\{s \in S | \ f(s) \geq t\}) - 1] \, dt.$$

[35] Schmeidler (1989, [20]), Gilboa (1987, [11]), and Sarin and Wakker (1992, [18]) have axiomatized the Choquet integral as a representation of a decision maker's preferences in the Anscomb-Aumann and the Savage framework, respectively.

To Section 3.1 Investment without intermediation

Remark 2. *Omitting reference to the additive probability distribution p, the Choquet expected indirect utility from an investment decision M in Period 0 takes the following form:*

$$\begin{aligned}\mathcal{V}_n(M;\gamma) &= \gamma \cdot [\pi \cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)] + (1-\gamma)\cdot \min\{v_n(M;h),\ v_n(M;\ell)\}\\ &= M \cdot [\gamma \cdot [\pi \cdot \beta_h + (1-\pi)\cdot \beta_\ell] + (1-\gamma)\cdot \beta_\ell] + (1-M)\cdot \alpha_2\\ &= M \cdot [\gamma \cdot \pi \cdot \beta_h + (1-\gamma\cdot\pi)\cdot \beta_\ell] + (1-M)\cdot \alpha_2.\end{aligned}$$

To Section 3.2 Optimal contract

Assumption 2 implies that the optimal contract never liquidates existing assets in Period 1. This this is stated in the following Lemma.

Lemma 12. *For any $\mu \in [0,1]$ and $\tau \in [0,1]$, the maximal average utility is obtained for*

$$\lambda(\tau,\mu) = 0.$$

Since both types of consumers put the same value on wealth in Period 2, the optimal allocation fails to be unique at an optimum where the incentive compatibility constraints are not binding. In this case, the Period 2 payout of one type may be decreased and the proceeds can be used to increase the payout of the other type in Period 2 without violating the incentive compatibility constraints. The following proposition characterizes an optimal payout scheme.

Proposition 13. *For given μ let*

$$\underline{\tau}(\mu) := \frac{\beta_\ell \cdot \mu}{\beta_\ell \cdot \mu + \alpha_2 \cdot (1-\mu)},$$
$$\overline{\tau}(\mu) := \frac{\beta_h \cdot \mu}{\beta_h \cdot \mu + \alpha_2 \cdot (1-\mu)}.$$

An optimal incentive compatible payout scheme for μ and τ is characterized by

$$z_{1h}(\tau,\mu) = \begin{cases} \dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ \dfrac{\mu}{\tau} & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{1\ell}(\tau,\mu) = \begin{cases} \mu - \left(\dfrac{\tau}{1-\tau}\right) \cdot \left(\dfrac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}\right) & \text{if } \tau \in [0, \underline{\tau}(\mu)] \\ 0 & \text{if } \tau \in (\underline{\tau}(\mu), 1], \end{cases}$$

$$z_{2h}(\tau,\mu) = \begin{cases} 0 & \textit{if } \tau \in [0, \overline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) - \left(\frac{1-\tau}{\tau}\right) \cdot (\mu \cdot \beta_h) & \textit{if } \tau \in [\overline{\tau}(\mu), 1], \end{cases}$$

$$z_{2\ell}(\tau,\mu) = \begin{cases} \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} & \textit{if } \tau \in [0, \overline{\tau}(\mu)] \\ \alpha_2 \cdot (1-\mu) + \mu \cdot \beta_h & \textit{if } \tau \in (\overline{\tau}(\mu), 1]. \end{cases}$$

Remark 3. *Due to assumptions 3 and 4, the Choquet expected value of the indirect utility from the investment* μ, $\mathcal{V}_o(\mu;\gamma)$, *has the following form:*

$$\begin{aligned} \mathcal{V}_o(\mu;\gamma) := \quad & \gamma \cdot \int_0^1 [\tau \cdot v_o(\tau,\mu;h) + (1-\tau) \cdot v_o(\tau,\mu;\ell)] \cdot p(\tau)\ d\tau \\ & \qquad + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t) \\ = \quad & \gamma \cdot [\pi \cdot v_o(\pi,\mu;h) + (1-\pi) \cdot v_o(\pi,\mu;\ell)] + (1-\gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_o(\tau,\mu;t). \end{aligned}$$

From Proposition 13, straightforward calculation yields the following result.

Lemma 14. *For any* $\mu \in [0,1]$ *we have*

$$\mathcal{V}_o(\mu;\gamma) = \begin{cases} [1 + \gamma \cdot (\frac{\beta_h}{\beta_\ell} - 1) \cdot \tau] \cdot [\alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu] & \textit{if} \quad \pi \in [0, \underline{\tau}(\mu)) \\ \alpha_2 \cdot (1-\mu) + [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] \cdot \mu & \textit{if} \quad \pi \in [\underline{\tau}(\mu), 1]. \end{cases}$$

Remark 4. *The optimal investment policy* μ *is determined as the solution to the following optimization problem:*

$$\max_{\mu\in[0,1]} \quad \mathcal{V}_o(\mu;\gamma),$$

where $\mathcal{V}_o(\mu;\gamma)$ *denotes the Choquet expected value of the indirect utility from the investment policy* μ. *This problem is complicated because the critical population shares of type* h *consumers,* $\underline{\tau}(\mu)$ *and* $\overline{\tau}(\mu)$, *depend on the investment policy.*

Remark 5. *For levels of confidence that exceed a threshold value, the optimal investment policy is characterized by the maximal level of money holdings for which the self selections constraint of type ℓ consumers still holds when:*

- *the proportion of h types is π,*
- *the money holdings are distributed evenly among the h types in Period 1 and*
- *the illiquid assets are distributed evenly among the ℓ types in Period 2.*

This investment policy is denoted by:

$$\mu_\ell := \frac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}.$$

Lemma 15. *The corresponding ex-ante expected utility of the consumers is*

$$\begin{aligned} \mathcal{V}_o^*(\gamma) :=& \quad \mathcal{V}_o(\mu^*(\gamma); \gamma) \\ =& \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \le \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases} \end{aligned}$$

To Section 4.2 Asset market

Solving for the equilibrium price q_a^E for the parameters of the model requires the solving of a quadratic equation. The function $\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$ provides the unique positive root of this quadratic equation:

Remark 6. *The equilibrium price in the asset market in Period 1 is*

$$\begin{aligned} &\widehat{q}_a^E(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell) := \frac{1}{2 \cdot \pi \cdot \beta_h} \cdot \\ &\quad \Big[- \Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big) \\ &\quad + \sqrt{\Big(\alpha_2 - \pi \cdot (\alpha_2 + \beta_h) + \tfrac{1-\gamma}{\gamma} \cdot (\alpha_2 - \beta_\ell) \Big)^2 + 4 \cdot \pi \cdot (1 - \pi) \cdot \alpha_2 \cdot \beta_h} \Big]. \end{aligned}$$

For the expression from which this equilibrium price is derived, see Appendix B, Proof of Proposition 16, case (iii).

Proposition 16 characterizes the equilibrium in the asset market and the resulting ex-ante Choquet expected utility of the consumers.

Proposition 16. Equilibrium in the asset market

An equilibrium $(q_a^*(\gamma),\ \mu_a^*(\gamma))$ *in the asset market exists and the price*

$$q_a^*(\gamma) = \min\left\{\frac{\alpha_2}{\beta_\ell}, \max\{\frac{\alpha_2}{\beta_h}, f(\gamma, \pi, \alpha_2, \beta_h, \beta_\ell)\}\right\}$$

is unique. The equilibrium satisfies:

- $\gamma = 1,$ *then* $q_a^*(\gamma) = 1$ *and* $\mu_a^*(\gamma) = \pi,$
- $\gamma \in (\frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}, 1),$ *then* $q_a^*(\gamma) \in (\frac{\alpha_2}{\beta_h}, 1)$ *and* $\mu_a^*(\gamma) \in (\frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}, \pi),$
- $\gamma \in [0, \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}],$ *then* $q_a^*(\gamma) = \frac{\alpha_2}{\beta_h}$ *and* $\mu_a^*(\gamma) \in [0, \frac{\pi\cdot\alpha_2}{\pi\cdot\alpha_2+(1-\pi)\cdot\beta_h}].$

The ex-ante Choquet expected utility of a consumer is

$$\begin{aligned}
\mathcal{V}_a^*(\gamma) := \;& \mathcal{V}_a(\mu_a^*(\gamma), \mu_a^*(\gamma); \gamma) \\
= \;& \max\{\gamma \cdot \left[\pi \cdot \max\{\beta_h, \tfrac{\alpha_2}{q_a^*(\gamma)}\} + (1-\pi)\cdot \max\{\beta_\ell, \tfrac{\alpha_2}{q_a^*(\gamma)}\}\right] + (1-\gamma)\cdot\beta_\ell, \\
& \gamma \cdot [\pi \cdot \max\{\beta_h \cdot q_a^*(\gamma), \alpha_2\} + (1-\pi)\cdot \max\{\beta_\ell \cdot q_a^*(\gamma), \alpha_2\}] + (1-\gamma)\cdot \alpha_2\}.
\end{aligned}$$

Remark 7. *The ex-ante Choquet expected indirect utility from one unit of money holdings is*

$$\widehat{\mathcal{V}}_a^m(q_a; \gamma) = \gamma \cdot [\pi \cdot R^m(q_a; h) + (1-\pi)\cdot R^m(q_a; \ell)] + (1-\gamma)\cdot \beta_\ell.$$

Similarly we obtain as the ex-ante utility for one unit of the illiquid asset

$$\widehat{\mathcal{V}}_a^a(q_a; \gamma) = \gamma \cdot [\pi \cdot R^a(q_a; h) + (1-\pi)\cdot R^a(q_a; \ell)] + (1-\gamma)\cdot \alpha_2.$$

The worst case for holding money occurs for $(t, \tau) = (\ell, 0)$. *For investing in the illiquid asset it occurs for* $(t, \tau) = (\ell, \tau)$ *for any* $\tau \in [0, 1]$. *From this it follows that for an asset price* q_a *the ex-ante Choquet expected utility for a consumer who holds a fraction* M *of his initial one unit of wealth as money is*

$$M \cdot \widehat{\mathcal{V}}_a^m(q_a; \gamma) + (1-M)\cdot \widehat{\mathcal{V}}_a^a(q_a; \gamma).$$

Note that it is important that the worst case for holding all wealth as money and for investing all wealth in the asset occurs for the same combination $(t,\tau) = (\ell, 0)$. *When this is not the case, investment policy* M *may affect the state* (t,τ) *for which the worst case is obtained. As a consequence, the resulting Choquet expected utility may fail to be the simple convex combination of* $\widehat{\mathcal{V}}_a^m$ *and* $\widehat{\mathcal{V}}_a^a$.

To Section 4.3 Mutual fund

Remark 8. *Suppose that short selling of shares is not permitted. Consumers who invest the fraction* F *of their wealth in the mutual fund, hold the fraction* M *as money, and invest the remaining fraction* $(1 - F - M)$ *in the illiquid asset, realize an interim utility of*

$$v_f(F, M; \tau, \psi, \varphi, r; h) := \beta_h \cdot [r \cdot F + q_f^*(\tau; \psi, \varphi, r) \cdot F + M] + \alpha_2 \cdot (1 - F - M),$$

if their type is h, *and*

$$v_f(F, M; \tau, \psi, \varphi, r; \ell) := \alpha_2 \cdot [\frac{(M + r \cdot \varphi)}{q_f^*(\tau; \psi, \varphi, r)} + (1 - r)] + \alpha_2 \cdot (1 - F - M),$$

if their type is ℓ.

Remark 9. *Given the consumers' ambiguity about their own type and the true proportion of* $h-$*types, the ex-ante Choquet expected indirect utility of the consumers is:*

$$\widetilde{\mathcal{V}}_f(r; \psi, \varphi; \gamma) := \max_{\substack{(F,M)\in[0,1]\times[0,1] \\ s.t.\ F+M\leq 1}} \gamma \cdot [\pi \cdot v_f(F, M; \pi, \psi, \varphi, r; h) + (1 - \pi) \cdot v_f(F, M; \pi, \psi, \varphi, r; \ell)]$$

$$+(1 - \gamma) \cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_f(F, M; \tau, \psi, \varphi, r; t).$$

For type ℓ *consumers, the worst case occurs when* $q_f^*(\tau; \psi, \varphi, r)$ *attains its upper bound at* $\frac{\alpha_2\cdot(1-r)}{\beta_\ell}$. *In this case a consumer's utility equals*

$$\alpha_2 + (\beta_\ell - \alpha_2) \cdot M + (r \cdot \beta_\ell - \alpha_2) \cdot F.$$

This expression is decreasing both in M *and in* F. *Consumers of type* h *can always obtain a utility level at least as high as the consumers of type* ℓ *by copying the latter's behaviour.*

Lemma 17. *For any $\psi, \varphi \in [0,1]$ we have*

$$\widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma) = \max_{r\in[0,1]} \gamma \cdot [\pi \cdot \beta_h \cdot (r + q_f^*(\pi;\psi,\varphi,r)) + (1-\pi)\cdot \alpha_2 \cdot [\tfrac{r\cdot(1-r)}{q_f^*(\pi;\psi,\varphi,r)} + (1-r)]$$
$$+(1-\gamma)\cdot[r\cdot\beta_\ell + \alpha_2 \cdot (1-r)].$$

Remark 10. *The mutual fund chooses its reserve holdings r so that it maximizes the ex-ante utility of its investors for given (ψ,φ). Thus, its decision problem for given (ψ,φ) is*

$$\max_{r\in[0,1]} \widetilde{\mathcal{V}}_f(r;\psi,\varphi;\gamma).$$

As a result, we consider a Nash-equilibrium in the implied game between the mutual fund and the consumers.

An alternative way to model the behaviour of the mutual fund would be to assume that it chooses its investment policy, anticipating individual investment policies that are a Nash-equilibrium given the investment policy of the mutual fund. This alternative way of modelling would affect the outcomes as follows:

- *when short-selling is not allowed, the optimal outcome is the unique equilibrium, whereas in the model considered in this paper, there is another equilibrium, in which no consumers pays into a mutual fund which chooses either $r^* = 0$ or $r^* = 1$.*
- *when short-selling is allowed, any reserve holding r^* of the mutual fund is compatible with equilibrium. For the aggregate investment policy, it would continue to hold that $\mu^* := \varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$.*

To Section 5 Banks

Remark 11. *The actual payouts of a deposit contract with contracted interest rates (i_1, i_2) for reserve holdings r and an actual fraction of withdrawals w are*

$$\widehat{\rho}_1(w;i_1,i_2,r) := \begin{cases} (1+i_1) & \text{if} \quad w \le \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \\[2ex] \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{w} & \text{if} \quad w > \dfrac{\alpha_1 + (1-\alpha_1)\cdot r}{1+i_1} \end{cases}$$

and

$$\widehat{\rho}_2(w;i_1,i_2,r) := \begin{cases} \dfrac{\alpha_2\cdot(1-r)+r-(1+i_1)\cdot w}{(1-w)} & \text{if} \quad w < \dfrac{r}{1+i_1} \\ 1+i_2 & \text{if} \quad w = \dfrac{r}{1+i_1} \\ \dfrac{\alpha_2}{(1-w)}\cdot[(1-r)-\dfrac{(w-r)}{\alpha_1}] & \text{if} \quad w \in \left(\frac{r}{1+i_1}, \frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}\right) \\ 0 & \text{if} \quad w \in \left[\frac{\alpha_1+(1-\alpha_1)\cdot r}{1+i_1}, 1\right]. \end{cases}$$

Notice that these actual payouts on deposits are feasible for any level of withdrawals. Moreover, this payout scheme guarantees zero profits of the bank.

Since the contracted interest rates (i_1, i_2) are functions of the reserve holding r and the predicted withdrawals w_0, one can write the actual return functions $\widehat{\rho}_1(\cdot)$ and $\widehat{\rho}_2(\cdot)$ also as functions of these variables only,

$$\rho_1(w;r,w_0) := \quad \widehat{\rho}_1(w;i_1(r,w_0),i_2(r,w_0),r),$$

$$\rho_2(w;r,w_0) := \quad \widehat{\rho}_2(w;i_1(r,w_0),i_2(r,w_0),r).$$

Remark 12. *To ensure a situation that only type h consumers withdraw their funds in Period 1, i.e., $w=\tau$, the following self selection constraints must hold:* [36]

$$\rho_2(\tau;r,w_0) \le \beta_h\cdot\rho_1(\tau;r,w_0) \qquad S_h$$

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) \le \rho_2(\tau;r,w_0) \qquad S_\ell.$$

Self selection requires the actual return in Period 2 to lie between the type-contingent valuations of the actual return in Period 1. When these self selection constraints fail to be met, all consumers make withdrawals in the same period. In particular, when for $\tau<1$ the constraint

$$\beta_\ell\cdot\rho_1(\tau;r,w_0) > \rho_2(\tau;r,w_0),$$

is violated, a bankrun is inevitable.

We assume a bankrun only occurs when it is inevitable. Under this assumption the withdrawal behaviour of type h consumers is:

$$W(\tau;r,w_0;h) = \quad \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0)\le\beta_h\cdot\rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0)>\beta_h\cdot\rho_1(\tau;r,w_0) \end{cases}$$

[36] These constraints guarantee that $h-$type consumers withdrawing their deposits, $W(\tau;h)=1$, and $\ell-$type consumers leaving their deposits in the bank, $W(\tau;\ell)=0$, constitutes a Nash equilibrium. It is well-known that there are other equilibria, e.g. an equilibrium where all consumers withdraw their funds. These other equilibria are ruled out in order to compare the intermediary institution "competing banks" under conditions that are most favourable for consumers.

and for $\ell-$types

$$W(\tau;r,w_0;\ell) = \begin{cases} 1 & \text{if } \rho_2(\tau;r,w_0) < \beta_\ell \cdot \rho_1(\tau;r,w_0) \\ 0 & \text{if } \rho_2(\tau;r,w_0) \geq \beta_\ell \cdot \rho_1(\tau;r,w_0). \end{cases}$$

Remark 13. *Given a deposit contract $(i_1(r,w_0), i_2(r,w_0))$, a consumer's ex-ante Choquet expected indirect utility from depositing the fraction D of his wealth with a bank, holding a fraction M as money and investing the remainder in the illiquid asset is*

$$\begin{aligned} &\mathcal{V}_b(D,M;r,w_0;\gamma) \\ &:= \gamma \cdot \int [\tau \cdot v_b(D,M;\tau,r,w_0;h) + (1-\tau)\cdot v_b(D,M;\tau,r,w_0;\ell)] \cdot p(\tau)\ d\tau \\ &\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} v_b(D,M;\tau,r,w_0;t). \end{aligned}$$

If the consumer chooses $D, M \in [0,1]$ such that $\widetilde{\mathcal{V}}_b(D,M;r,w_0;\gamma)$ is maximized, one obtains

Lemma 18.

$$\begin{aligned} \widetilde{\mathcal{V}}_b(r,w_0;\gamma) &:= \max_{\substack{(D,M)\in[0,1]\times[0,1] \\ s.t.\ D+M\leq 1}} \mathcal{V}_b(D,M;r,w_0;\gamma) \\ &= \max\{[\gamma\cdot\{\pi\cdot v_b(\pi;r,w_0;h) + (1-\pi)\cdot v_b(\pi;r,w_0;\ell)\} \\ &\qquad + (1-\gamma)\cdot \min_{\tau\in[0,1]} v_b(\tau;r,w_0;t)], \mathcal{V}_n^*(\gamma)\}, \end{aligned}$$

where $\mathcal{V}_n^*(\gamma)$ is defined in Proposition 5.

Remark 14. *It remains to determine the optimal parameters (r,w_0) of the deposit contract. Competition forces banks to choose the most favourable contract for the consumers. Hence, (r,w_0) is chosen to maximize the representative consumer's ex-ante Choquet expected utility, $\widetilde{\mathcal{V}}_b(r,w_0;\gamma)$, taking into account the withdrawal behaviour $W(\tau;r,w_0;t)$ as stated in Remark 12.*

From Lemma 18 it follows, that only two values for τ play a role in determining the deposit contract, viz. $\tau=\pi$ and $\tau=1$. The first value occurs when the additive probability assessment applies, for the second value a bankrun and the liquidation of illiquid assets is inevitable for

any fraction of reserves $r < 1$. As a consequence, the only self selection constraints that are relevant for a deposit contract with $r < 1$ are those for $\tau = \pi$, i.e.

$$\beta_\ell \cdot \rho_1(\pi; r, w_0) \leq \rho_2(\pi; r, w_0) \leq \beta_h \cdot \rho_1(\pi; r, w_0).$$

As a consequence, the banks offer deposit contracts based on predicted withdrawals $w_0 = \pi$. The following lemma provides the solution to this problem.

Lemma 19. *Suppose the level of confidence γ exceeds some threshold value and all consumers deposit all their wealth with the bank, i.e. $\delta^* = 1$. Then the optimal reserve holdings are*

$$r^* = \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1 - \pi) \cdot \beta_\ell} = \mu_o^*(\gamma)$$

and the optimal prediction of withdrawals is

$$w_0^* = \pi.$$

It is worth noting that neither the optimal reserve holdings r^* nor the optimal prediction w_0^* depend on the degree of confidence γ, provided it exceeds the threshold value.

It remains to characterize the ex-ante Choquet expected utility which arises from the deposit contract provided by competing banks.

Proposition 20. *Consider the equilibrium deposit contract (r^*, w_0^*) determined in Lemma 19. Then there exists some*

$$\overline{\gamma} \in \left(\frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1 - \pi) \cdot \beta_\ell}, 1 \right)$$

such that the aggregate level of deposits in equilibrium is

$$\delta(\gamma) := \begin{cases} 1 & \text{if} \quad \gamma > \overline{\gamma} \\ \in [0, 1] & \text{if} \quad \gamma = \overline{\gamma} \\ 0 & \text{otherwise.} \end{cases}$$

The resulting ex-ante Choquet expected utility is

$$\begin{aligned} \mathcal{V}_b^*(\gamma) := \quad & \widetilde{\mathcal{V}}_b(r^*, \pi; \gamma) \\ = \quad & \max\{\mathcal{V}_n^*(\gamma),\ \gamma \cdot [\beta_h \cdot r^* + \alpha_2 \cdot (1 - r^*)] + (1 - \gamma) \cdot [r^* + \alpha_1 \cdot (1 - r^*)] \cdot \beta_\ell]\}. \end{aligned}$$

Appendix B: Proofs

In this appendix, the proofs are ordered according to the number of the respective Lemma, Proposition or Theorem they refer to.

Proof of Proposition 8

(i) Price for shares

Denote

$$\mu_h(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_h + \tau \cdot \alpha_2}$$

and

$$\mu_\ell(\tau) := \frac{\tau \cdot \alpha_2}{(1-\tau)\cdot \beta_\ell + \tau \cdot \alpha_2}.$$

The ad interim price of ex-dividend shares in the mutual fund is

$$q_f^E(\tau;\psi,\varphi;r) := \begin{cases} \frac{\alpha_2}{\beta_h}\cdot(1-r) & \text{for} \quad \mu < \mu_h(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_h+\tau\cdot\alpha_2} \\ \frac{(1-\tau)}{\tau}\cdot\frac{\mu}{1-\mu}\cdot(1-r) & \text{for} \quad \mu \in [\mu_h(\tau),\mu_\ell(\tau)] \\ \frac{\alpha_2}{\beta_\ell}\cdot(1-r) & \text{for} \quad \mu > \mu_\ell(\tau) := \frac{\tau\cdot\alpha_2}{(1-\tau)\cdot\beta_\ell+\tau\cdot\alpha_2}, \end{cases}$$

where $\mu := r\cdot\varphi + \psi$ denotes the total aggregate money holdings in the economy. For the remainder of this proof, denote

$$q(\tau) := q_f^E(\tau; 0, 1; \mu_\ell)$$

(ii) Every equilibrium with $\varphi = 1$ is efficient

Obviously, this requires $\mu = \mu_o^*(\gamma)$. So assume that $r = \mu_o^*(\gamma) = \mu_\ell(\pi)$. Three cases have to be distinguished. Let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13.

(iia) $\tau \leq \underline{\tau}(\mu_\ell)$.

In this case we have $\mu \geq \mu_\ell(\tau)$. From *(i)* we obtain

$$q(\tau) = \frac{\alpha_2}{\beta_\ell}\cdot(1-r) = \frac{\alpha_2}{\beta_\ell}\cdot\left(\frac{(1-\pi)\cdot\beta_\ell}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2}\right) = \mu_\ell.$$

Each type h consumer sells his shares in the fund in Period 1. His resulting consumption is

$$z_{1h}(\tau) = \frac{\pi\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} + \frac{(1-\pi)\cdot\alpha_2}{(1-\pi)\cdot\beta_\ell+\pi\cdot\alpha_2} = \frac{\mu_\ell}{\pi}$$

and $z_{2h}(\tau) = 0$. The shares offered by $h-$types are bought by $\ell-$types. The value of the shares on offer is $\tau\cdot\frac{\alpha_2}{\beta_\ell}\cdot(1-r)$, which by assumption *(iia)* is less than the dividend payments to type

ℓ consumers. For the resulting price $q(\tau)$ the type ℓ consumers are indifferent between buying and not buying shares. Therefore, they will consume the surplus of dividends over the value of shares offered by $h-$types. This leads to

$$z_{1\ell}(\tau) := \mu_\ell - \frac{\alpha_2}{\beta_\ell} \cdot (1-\mu_\ell) \cdot \frac{\tau}{1-\tau}$$

and

$$z_{2\ell}(\tau) := \alpha_2 \cdot \frac{(1-\mu_\ell)}{1-\tau}.$$

This equals the payouts of the optimal contract in Proposition 13 for $\tau \in [0, \underline{\tau}(\mu_\ell)]$.

(iib) $\tau \in (\underline{\tau}(\mu_\ell), \overline{\tau}(\mu_\ell))$

From *(i)* we have $q(\tau) = \frac{(1-\tau)}{\tau} \cdot \frac{\mu_\ell}{1-\mu_\ell} \cdot (1-\mu_\ell) = \frac{(1-\tau)}{\tau} \cdot \mu_\ell$. The value of the total shares held by type h consumers is

$$\tau \cdot q(\tau) = (1-\tau) \cdot \mu_\ell,$$

which equals the value of the dividend payment to $\ell-$types. This results in allocation $z_{1h}(\tau) = \frac{\mu_\ell}{\tau}$, $z_{2h}(\tau) = 0$, $z_{1\ell}(\tau) = 0$ and $z_{2\ell}(\tau) = \alpha_2 \cdot \frac{1-\mu_\ell}{\pi}$. According to Proposition 13 and Proposition 10 this consumption plan is optimal.

(iic) $\tau \in [\overline{\tau}(\mu_\ell), 1]$

From *(i)* we have $q(\tau) = \frac{\alpha_2}{\beta_h} \cdot (1-\mu_\ell)$. Each $\ell-$type consumer uses all his dividend payments to buy shares. The resulting aggregate demand of shares is

$$(1-\tau) \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}.$$

At the the ruling share price $q(\tau)$, type h consumers are indifferent between selling their shares and holding them. They will sell just as many shares as are demanded at this price. The remaining shares are held until they pay out in Period 2. This results in the allocation $z_{1h}(\tau) := \frac{\mu_\ell}{\tau}$, $z_{1\ell}(\tau) := 0$,

$$\begin{aligned} z_{2\ell}(\tau) := &\quad \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 + \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = &\quad \alpha_2 \cdot (1-\mu_\ell) + \mu_\ell \cdot \beta_h, \end{aligned}$$

and

$$\begin{aligned} z_{2h}(\tau) := \quad & \alpha_2 \cdot (1-\mu_\ell) \cdot \left[1 - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \frac{\beta_h}{\alpha_2 \cdot (1-\mu_\ell)}\right] \\ = \quad & \alpha_2 \cdot (1-\mu_\ell) - \frac{1-\tau}{\tau} \cdot \mu_\ell \cdot \beta_h. \end{aligned}$$

From Proposition 13 and Proposition 10 it follows that this is the optimal liquidity allocation for $\tau \in [\overline{\tau}(\mu_\ell), 1]$.

(iii) $F = 1$ *is a best reply for* $r = \mu_\ell$ *and* $\varphi = 1$

We show that consumers cannot improve by unilaterally holding some of their initial wealth as money. For $\gamma > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ the ex-ante Choquet expected utility per unit of money holdings, when in Period 1 for $\tau = \pi$ the asset price $q(\pi)$ results, is

$$\gamma \cdot \left[\pi \cdot \beta_h \cdot 1 + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{1}{q(\pi)}\right] + (1-\gamma) \cdot \beta_\ell$$

which equals

$$\begin{aligned} & \gamma \cdot \left[\pi \cdot \beta_h + (1-\pi) \cdot \alpha_2 \cdot (1-\mu_\ell) \cdot \frac{\beta_\ell}{\alpha_2 \cdot (1-\mu_\ell)}\right] + (1-\gamma) \cdot \beta_\ell \\ = \quad & \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell. \end{aligned}$$

This expression is less than the corresponding ex-ante Choquet expected utility of investing in the mutual fund. Therefore, investing in the mutual fund is better than holding money. For $\gamma \leq \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$ aggregate money holdings $r = 0$ are optimal and yield an ex-ante utility of α_2. From Proposition 5 it follows that this is the level of utility a consumer can obtain by himself in the absence of financial institutions.

■

Proof of Proposition 9

In this proof we consider three cases. First, we assume aggregate money holdings exceed $\mu_a^*(\gamma)$, followed by assuming the aggregate money holdings fall short of $\mu_a^*(\gamma)$. It is shown that neither case is compatible with equilibrium. Finally, we show that aggregate money holdings which are equal to $\mu_a^*(\gamma)$ are compatible with equilibrium.

(i) $\varphi^* \cdot r^* + \psi^* > \mu_a^*(\gamma)$

From assumption *(i)* it follows, that $\mu_a^*(\gamma) < 1$ and therefore that in equilibrium the Choquet expected utility from holding one unit of wealth as money must equal the Choquet expected utility of investing it in the illiquid asset. This must hold both directly for investment in the illiquid asset and indirectly through investment in the mutual fund. Since the mutual fund only coordinates (part of) the investment decisions of its owners, it does not affect the price of the asset in Period 1 that follows from a given aggregate investment policy μ. Therefore, the price $q_a = q_a^*(\gamma)$ must be such that

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

As the aggregate reserve holdings under the mutual fund exceeds that under the asset market, the Choquet expected return of assets exceeds that of money holding. So all consumers with $F^* \cdot r^* + M^* > \mu_a^*(\gamma)$ can improve by investing all their wealth in the illiquid asset. As the amount of such consumers has a positive mass, the situation under assumption *(i)* cannot occur in equilibrium.

(ii) $\varphi^* \cdot r^* + \psi^* < \mu_a^*(\gamma)$

From assumption *(ii)* is follows that $\mu_a^*(\gamma) > 0$. By reasoning similar to that of case *(i)*, consumers with $F^* \cdot r^* + M^* < \mu_a^*(\gamma)$ can improve holding all their wealth as money and the situation under assumption *(ii)* cannot occur in equilibrium.

(iii) $\varphi^* \cdot r^* + \psi^* = \mu_a^*(\gamma)$

In this case, the mutual fund leads to the same aggregate investment policy as the asset market. These reserve holdings equalize the Choquet expected returns of money holdings and investing in the illiquid asset, so consumers are indifferent between the investment decisions available to them. In particular, the mutual fund choosing reserves r^* and each consumer choosing $F^* = \varphi^*$ and $M^* = \psi^*$ constitutes an equilibrium.

(iv) $\varphi^* + \psi^* = \mu_a^*(\gamma)$

This assumption is equivalent to $1 - \varphi^* - \psi^* = 1 - \mu_a^*(\gamma)$, which states that all investment in assets takes place by individual consumers. Suppose, for contradiction, a positive fraction

of the aggregate investment in assets takes place by the mutual fund, i.e. $\varphi^* \cdot (1 - r^*) > 0$. Given φ^* and ψ^*, the mutual fund could now increase the ex-ante Choquet expected utility of its share holders by slightly increasing its money holdings and slightly reducing its investment in the illiquid asset, thus increasing the implicit Period 1 price of the asset. This contradicts the initial investment policy being optimal.

■

Proof of Proposition 10

The planner's decision problem is to maximise the representative consumer's ex-ante indirect utility by choosing the investment policy μ, i.e.

$$\max_{\mu \in [0,1]} \mathcal{V}_o(\mu; \gamma).$$

As follows from Lemma 14, for this we must consider the following cases.

(i) $\mu \leq \mu_\ell$

For these investment policies, according to Lemma 14, the planner maximises

$$\alpha_2 + (\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell - \alpha_2) \cdot \mu.$$

Whenever $\gamma \cdot \beta_h + (1 - \gamma) \cdot \beta_\ell > \alpha_2$, this expression is increasing in μ, from which it follows that μ obtains its maximal value its μ_ℓ whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.
For $\gamma < \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$ the maximum ex-ante indirect utility is obtained for $\mu = 0$.

(ii) $\mu \geq \mu_\ell$

For these values, Lemma 14 indicates the objective function to be maximised is

$$(1 + \gamma \cdot \left(\frac{\beta_h}{\beta_\ell} - 1\right) \cdot \pi) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu).$$

This expression is decreasing μ, since by Assumption 2 we have $\alpha_2 > \beta_\ell$. Therefore, the ex-ante indirect utility is maximized when μ obtains its lower bound, i.e.

$$\mu = \mu_\ell.$$

(iii) The optimum

Comparing the results of the cases *(i)* and *(ii)* shows that $\mathcal{V}_o(\mu;\gamma)$ is continuous in μ and that the maximum in case *(i)* dominates that in case *(ii)*. Thus, the results from case *(i)* determine the optimal investment policy, which leads to

$$\mu_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \in [0, \dfrac{\alpha_2 \cdot \pi}{\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi}] & \text{if} \quad \gamma = \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ 0 & \text{if} \quad \gamma < \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

For the corresponding ex-ante Choquet expected indirect utility of the representative consumer we obtain

$$\mathcal{V}_o^*(\gamma) := \begin{cases} \dfrac{\alpha_2 \cdot [\pi \cdot \gamma \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell]}{(\beta_\ell + (\alpha_2 - \beta_\ell) \cdot \pi)} & \text{if} \quad \gamma > \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell} \\ \alpha_2 & \text{if} \quad \gamma \leq \dfrac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}. \end{cases}$$

■

Proof of Theorem 11

(i) $\mathcal{V}_a^*(\gamma)$ is convex

The ex-ante Choquet expected utility for an equilibrium in the asset market is

$$\widehat{\mathcal{V}}_a^m(q_a^*(\gamma);\gamma) = \widehat{\mathcal{V}}_a^a(q_a^*(\gamma);\gamma) = \gamma \cdot q_a^*(\gamma) \cdot \pi \cdot \beta_h + \gamma \cdot (1-\pi) \cdot \alpha_2 + (1-\gamma) \cdot \alpha_2.$$

In determining the curvature of this function w.r.t. γ, the last two terms can be disregarded as they are linear in γ. After substituting for $q_a^*(\gamma)$ the first term is

$$\begin{aligned} \gamma \cdot q^*(\gamma) &= \gamma \cdot \frac{1}{a} \left(-(b_1 + b_2 \cdot \frac{1}{\gamma}) + \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c} \right) \\ &\sim -(b_1 \cdot \gamma + b_2) + \gamma \cdot \sqrt{(b_1 + b_2 \cdot \frac{1}{\gamma})^2 + c}, \end{aligned}$$

where

$$\begin{aligned} a &:= 2 \cdot \pi \cdot \beta_h \\ b_1 &:= \alpha_2 - \pi \cdot (\alpha_2 + \beta_h) \\ b_2 &:= \gamma \cdot (\alpha_2 - \beta_\ell) \\ c &:= 4 \cdot \pi \cdot (1-\pi) \cdot \alpha_2 \cdot \beta_h. \end{aligned}$$

As the first part of this expression is linear, it is the second part that determines its curvature. This second term is obtained as the square root of

$$\begin{aligned} & (b_1 \cdot \gamma + b_2)^2 + c \cdot \gamma^2 \\ = \; & b_1^2 \cdot \gamma^2 + 2 \cdot b_1 \cdot b_2 \cdot \gamma + b_2{}^2 + c \cdot \gamma^2 \\ = \; & (b_1^2 + c) \cdot \gamma^2 + 2 \cdot \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}} \cdot \sqrt{(b_1^2 + c)} \cdot \gamma + (\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 + b_2{}^2 \\ & -(\frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}})^2 \\ = \; & [\sqrt{b_1^2 + c} \cdot \gamma + \frac{b_1 \cdot b_2}{\sqrt{(b_1^2 + c)}}]^2 + (1 - \frac{b_1^2}{b_1^2 + c}) \cdot b_2^2. \end{aligned}$$

As $c = 4 \cdot (1 - \pi) \cdot \pi \cdot \alpha_2 \cdot \beta_h > 0$ and the square root we are considering is of the form $\sqrt{(x^2 + c)}$ this square root is convex, as its first derivative with respect to x is

$$[x^2 + c]^{-\frac{1}{2}} \cdot x = [1 + \frac{c}{x^2}]^{-\frac{1}{2}}$$

and its second derivative is

$$-\frac{1}{2} \cdot [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (-2 \cdot \frac{c}{x^3}) = [1 + \frac{c}{x^2}]^{\frac{3}{2}} \cdot (\frac{c}{x^3}) > 0.$$

Therefore the ex-ante Choquet expected indirect utility function $\mathcal{V}_a^*(\gamma)$ is convex.

(ii) $\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_b^*(\widehat{\gamma}')$ and $\mathcal{V}_a^*(1) < \mathcal{V}_b^*(1)$

Denote

$$\widehat{\gamma}' = \frac{\pi \cdot (\alpha_2 - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}{\pi \cdot (\beta_h - \beta_\ell) \cdot \alpha_2 + (1 - \pi) \cdot (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot \beta_\ell}$$

as in the proof of Proposition 20. Since $\widehat{\gamma}' > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}$, we have

$$\mathcal{V}_a^*(\widehat{\gamma}') > \mathcal{V}_a^*(\frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}) = \mathcal{V}_b^*(\widehat{\gamma}').$$

For $\gamma = 1$ the competing banks implement the optimal liquidity allocation and from Proposition 10 we have $\mu_o^*(\gamma) = \frac{\pi \cdot \alpha_2}{\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell)}$. Therefore

$$\mathcal{V}_b^*(1) := \alpha_2 + (\beta_h - \alpha_2) \cdot \frac{\pi \cdot \alpha_2}{\pi \cdot \alpha_2 + (1 - \pi) \cdot \beta_\ell}.$$

From $\beta_\ell + \pi \cdot (\alpha_2 - \beta_\ell) < \alpha_2$ it follows that

$$\mathcal{V}_a^*(1) = \alpha_2 + \pi \cdot (\beta_h - \alpha_2) < \mathcal{V}_b^*(1).$$

(iii) $\mathcal{V}_b^*(\widetilde{\gamma}) = \mathcal{V}_a^*(\widetilde{\gamma})$ *has a unique solution*

Consider a continuous convex function f from a compact interval $[a,b] \subseteq \mathbb{R}$ in $\mathbb{R}$ and a linear function l such, that $f(a) > l(a)$ and $f(b) < l(b)$. It is shown that there exists a unique $c \in [a,b]$ such that $f(a) = l(a)$. According to the Mean Value Theorem, at least on such c exists.

It remains to show that c is unique. For contradiction, assume c is not unique, i.e. there exist at least two such values c_1 and $c_2 > c_1$. From the convexity of f it follows that $f'(c_1) < l' < f'(c_2)$. The derivative l' of the linear function l is constant. Therefore we have for all $c_3 > c_2$, that $f'(c_3) \geq f'(c_2)$. As, however, $f(c_2) = l(c_2)$ and f is a continuous function on the interval $[a,b]$, this contradicts $f(b) < l(b)$.

■

Proof of Lemma 12

Consider an optimal solution $(\lambda^*, z_{1h}^*, x_{2h}^*, z_{1\ell}^*, z_{2\ell}^*)$. Such a solution exists. In this solution z_{1h}^* must be strictly positive, as $\beta_h > \alpha_2 > \beta_\ell$. For any optimal solution, at least one of the self selection constraints is not binding. For $\lambda^* = 0$ the lemma holds, so assume, $\lambda^* > 0$ for contradiction.

(i) S_ℓ *is not binding*

For $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{2h}^* + \alpha_2 \cdot (1-\mu) \cdot \varepsilon,\ z_{1\ell}^*,\ z_{2\ell}^*)$$

is attainable. Since we have $\alpha_2 > \alpha_1 \cdot \beta_h$ by Assumption 2, the average utility obtained by this allocation exceeds that of the optimal allocation, which constitutes a contradiction.

(ii) S_h *is not binding*

As in case *(i)*, for $\varepsilon > 0$ sufficiently small, the allocation

$$(\lambda^* - \varepsilon,\ z_{1h}^* - \frac{\alpha_1 \cdot (1-\mu)}{\tau} \cdot \varepsilon,\ z_{2h}^*,\ z_{1\ell}^*,\ z_{2\ell}^* + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} \cdot \varepsilon)$$

is attainable and obtains an average utility which exceeds the utility of the optimal allocation.

■

Proof of Proposition 13

Using Lemma 12, for any (τ, μ) the optimal payouts satisfy the feasibility conditions

$$\begin{aligned} \tau \cdot z_{1h}(\tau,\mu) + (1-\tau) \cdot z_{1\ell}(\tau,\mu) &= \mu, \\ \tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) &= \alpha_2 \cdot (1-\mu). \end{aligned}$$

(i) $\tau \in [0, \underline{\tau}]$

For these small values of τ there are "too few" $h-$types in the economy and the payouts $z_{1h}(\tau,\mu) := \frac{\mu}{\tau}$ and $z_{2h}(\tau,\mu) = 0$ violate the self selection constraint of type h consumers, S_h. For given μ, the most efficient way to restore this constraint is to increase $z_{1\ell}(\tau,\mu)$ and decrease $z_{1h}(\tau,\mu)$ such that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu) = \beta_\ell \cdot z_{1h}(\tau,\mu)$. From this it follows that $\beta_\ell \cdot z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{1-\tau} = \beta_\ell \cdot z_{1h}(\tau,\mu)$ and therefore

$$z_{1\ell}(\tau,\mu) = z_{1h}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}.$$

Substituting for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu)$ we obtain $z_{1\ell}(\tau,\mu) = \frac{\mu}{\tau} - \frac{(1-\tau)}{\tau} \cdot z_{1\ell}(\tau,\mu) - \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)}$ which simplifies to

$$z_{1\ell}(\tau,\mu) = \mu - \frac{\tau}{1-\tau} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

which leads to

$$z_{1h}(\tau,\mu) = z_{1\ell}(\tau,\mu) + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell \cdot (1-\tau)} = \mu + \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell}.$$

(ii) $\tau \in (\underline{\tau}, \overline{\tau})$

For these proportions of $h-$types, neither of the self selection constraints is binding for $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$, $z_{2h}(\tau,\mu) = z_{1\ell}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$. From $\beta_h > \beta_\ell$ it follows that the payouts $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$ are optimal. Consumption in Period 2 is valued equally by both agents and therefore any combination of $z_{2h}(\tau,\mu)$ and $z_{2\ell}(\tau,\mu)$ with

$$\tau \cdot z_{2h}(\tau,\mu) + (1-\tau) \cdot z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu)$$

obtains the same average interim utility. Thus, any such combination that satisfies the self selection S_ℓ is a solution for the optimization problem. In particular, this holds for the payouts $z_{2h}(\tau,\mu) = 0$ and $z_{2\ell}(\tau,\mu) = \frac{\alpha_2 \cdot (1-\mu)}{1-\tau}$.

(iii) $\tau \in [\overline{\tau}, 1]$

For these values of τ there are "too many" $h-$types in the economy. So it is optimal to choose $z_{1h}(\tau,\mu) = \frac{\mu}{\tau}$ and $z_{1\ell}(\tau,\mu) = 0$. Even for this choice, however, the proportion of $h-$types is so high, that for $z_{2h}(\tau,\mu) = 0$ the self selection constraint S_h is violated. The payout $z_{2h}(\tau,\mu)$ obtains its lowest value when S_h binds and its highest value when S_ℓ binds. The self selection constraint S_h reads

$$\beta_h \cdot z_{1h}(\tau,\mu) + z_{2h}(\tau,\mu) \geq \beta_h \cdot z_{1\ell}(\tau,\mu) + z_{2\ell}(\tau,\mu)$$

and after substitution and rearranging we obtain

$$z_{2h}(\tau,\mu) = z_{2\ell}(\tau,\mu) - \beta_h \cdot \frac{\mu}{\tau}.$$

Using $z_{2\ell}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)]$ we find $z_{2h}(\tau,\mu) = \frac{1}{(1-\tau)} \cdot [\alpha_2 \cdot (1-\mu) - \tau \cdot z_{2h}(\tau,\mu)] - \beta_h \cdot \frac{\mu}{\tau}$, from which

$$z_{2h}(\tau,\mu) = \alpha_2 \cdot (1-\mu) - \frac{(1-\tau)}{\tau} \cdot \beta_h \cdot \mu = \alpha_2 - \frac{\mu}{\tau} \cdot (\tau \cdot \alpha_2 + (1-\tau) \cdot \beta_h)$$

and

$$z_{2\ell}(\tau,\mu) = \alpha_2 \cdot (1-\mu) + \beta_h \cdot \mu$$

follow.

■

Proof of Lemma 14

The structure of this proof is as follows. First the average interim utility for the investment policy μ is determined for $\tau = \pi$. Then the interim utility of the worst case is determined and both are combined to calculate the ex-ante Choquet expected indirect utility of investment policy μ.

(i) Average interim utility

For any $\mu \in [0,1]$ let $\underline{\tau}(\mu)$ and $\overline{\tau}(\mu)$ be as in Proposition 13. For any investment policy μ and for $\tau = \pi$ the average interim utility is

$$\widetilde{U}(\pi,\mu) := \pi \cdot \beta_h \cdot z_{1h}(\pi,\mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi,\mu) + \pi \cdot z_{2h}(\pi,\mu) + (1-\pi) \cdot z_{2\ell}(\pi,\mu).$$

From Lemma 12 we have

$$\pi \cdot z_{2h}(\pi, \mu) + (1-\pi) \cdot z_{2\ell}(\pi, \mu) = \alpha_2 \cdot (1-\mu)$$

substitution of which yields

$$\widetilde{U}(\pi, \mu) = \pi \cdot \beta_h \cdot z_{1h}(\pi, \mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi, \mu) + \alpha_2 \cdot (1-\mu).$$

(ia) $\pi \in [0, \underline{\tau}(\mu))$

Using Lemma 12 we obtain

$$\begin{aligned} \widetilde{U}(\pi, \mu) &= \pi \cdot \beta_h \cdot \left(\frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} + \mu \right) \\ &\quad + (1-\pi) \cdot \beta_\ell \cdot \left(\mu - \frac{\pi}{(1-\pi)} \cdot \frac{\alpha_2 \cdot (1-\mu)}{\beta_\ell} \right) + \alpha_2 \cdot (1-\mu) \\ &= \left(1 + (\frac{\beta_h}{\beta_\ell} - 1) \cdot \pi \right) \cdot (\alpha_2 + (\beta_\ell - \alpha_2) \cdot \mu). \end{aligned}$$

(ib) $\pi \in [\underline{\tau}(\mu), 1]$

Using Lemma 12 we obtain

$$\begin{aligned} \widetilde{U}(\pi, \mu) &= \pi \cdot \beta_h \cdot z_{1h}(\pi, \mu) + (1-\pi) \cdot \beta_\ell \cdot z_{1\ell}(\pi, \mu) + \alpha_2 \cdot (1-\mu) \\ &= \pi \cdot \beta_h \cdot \frac{\mu}{\pi} + \alpha_2 \cdot (1-\mu) = \alpha_2 + (\beta_h - \alpha_2) \cdot \mu, \end{aligned}$$

which complete this part of the proof.

(ii) The worst case

The interim utility a type h consumer obtains from payouts (z_h, z_ℓ) that satisfy the self selection constrains is as least as last as the corresponding utility of a type ℓ consumer since $\beta_h > \beta_\ell$. To determine the worst case, we can therefore restrict our attention to a consumer with a low preference for liquidity. The utility of $\ell-$types from payouts that satisfy the self selection constraints for τ is increasing with τ for given μ.

Therefore, for any value of μ the utility of $\ell-$types obtains its minimum value for $\tau = 0$. Using this, we obtain

$$\min_{(t,\tau) \in \{h,\ell\} \times [0,1]} v_o(\mu, \tau; t) = \beta_\ell \cdot z_{1\ell}(0, \mu) + z_{2\ell}(0, \mu) = \alpha_2 \cdot (1-\mu) + \beta_\ell \cdot \mu.$$

(iii) The ex-ante Choquet expected utility

After rearranging terms, from $\pi = \underline{\tau}(\mu)$ we obtain

$$\mu = \frac{\pi \cdot \alpha_2}{(1-\pi)\cdot \beta_\ell + \pi \cdot \alpha_2} = \mu_\ell.$$

For given investment policy μ and given level of confidence γ, substitution of the results of part *(i)* and part *(ii)* of this proof yields

$$\begin{aligned}\mathcal{V}_o(\mu;\gamma) &:= \gamma \cdot \widetilde{U}(\pi,\mu) + (1-\gamma)\cdot \widetilde{U}(0,\mu)\\ &= \begin{cases} (1+\gamma\cdot\left(\dfrac{\beta_h}{\beta_\ell}-1\right)\cdot\pi)\cdot(\alpha_2+(\beta_\ell-\alpha_2)\cdot\mu) & \text{if} \quad \mu > \mu_\ell \\ \\ \alpha_2 + (\gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell - \alpha_2)\cdot\mu & \text{if} \quad \mu \le \mu_\ell, \end{cases}\end{aligned}$$

which completes the proof.

■

Proof of Proposition 16

Denote

$$\hat{v}_a(M;q;t) := M \cdot R^m(q;t) + (1-M)\cdot R^a(q;t).$$

For all $q \in [\frac{\alpha_2}{\beta h}, \frac{\alpha_2}{\beta_\ell}]$ we have $\hat{v}_a(M;q;h) > \hat{v}_a(M;q;\ell)$. Furthermore, $\hat{v}_a(M;q;\ell)$ is decreasing in q. From $q_a^E(\tau,\mu) \le \frac{\alpha_2}{\beta_\ell}$ it now follows that

$$\begin{aligned}&\min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \left[\hat{v}_a(M;q_a^E(\tau,\mu);t)\right]\\ =\;& \min_{q\in[\frac{\alpha_2}{\beta_h},\frac{\alpha_2}{\beta_\ell}]} \hat{v}_a(M;q;\ell) = M\cdot\beta_\ell + (1-M)\cdot\alpha_2.\end{aligned}$$

To simplify notation, we denote $q_a := q_a^E(\pi,\mu)$. From Remark 7 it follows that in an equilibrium with $\mu \in (0,1)$, the asset price $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$ must satisfy

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

(i) $q_a = \frac{\alpha_2}{\beta_\ell}$

In this case consumers anticipate the asset price to be at its upper bound for $\tau = \pi$. By deriving a contradiction, we show this cannot occur in equilibrium. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma\cdot[\pi\cdot\beta_h + (1-\pi)\cdot\beta_\ell] + (1-\gamma)\cdot\beta_\ell = \gamma\cdot\pi\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma \cdot [\pi \cdot \frac{\alpha_2}{\beta_\ell} \cdot \beta_h + (1-\pi)\cdot\alpha_2] + (1-\gamma)\cdot\alpha_2 = \gamma\cdot\pi\cdot\frac{\alpha_2}{\beta_\ell}\cdot\beta_h + (1-\gamma\cdot\pi)\cdot\alpha_2.$$

This implies $\widehat{\mathcal{V}}_a^m(q_a;\gamma) < \widehat{\mathcal{V}}_a^a(q_a;\gamma)$. In this case, any consumer would invest all his wealth in the illiquid asset, i.e. $\mu = 0$. This aggregate investment policy, however, leads to the asset price $q_a = \frac{\alpha_2}{\beta_h}$, which contradicts assumption *(i).*

(ii) $q_a = \frac{\alpha_2}{\beta_h}$

In this case, the consumers anticipate the asset price to obtain its lower bound for $\tau = \pi$. Substituting for q_a yields

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) := \gamma\cdot[\pi\cdot\beta_h + (1-\pi)\cdot\beta_h] + (1-\gamma)\cdot\beta_\ell = \gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell$$

and

$$\widehat{\mathcal{V}}_a^a(q_a;\gamma) := \gamma\cdot[\pi\cdot\alpha_2 + (1-\pi)\cdot\alpha_2] + (1-\gamma)\cdot\alpha_2 = \alpha_2.$$

This lower bound for the asset price will only be obtained in equilibrium if

$$\alpha_2 \geq \gamma\cdot\beta_h + (1-\gamma)\cdot\beta_\ell.$$

Rearranging terms, this inequality leads to $\gamma \leq \frac{\alpha_2-\beta_\ell}{\beta_h-\beta_\ell}$. From Proposition 10 it follows that for these levels of confidence financial institutions are superfluous form the ex-ante point of view.

(iii) $q_a \in (\frac{\alpha_2}{\beta_h}, \frac{\alpha_2}{\beta_\ell})$

For any such q_a to be the anticipated equilibrium price in Period 1 for $\tau = \pi$, we must have

$$\widehat{\mathcal{V}}_a^m(q_a;\gamma) = \widehat{\mathcal{V}}_a^a(q_a;\gamma).$$

Substitution of $R^m(q_a;h) = \beta_h$, $R^m(q_a;\ell) = \frac{\alpha_2}{q_a}$, $R^a(q_a;h) = \beta_h\cdot q_a$ and $R^a(q_a;\ell) = \alpha_2$ leads to

$$\begin{aligned} &\gamma\cdot\pi\cdot\beta_h + \gamma\cdot(1-\pi)\cdot\frac{1}{q_a}\cdot\alpha_2 + (1-\gamma)\cdot\beta_\ell \\ = \;&\gamma\cdot\pi\cdot\beta_h\cdot q_a + \gamma\cdot(1-\pi)\cdot\alpha_2 + (1-\gamma)\cdot\alpha_2. \end{aligned}$$

Rearranging yields

$$(1-\gamma)\cdot[\alpha_2-\beta_\ell] = \left[\pi\cdot\beta_h + (1-\pi)\cdot\alpha_2\cdot\frac{1}{q_a}\right]\cdot\gamma\cdot(1-q_a).$$

For $\gamma < 1$ we have $q_a < 1$, from which we obtain

$$\pi \cdot \beta_h - (1-\pi) \cdot \alpha_2 + \frac{1}{q_a} \cdot (1-\pi) \cdot \alpha_2 - q_a \cdot \pi \cdot \beta_h = (\frac{1}{\gamma} - 1) \cdot (\alpha_2 - \beta_\ell).$$

Denote the solution of this equation w.r.t. q_a by $\widehat{q}_a(\gamma; \pi, \alpha_2, \beta_h, \beta_\ell)$.

The derivative of the left hand side of this expression with respect to q_a is

$$-\frac{(1-\pi) \cdot \alpha_2}{(q_a)^2} - \pi \cdot \beta_h < 0.$$

Therefore, the left hand side of the expression is decreasing in q_a.

The right hand side of the expression is decreasing in γ. An increase in the level of ambiguity, which corresponds to a decrease in the level of confidence, increases the right hand side of the equation. To maintain equality, the left hand side of the equation has to increase as well, which leads to a decrease in the asset price q_a, keeping all other parameters constant. The asset price is, however, bound from below by the value $\frac{\alpha_2}{\beta_h}$. After substituting this lower bound for the asset price and solving for the level of confidence γ, we find that $q_a > \frac{\alpha_2}{\beta_h}$, whenever $\gamma > \frac{(\alpha_2 - \beta_\ell)}{(\beta_h - \beta_\ell)}$.

■

Proof of Lemma 18

If the self selection constraint for type ℓ consumers is violated for the deposit contract (r, π) and withdrawals $w = \pi$, then a bankrun in Period 1 can not be avoided. Hence, all illiquid assets will be liquidated. In this case, it is optimal for the consumers not to deposit their wealth in the bank. This leads to an ex-ante utility of $\mathcal{V}_n^*(\gamma)$.

Therefore, consider deposit contracts (r, π) such that for withdrawals $w = \pi$ the self selection constraint of type ℓ consumers is satisfied. Using $w = \pi$ this leads to the ex-ante Choquet expected utility of

$$
\begin{aligned}
&\mathcal{V}_b(D,M;r,\pi;\gamma) := \\
&\quad \gamma \cdot [\mathbb{E}_\tau\{\tau \cdot v_b(D,M;\tau,r,\pi;h) + (1-\tau)\cdot v_b(D,M;\tau,r,\pi;\ell)\}] \\
&\qquad\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \cdot v_b(D,M;\tau,r,\pi;t) \\
&\quad = \gamma\cdot[\pi\cdot v_b(D,M;\pi,r,\pi;h) + (1-\pi)\cdot v_b(D,M;\pi,r,\pi;\ell)] \\
&\qquad\qquad + (1-\gamma)\cdot \min_{(t,\tau)\in\{h,\ell\}\times[0,1]} \cdot v_b(D,M;\tau,r,\pi;t) \\
&\quad = \gamma\cdot[\pi\cdot[D\cdot\widetilde{v}_b(\pi;r,\pi;h) + (1-D)\cdot v_n(M;h)] \\
&\qquad\qquad + (1-\pi)\cdot[D\cdot\widetilde{v}_b(\pi;r,\pi;\ell) + (1-D)\cdot v_n(M;\ell)]] \\
&\quad + (1-\gamma)\cdot \min_{\tau\in[0,1]}\min_{t\in\{h,\ell\}}\{D\cdot\widetilde{v}_b(\tau;r,\pi;t) + (1-D)\cdot v_n(M;t)\} \\
&\quad = \gamma\cdot[D\cdot\mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\} + (1-D)\cdot\mathbb{E}_t\{v_n(M;t)\}] \\
&\quad + (1-\gamma)\cdot\min_{\tau\in[0,1]}[D\cdot\widetilde{v}_b(\tau;r,\pi;\ell) + (1-D)\cdot v_n(M;\ell)] \\
&\quad = D\cdot\left[\gamma\cdot\mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\} + (1-\gamma)\cdot\min_{\tau\in[0,1]}\widetilde{v}_b(\tau;r,\pi;\ell)\right] \\
&\qquad + (1-D)\cdot[\gamma\cdot\mathbb{E}_t\{v_n(M;t)\} + (1-\gamma)\cdot v_n(M;\ell)].
\end{aligned}
$$

For any $\tau \in [0,1]$ we have that $\widetilde{v}_b(\tau;r,\pi;h) > \widetilde{v}_b(\tau;r,\pi;\ell)$ and for every $M \in [0,1]$, that $v_n(M;h) \geq v_n(M;\ell)$. Thus we obtain:

$$
\begin{aligned}
\widetilde{\mathcal{V}}_b(r,\pi;\gamma) := \max_{(D,M)\in[0,1]\times[0,1]} &\quad \mathcal{V}_b(D,M;r,\pi;\gamma) \\
\text{s.t.} &\quad D+M \leq 1
\end{aligned}
$$

$$
\begin{aligned}
&= \max_{D\in[0,1]} D\cdot[\gamma\cdot(\pi\cdot\widetilde{v}_b(\pi;r,\pi;h) + (1-\pi)\cdot\widetilde{v}_b(\pi;r,\pi;\ell)) \\
&\qquad\qquad\qquad + (1-\gamma)\cdot\min_{\tau\in[0,1]}\widetilde{v}_b(\tau;r,\pi;\ell)\Big] \\
&+ \max_{M\in[0,1-D]}[\gamma\cdot(\pi\cdot v_n(M;h) + (1-\pi)\cdot v_n(M;\ell)\}) + (1-\gamma)\cdot v_n(M;\ell)] \\
&= \max_{D\in[0,1]} D\cdot\left[\gamma\cdot\mathbb{E}_t\{\widetilde{v}_b(\pi;r,\pi;t)\} + (1-\gamma)\cdot\min_{\tau\in[0,1]}\widetilde{v}_b(\tau;r,\pi;\ell)\right] + (1-D)\cdot\mathcal{V}_n^*(\gamma).
\end{aligned}
$$

This expression obtains its maximum either for $D = 0$ or for $D = 1$ or for any value $D \in [0,1]$. From this the result of the Lemma follows immediately.

■

Proof of Lemma 19

This proof consists of two steps. Firstly, for any reserves r, the optimal payouts in Period 1 are determined. The second step consist of finding the optimal reserve policy, under the assumption that separating the two types of consumers is worthwhile. If the separation of types is not worth while, one either obtains the reserves as derived here and deposits $\delta = 0$, or the bank hold reserves $r = \mu_n^*(\gamma)$.

(i) $w_0 = \pi$.

For given reserves r, the representative bank is to choose the interest payments $(i_1(r, w_0))$, $(i_2(r, w_0))$. Under the zero-profit condition, this is equivalent to choosing its predicted withdrawals w_0. Therefore, its decision problem is

$$\max_{w_0 \in [\underline{\tau}(r), \overline{\tau}(r)]} \gamma \cdot [\pi \cdot \beta_h \cdot \rho_1(\pi; r, w_0) + (1-\pi) \cdot \rho_2(\pi; r, w_0)] + (1-\gamma) \cdot \beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r).$$

Rearranging, dividing by γ and disregarding the constant term to $(1-\gamma)$ yields

$$\max_{w_0 \in [\underline{\tau}(r), \overline{\tau}(r)]} \pi \cdot \beta_h \cdot \rho_1(\pi; r, w_0) + (1-\pi) \cdot \rho_2(\pi; r, w_0).$$

This problem has an interior solution, which does not depend on γ and which is obtained for $w_0 = \pi$.

(ii) Optimal reserves r

Denote

$$\mu_h := \frac{\pi \cdot \alpha_2}{(1-\pi) \cdot \beta_\ell + \pi \cdot \alpha_2}.$$

Using the result from part *(i)* of this proof, the decision problem of the bank reduces to

$$\max_{r \in [\mu_h, \mu_\ell]} \gamma \cdot [\pi \cdot \beta_h \cdot \frac{r}{\pi} + (1-\pi) \cdot \frac{\alpha_2 \cdot (1-r)}{(1-\pi)}] + (1-\gamma) \cdot \beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r),$$

where $[\mu_h, \mu_\ell]$ denotes the set of reserves that are compatible with self selection if $\tau = \pi$. After rearranging the objective function can be written as

$$[\gamma \cdot (\beta_h - \alpha_2) + (1-\gamma) \cdot \beta_\ell \cdot (1-\alpha_1)] \cdot r + \gamma \cdot \alpha_2 + (1-\gamma) \cdot \beta_\ell \cdot \alpha_1.$$

This expression is maximised when r is at its upper bound, whenever

$$\gamma \cdot (\beta_h - \alpha_2) + (1-\gamma) \cdot \beta_\ell \cdot (1-\alpha_1) > 0.$$

From Assumption 2 we know that $\beta_h > \alpha_2$ and $\alpha_1 < 1$. Therefore the inequality holds and we have

$$r = \mu_\ell := \frac{\alpha_2 \cdot \pi}{\alpha_2 \cdot \pi + (1-\pi) \cdot \beta_\ell}.$$

■

Proof of Proposition 20

The ex-ante Choquet expected indirect utility of the consumers from the deposit contract (r, π) is obtained as

$$\max_{D \in [0,1]} \quad D \cdot \gamma \cdot [\beta_h \cdot r + \alpha_2 \cdot (1-r)] + (1-\gamma) \cdot D \cdot [\beta_\ell \cdot (\alpha_1 \cdot (1-r) + r)]$$
$$+ (1-D) \cdot \mathcal{V}_n^*(\gamma).$$

After rearranging the objective function, it follows directly that a consumer deposits all his wealth in the bank, i.e. $D^* = 1$, if

$$\gamma \cdot [(\beta_h - \alpha_2) \cdot r + \alpha_2] + (1-\gamma) \cdot [\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot r)] - \mathcal{V}_n^*(\gamma) > 0.$$

To prove the Proposition, two cases are considered.

(i) $\alpha_2 \geq \gamma \cdot \pi \cdot \beta_h + (1-\gamma \cdot \pi) \cdot \beta_\ell$

For these parameter values we have $\mathcal{V}_n^*(\gamma) = \alpha_2$ and substitution yields

$$\gamma \cdot [(\beta_h - \alpha_2) \cdot \mu + \alpha_2] + (1-\gamma) \cdot [\beta_\ell \cdot (\alpha_1 + (1-\alpha_1) \cdot \mu)] - \alpha_2 > 0.$$

For the left hand side of this expression we obtain

$$\gamma \cdot [(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell]$$
$$-(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi - (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell.$$

This equals zero for the level of confidence

$$\widehat{\gamma}' = \frac{(\alpha_2 - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell}{(\beta_h - \beta_\ell) \cdot \alpha_2 \cdot \pi + (\alpha_2 - \beta_\ell \cdot \alpha_1) \cdot (1-\pi) \cdot \beta_\ell} > \frac{\alpha_2 - \beta_\ell}{\beta_h - \beta_\ell}.$$

So $D^* = 1$, if $\gamma > \widehat{\gamma}'$.

(ii) $\alpha_2 < \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell$

For these parameter values we have $\mathcal{V}_n^*(\gamma) = \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell$. The critical level of confidence now exceeds the value $\widehat{\gamma}'$ from part *(i)* of the proof, as individual money holding yields a higher ex-ante Choquet expected utility as investing in the illiquid asset. Furthermore, the critical values is less than one, as in the absence of ambiguity depositing wealth in the deposit contract strictly dominates holding the individual wealth as money. After substituting $\mathcal{V}_n^*(\gamma) = \gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell$ it directly follows, that is $D^* = 1$, whenever

$$\gamma \cdot [(\beta_h - \alpha_2) \cdot \mu + \alpha_2] + (1 - \gamma) \cdot [\beta_\ell \cdot (\alpha_1 + (1 - \alpha_1) \cdot \mu)] - [\gamma \cdot \pi \cdot \beta_h + (1 - \gamma \cdot \pi) \cdot \beta_\ell] > 0.$$

As the left hand side of this expression in linear in γ, there exists a unique critical level of confidence $\widehat{\gamma}''$ such that $D^* = 1$, if $\gamma > \widehat{\gamma}'' > \widehat{\gamma}'$.

(iii) The critical value $\widehat{\gamma}$

From part *(i)* and part *(ii)* it follows that

$$\widehat{\gamma} := \begin{cases} \widehat{\gamma}' & \text{if} \quad \alpha_2 \geq \widehat{\gamma}'' \cdot \pi \cdot \beta_h + (1 - \widehat{\gamma}'' \cdot \pi) \cdot \beta_\ell) \\ \widehat{\gamma}'' & \text{if} \quad \alpha_2 < \widehat{\gamma}'' \cdot \pi \cdot \beta_h + (1 - \widehat{\gamma}'' \cdot \pi) \cdot \beta_\ell. \end{cases}$$

■